OPTIMIZATION OF WAVELENGTHS FOR QUADRI-SPECTRAL PYROMETER IN VISIBLE AND NEAR INFRARED RADIATION RANGE USED FOR HEAT TREATMENTS OF STEELS

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ABSTRACT

Optimum wavelengths for a pyrometer that can be used during the heat treatment of steels will be selected at the end of this article. Thanks to one of the physical properties of hot steels which has the possibility to radiate, it is possible to remotely measure its temperature on the surface. This temperature is proportional to the infrared or visible radiation emitted. The system is equipped with optical filters to control electromagnetic radiation and converge them to the appropriate detector. These optical filters are characterized according to the wavelengths used. The pyrometer we talked about will be a quadri-spectral pyrometer. Steel is one of the nonlinear emissivity metals, so the measurement of temperature requires a technic which is able to overcome this nonlinearity. A model called TNL.Tabc that means Temperature by Non Linear model with T, a, b, and c the parameters to estimate will be used to select the optimal wavelengths. It will focus on minimizing a cost function by the ordinary least squares method. With this model we will sequentially choose the optimal wavelengths one after the other by inverse method. This method consists in fixing the temperature and finding the wavelength corresponding to the temperature set. The first wavelength obtained will be used to calculate the second. And this principle will be applied to find the third as well as the fourth. Optimum wavelengths will be obtained from five (5) selected temperatures in the temperature range of heat treatment of steels. Each of those wavelengths must pass the test of the various criteria to minimize the errors of measurements on the temperature. The wavelength groups that will meet these criteria will be the optimum wavelengths for a pyrometer for the heat treatment of steels.

Keyword: *Quadri-spectral pyrometer, optimal wavelength, electromagnetic radiation, Temperature, and Heat treatment of Steel*

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1. INTRODUCTION

The temperature is the physical quantity very essential in the fields of productions. And most of the time, to know it, we use thermometers that are in direct contact with the object whose temperature is measured. But this technique is not applicable at all for moving objects, located in a hazardous area, for objects with poor thermal conductivity, deformable surface and especially for very high temperature. The metallurgical industries are the most affected by these problems, such as the heat treatment of metals. It is for this reason that the radiative property of materials is exploited so that its temperature can be measured remotely [1].

There are several techniques used for the realization of a pyrometer. One of these techniques is the quad spectral method that uses several wavelengths. A major problem in the design of such a multi-spectral pyrometer is the choice of wavelengths to be used because it is essential and keeps a very important role in the temperature calculation. In addition, metals are characterized by their nonlinear emissivity, low in the range of visible and near infrared waves. This non-linearity of the emissivity makes it difficult to measure the temperature and causes serious errors. The choice of wavelengths is very important in minimizing temperature errors and relative errors due to the spectral emissivity of metals.

In this article, we will try to find the optimal wavelengths used for a four-band multi spectral pyrometer in visible and neat infrared range for the heat treatment of steels. The goal is to have the four wavelengths whose error on the temperature of the fluxes obtained and the error relating to the spectral emissivity of the steels are minimal.

2. LAW OF ELECTROMAGNETIC RADIATION

2.1. Law of Planck

Let a black body at the temperature T, the energy density of the radiation of this body can be calculated. The calculations are based on the assumption that the electromagnetic field in the limited cavity of the black body is equivalent to a set of independent harmonic oscillators in thermodynamic equilibrium at temperature T and obeying the Boltzmann statistic. It is shown that the luminance $L_\lambda^0(T)$ of the black body is equal to the energy density of the radiation multiplied by $\frac{4\pi}{c}$, where the luminance is the ratio of the luminous intensity or energy density of the radiation to the emission surface [2].

$$L_\lambda^0(T) = \frac{2hc^2\lambda^{-5}}{exp\left(\frac{hc}{k\lambda T}\right)-1}$$

Where h= 6.6255x10^{-34}Js Planck constant, k = 1.38x10^{-23} JK^{-1} Boltzmann constant, c = 2.996x10^{8} ms^{-1} speed of electromagnetic waves in vacuum.

This formula is also used with the so-called Planck constants C_1 and C_2:

$$L_\lambda^0(T) = \frac{C_1\lambda_i^{-5}}{\exp\left(\frac{C_2}{\lambda_i T}\right)-1} \text{ with } C_1 = 2hc^2 \text{ and } C_2 = \frac{hc}{k}$$

2.2. Definition of spectral emissivity

The ratio between the monochromatic luminance of the real source $L_\lambda(T)$ and that of the black body $L_\lambda^0(T)$, for the same values of the wavelength λ and the temperature T, defines the monochromatic emissivity or spectral emissivity ε_λ of the source [3].

$$\varepsilon_\lambda = \frac{L_\lambda(T)}{L_\lambda^0(T)}$$

In the general case, ε_λ depends on the source, the wavelength λ and the temperature T and the direction of emission. Whereas the total emissivity is defined in the same way by the following relation:

$$\varepsilon_t = \int_0^\infty \varepsilon_\lambda d\lambda$$

The luminance of the black body does not depend on the direction of emission, and if it is the same for the real source (source radiating according to Lambert's law), the spectral emissivity does not depend either on the direction of emission.

2.3. Spectral emissivity of metals

The dependence on wavelength emissivity can be expressed in several forms, but we will consider those that can adjust experimental measurements or simplify the analysis. Most surfaces have emissivity that varies with wavelength and temperature. The emissivity of metal surfaces in the wavelengths of the visible and the near infrared often have a polynomial dependence on the wavelength [4].

$$\varepsilon_\lambda = c_0 + c_1\lambda + c_2\lambda^2 + \ldots + c_n\lambda^n$$

In our case, we use the polynomial model of order 2: $\varepsilon_\lambda = a + b\lambda + c\lambda^2$

But in theory, the emissivity depends on the material, the nature of its surface, the temperature, the wavelength and possibly the measurement configuration used. Since metals often reflect radiation, they are generally characterized by a low, non-linear emission level, which is highly dependent on the surface structure and tends towards long wavelengths. This dependence can lead to different and unreliable measurement results [4].

When choosing the appropriate thermal measuring devices, it should be ensured that the infrared radiation is measured with a certain wavelength and a temperature range for which the metals have a relatively high degree of emission.

3. MULTI-SPECTRAL METHOD BASED ON PLANCK LAW

3.1. Presentation of the TNL.Tabc model

The goal is to find the temperature of the steel during the heat treatment with its emissivity simultaneously. The model "TNL.Tabc" means Temperature by Non-Linear model with T, a, b and c, the parameters to be estimated. This model is unbiased and based on the estimation of flux expressed using Planck's law. It will also take into account the polynomial modeling of the emissivity up to the order 2 of the global spectral transfer function of the measurement chain using coefficients (a, b, c). The flux as a function of wavelength and temperature is $L_\lambda(T,a,b,c)$ [5].

$$L_{\lambda_i}(T,a,b,c) = \left(a + b\lambda_i + c\lambda_i^2\right)\frac{C_1\lambda_i^{-5}}{\exp\left(\dfrac{C_2}{\lambda_i T}\right) - 1}$$

With $\varepsilon_{\lambda_i} = a + b\lambda_i + c\lambda_i^2$ spectral emissivity.

The estimation of the parameters (T, a, b, c) will then be carried out by minimizing the function $J(T,a,b,c)$, in which $L_{\lambda_i}^{exp}$ denotes the spectral experimental flux measured at the wavelength λ_i, and $L_{\lambda_i}(T,a,b,c)$ is the theoretical spectral flux at the wavelength λ_i.

$$J(T,a,b,c) = \sum_{i=1}^{4}\left(L_{\lambda_i}^{exp} - L_{\lambda_i}(T,a,b,c)\right)^2$$

$$J(T,a,b,c) = \left(L_{\lambda_1}^{exp} - L_{\lambda_1}(T,a,b,c)\right)^2 + \ldots + \left(L_{\lambda_4}^{exp} - L_{\lambda_4}(T,a,b,c)\right)^2$$

Note: The index 4 designates the four (04) wavelengths for the estimation of the four parameters $\{T,a,b,c\}$. So we have four (04) equations for the theoretical flows $L_{\lambda_1}(T,a,b,c)$, $L_{\lambda_2}(T,a,b,c)$, $L_{\lambda_3}(T,a,b,c)$ et $L_{\lambda_4}(T,a,b,c)$.

3.2. Model with the method of sequential selection of wavelengths

The method used to estimate the temperature is based on the minimization of a cost function using ordinary least squares method. With this method we will define optimal wavelengths with the inverse method. This method consists of fixing the temperature and finding the wavelength corresponding to this temperature. These wavelengths minimize the standard deviation on the estimated temperature. The determination of the different optimal wavelengths will be carried out using the cost function associated with the model "TNL.Tabc" because this does not require the approximation of Wien, does not present any systematic bias in the presence of $J(T,a,b,c)$ additive noise to the flux and zero average [4][5][6]. The statistical properties of the parameter estimator associated with the TNL.Tabc model and the parameters provided by the least squares method are given by the Variance-Covariance matrix. The matrix from which one can determine the standard deviations σ_{β_i} of the different parameters, and in particular, that of the temperature σ_T. The TNL.Tabc model is a nonlinear model, we will then use the approximate expression of the Ordinary Least Squares variance-Covariance matrix, which is given for a parameter vector $\beta = (T,a,b,c)$, under assumptions of an additive noise, independent, identically distributed (variance σ^2_{noise} is constant, and zero mean), by:

$$\operatorname{cov}(\beta) = \begin{bmatrix} \sigma_T^2 & \operatorname{cov}(T,a) & \operatorname{cov}(T,b) & \operatorname{cov}(T,c) \\ \operatorname{cov}(T,a) & \sigma_a^2 & \operatorname{cov}(a,b) & \operatorname{cov}(a,c) \\ \operatorname{cov}(T,b) & \operatorname{cov}(a,b) & \sigma_b^2 & \operatorname{cov}(b,c) \\ \operatorname{cov}(T,c) & \operatorname{cov}(a,c) & \operatorname{cov}(b,c) & \sigma_c^2 \end{bmatrix} = \left(X^t X\right)^{-1} \sigma^2_{noise}$$

With *X* the sensitivity matrix associated with the variance-covariance matrix, defined by:

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial c} \end{bmatrix}$$

And the standard deviation on the temperature σ_T is given according to the standard deviation on the noise σ_{noise}, by:

$$\sigma_T = \sqrt{\left(X^t X\right)^{-1}}\sigma_{noise}$$

We will take as value the standard deviation of the noise, which we have experimentally with the infrared camera, and having for value $\sigma_{noise} \approx 8,97.10^4 Wm^{-2}$, That is to say 7.43 10^{-3} % of the maximum of Planck's law [5].

3.3. Pseudo-optimal method for the selection of wavelengths

The pseudo-optimal method consists in sequentially selecting the wavelengths while respecting all the different criteria. These wavelengths are those that minimize the standard deviations on the temperature at a fixed temperature finding the temperature range of the heat treatment of the steels. In our case, we will use a temperature set (T_S) for the calculation at 1073.15 °K, 1173.15 °K, 1223.15 °K, 1273.15 °K and 1373.15 °K.

- Selection of the first optimal wavelength

The method of sequential selection of "pseudo-optimal" wavelengths consists of choosing for the first wavelength filter λ_{OP1}, the one which minimizes the standard deviation σ_T on the temperature, assuming that the measurement is mono spectral. The cost function $J(\beta)$ consists of only one parameter: the temperature T.

$$J(T)=\left(L_{\lambda_1}^{exp}-L_{\lambda_1}(T,a,b,c)\right)^2$$

The temperature T is then the only parameter to estimate. The sensitivity matrix *X* is composed only of the first column and first row.

$$X=\left[\frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial T}\right]$$

Expression of the standard deviation of temperature:

$$\sigma_T=\sqrt{\frac{e^{-\frac{2C_2}{T\lambda_1}}\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^4 T^4\lambda_1^{12}}{C_1^2 C_2^2}}\sigma_{noise}$$

The minimization of the cost function $J(T)$, involves the sensitivity matrix *X* of the flux at the various parameters to be estimated. The first optimal wavelength λ_{OP1} will minimize this standard deviation.

- Selection of the second optimal wavelengths

The selection of the second filter is performed by setting a = 1, b = 1, and $\lambda_1=\lambda_{OP1}$. And looking for second wavelength, the shortest that minimizes the local standard deviation of temperature in the *TNL.Ta* model. The function cost $J(T,a)$ and the sensitivity matrix *X* are respectively composed as follows:

$$J(T,a)=\sum_{i=1}^{2}\left(L_{\lambda_i}^{exp}-L_{\lambda_i}(T,a,b,c)\right)^2=\left(L_{\lambda_1}^{exp}-L_{\lambda_1}(T,a,b,c)\right)^2+\left(L_{\lambda_2}^{exp}-L_{\lambda_2}(T,a,b,c)\right)^2$$

$$X=\begin{bmatrix}\frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial a}\\ \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial a}\end{bmatrix}$$

After the selection of the first raw and column of the matrix $\left(X^t X\right)^{-1}$, the expression of the standard deviation of temperature will be shown in the next relation.

$$\sigma_T = \sqrt{\frac{N}{-D_1 + D_2}}\sigma_{noise}$$

With

$$N = \frac{C_1^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^{10}} + \frac{C_1^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^{10}}$$

$$D_1 = \left(\frac{\mathrm{e}^{\frac{C_2}{T\lambda_1}} C_1^2 C_2 \left(a+\lambda_1\right)}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_1}}\right)^3 T^2 \lambda_1^{11}} + \frac{\mathrm{e}^{\frac{C_2}{T\lambda_2}} C_1^2 C_2 \left(a+\lambda_2\right)}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_2}}\right)^3 T^2 \lambda_2^{11}}\right)^2$$

$$D_2 = \left(\frac{C_1^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^{10}} + \frac{C_1^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^{10}}\right)\left(\frac{\mathrm{e}^{\frac{2C_2}{T\lambda_1}} C_1^2 C_2^2 \left(a+\lambda_1\right)^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_1}}\right)^4 T^4 \lambda_1^{12}} + \frac{\mathrm{e}^{\frac{2C_2}{T\lambda_2}} C_1^2 C_2^2 \left(a+\lambda_2\right)^2}{\left(-1+\mathrm{e}^{\frac{C_2}{T\lambda_2}}\right)^4 T^4 \lambda_2^{12}}\right)$$

- Selection of the third optimal wavelengths

For the third wavelength, it is obtained by minimizing the cost function $J\left(T,a,b\right)$ with the sensitivity matrix X associated with the model TNL.Tab by fixing $\lambda_1 = \lambda_{OP1}$ and $\lambda_2 = \lambda_{OP2}$.

$$J\left(T,a,b\right) = \sum_{i=1}^{3}\left(L_{\lambda_i}^{exp} - L_{\lambda_i}\left(T,a,b,c\right)\right)^2 = \left(L_{\lambda_1}^{exp} - L_{\lambda_1}\left(T,a,b,c\right)\right)^2 + \ldots + \left(L_{\lambda_3}^{exp} - L_{\lambda_3}\left(T,a,b,c\right)\right)^2$$

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}\left(T,a,b,c\right)}{\partial T} & \frac{\partial L_{\lambda_1}\left(T,a,b,c\right)}{\partial a} & \frac{\partial L_{\lambda_1}\left(T,a,b,c\right)}{\partial b} \\ \frac{\partial L_{\lambda_2}\left(T,a,b,c\right)}{\partial T} & \frac{\partial L_{\lambda_2}\left(T,a,b,c\right)}{\partial a} & \frac{\partial L_{\lambda_2}\left(T,a,b,c\right)}{\partial b} \\ \frac{\partial L_{\lambda_3}\left(T,a,b,c\right)}{\partial T} & \frac{\partial L_{\lambda_3}\left(T,a,b,c\right)}{\partial a} & \frac{\partial L_{\lambda_3}\left(T,a,b,c\right)}{\partial b} \end{bmatrix}$$

After selecting the first raw and column of the matrix $\left(X^t X\right)^{-1}$, the expression of the standard deviation of temperature will be shown in the next relation.

$$\sigma_T = \sqrt{\frac{-N_1 + N_2 N_3}{D_1\left(D_2 D_3 - D_4 D_1\right) + D_6\left(-D_7 + D_4 D_5\right) - D_2\left(-D_3 D_1 + D_2 D_5\right)}}\sigma_{noise}$$

With

$$N_1=\left(\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^9}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^9}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^9}\right)^2$$

$$N_2=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^{10}}$$

$$N_3=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^{8}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^{8}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^{8}}$$

$$D_1=\frac{e^{\frac{C_2}{T\lambda_1}}C_1^2C_2\left(a+b\lambda_1\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3T^2\lambda_1^{10}}+\frac{e^{\frac{C_2}{T\lambda_2}}C_1^2C_2\left(a+b\lambda_2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3T^2\lambda_2^{10}}+\frac{e^{\frac{C_2}{T\lambda_3}}C_1^2C_2\left(a+b\lambda_3\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3T^2\lambda_3^{10}}$$

$$D_2=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^{9}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^{9}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^{9}}$$

$$D_3=\frac{e^{\frac{C_2}{T\lambda_1}}C_1^2C_2\left(a+b\lambda_1\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3T^2\lambda_1^{11}}+\frac{e^{\frac{C_2}{T\lambda_2}}C_1^2C_2\left(a+b\lambda_2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3T^2\lambda_2^{11}}+\frac{e^{\frac{C_2}{T\lambda_3}}C_1^2C_2\left(a+b\lambda_3\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3T^2\lambda_3^{11}}$$

$$D_4=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^{10}}$$

$$D_5=\frac{e^{\frac{2C_2}{T\lambda_1}}C_1^2C_2^2\left(a+b\lambda_1\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^4T^4\lambda_1^{12}}+\frac{e^{\frac{2C_2}{T\lambda_2}}C_1^2C_2^2\left(a+b\lambda_2\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^4T^4\lambda_2^{12}}+\frac{e^{\frac{2C_2}{T\lambda_3}}C_1^2C_2^2\left(a+b\lambda_3\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^4T^4\lambda_3^{12}}$$

$$D_6 = \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^8} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^8} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2 \lambda_3^8}$$

$$D_7 = \left(\frac{e^{\frac{C_2}{T\lambda_1}} C_1^2 C_2 \left(a+b\lambda_1\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3 T^2 \lambda_1^{11}} + \frac{e^{\frac{C_2}{T\lambda_2}} C_1^2 C_2 \left(a+b\lambda_2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3 T^2 \lambda_2^{11}} + \frac{e^{\frac{C_2}{T\lambda_3}} C_1^2 C_2 \left(a+b\lambda_3\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3 T^2 \lambda_3^{11}} \right)^2$$

- Selection of the fourth and last optimal wavelengths

The fourth optimal wavelength will be obtained on the same principle as how to obtain the second and the third optimal wavelength by fixing a = 1, b = 1, c=1, $\lambda_1 = \lambda_{OP1}$, $\lambda_2 = \lambda_{OP2}$ and $\lambda_3 = \lambda_{OP3}$. The cost function $J\left(T,a,b,c\right)$ and the sensitivity matrix *X* associated with the model TNL.Tabc are respectively represented as follows:

$$J\left(T,a,b,c\right) = \sum_{i=1}^{4} \left(L_{\lambda_i}^{exp} - L_{\lambda_i}\left(T,a,b,c\right)\right)^2 = \left(L_{\lambda_1}^{exp} - L_{\lambda_1}\left(T,a,b,c\right)\right)^2 + \ldots + \left(L_{\lambda_4}^{exp} - L_{\lambda_4}\left(T,a,b,c\right)\right)^2$$

$$X = \begin{bmatrix} \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_1}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_2}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_3}(T,a,b,c)}{\partial c} \\ \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial T} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial a} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial b} & \frac{\partial L_{\lambda_4}(T,a,b,c)}{\partial c} \end{bmatrix}$$

After selecting the first raw and column of the matrix $\left(X^t X\right)^{-1}$, the expression of the standard deviation of temperature will be shown in the next relation.

$$\sigma_T = \sqrt{\frac{N}{D}} \sigma_{noise}$$

$$N = \begin{pmatrix} \left(-N_3^2 + \left(N_1\right)\left(N_2\right)\right)\left(N_3\right) - \\ \left(-\left(N_4\right)\left(N_3\right) + \left(N_5\right)\left(N_2\right)\right)\left(N_2\right) + \\ \left(-N_1^2 + \left(N_5\right)\left(N_3\right)\right)\left(N_6\right) \end{pmatrix}$$

$$D=\begin{pmatrix} -(D_1)\begin{pmatrix}(D_2)\left((D_3)(D_4)-(D_5)(D_6)\right)-(D_7)\left((D_7)(D_4)-(D_5)(D_1)\right)+\\ (D_3)\begin{pmatrix}(D_7)(D_6)\\ -(D_3)(D_1)\end{pmatrix}\end{pmatrix}+\\ (D_8)\begin{pmatrix}(D_6)\left((D_3)(D_4)-(D_5)(D_6)\right)+(D_7)\left(-D_4^2+(D_5)(D_9)\right)-\\ (D_3)\left(-(D_4)(D_6)+(D_3)(D_9)\right)\end{pmatrix}-\\ (D_2)\begin{pmatrix}(D_6)\left((D_7)(D_4)-(D_5)(D_1)\right)+(D_2)\left(-D_4^2+(D_5)(D_9)\right)\\ -(D_3)\left(-(D_4)(D_1)+(D_7)(D_9)\right)\end{pmatrix}+\\ (D_7)\begin{pmatrix}(D_6)\left((D_7)(D_6)-(D_3)(D_1)\right)+(D_2)\left(-(D_4)(D_6)+(D_3)(D_9)\right)\\ -(D_7)\left(-(D_4)(D_1)+(D_7)(D_9)\right)\end{pmatrix}\end{pmatrix}$$

Avec

$$N_1=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^9}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^9}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^9}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^9}$$

$$N_2=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^7}$$

$$N_3=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^8}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^8}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^8}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^8}$$

$$N_5=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^{10}}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^{10}}$$

$$N_7=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^6}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^6}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^6}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^6}$$

$$D_1=\frac{e^{\frac{C_2}{T\lambda_1}}C_1^2C_2\left(a+b\lambda_1+c\lambda_1^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3T^2\lambda_1^9}+\frac{e^{\frac{C_2}{T\lambda_2}}C_1^2C_2\left(a+b\lambda_2+c\lambda_2^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3T^2\lambda_2^9}+\frac{e^{\frac{C_2}{T\lambda_3}}C_1^2C_2\left(a+b\lambda_3+c\lambda_3^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3T^2\lambda_3^9}+\frac{e^{\frac{C_2}{T\lambda_4}}C_1^2C_2\left(a+b\lambda_4+c\lambda_4^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^3T^2\lambda_4^9}$$

$$D_2=\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2\lambda_1^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2\lambda_2^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2\lambda_3^7}+\frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2\lambda_4^7}$$

$$D_3 = \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^9} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^9} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2 \lambda_3^9} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2 \lambda_4^9}$$

$$D_4 = \frac{e^{\frac{C_2}{T\lambda_1}} C_1^2 C_2 \left(a+b\lambda_1+c\lambda_1^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3 T^2 \lambda_1^{11}} + \frac{e^{\frac{C_2}{T\lambda_2}} C_1^2 C_2 \left(a+b\lambda_2+c\lambda_2^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3 T^2 \lambda_2^{11}} + \frac{e^{\frac{C_2}{T\lambda_3}} C_1^2 C_2 \left(a+b\lambda_3+c\lambda_3^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3 T^2 \lambda_3^{11}} + \frac{e^{\frac{C_2}{T\lambda_4}} C_1^2 C_2 \left(a+b\lambda_4+c\lambda_4^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^3 T^2 \lambda_4^{11}}$$

$$D_5 = \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^{10}} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^{10}} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2 \lambda_3^{10}} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2 \lambda_4^{10}}$$

$$D_6 = \frac{e^{\frac{C_2}{T\lambda_1}} C_1^2 C_2 \left(a+b\lambda_1+c\lambda_1^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^3 T^2 \lambda_1^{10}} + \frac{e^{\frac{C_2}{T\lambda_2}} C_1^2 C_2 \left(a+b\lambda_2+c\lambda_2^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^3 T^2 \lambda_2^{10}} + \frac{e^{\frac{C_2}{T\lambda_3}} C_1^2 C_2 \left(a+b\lambda_3+c\lambda_3^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^3 T^2 \lambda_3^{10}} + \frac{e^{\frac{C_2}{T\lambda_4}} C_1^2 C_2 \left(a+b\lambda_4+c\lambda_4^2\right)}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^3 T^2 \lambda_4^{10}}$$

$$D_7 = \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^8} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^8} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2 \lambda_3^8} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2 \lambda_4^8}$$

$$D_8 = \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^2 \lambda_1^6} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^2 \lambda_2^6} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^2 \lambda_3^6} + \frac{C_1^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^2 \lambda_4^6}$$

$$D_9 = \frac{e^{\frac{2C_2}{T\lambda_1}} C_1^2 C_2^2 \left(a+b\lambda_1+c\lambda_1^2\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_1}}\right)^4 T^4 \lambda_1^{12}} + \frac{e^{\frac{2C_2}{T\lambda_2}} C_1^2 C_2^2 \left(a+b\lambda_2+c\lambda_2^2\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_2}}\right)^4 T^4 \lambda_2^{12}} + \frac{e^{\frac{2C_2}{T\lambda_3}} C_1^2 C_2^2 \left(a+b\lambda_3+c\lambda_3^2\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_3}}\right)^4 T^4 \lambda_3^{12}} + \frac{e^{\frac{2C_2}{T\lambda_4}} C_1^2 C_2^2 \left(a+b\lambda_4+c\lambda_4^2\right)^2}{\left(-1+e^{\frac{C_2}{T\lambda_4}}\right)^4 T^4 \lambda_4^{12}}$$

4. CRITERIA FOR THE SELECTION OF OBTAINED OPTIMUM WAVELENGTHS

Our pyrometer must be very sensitive to the temperature between 975.15 °K and 1473.15 °K. This range borders the temperature range of the heat treatment of steels which is between 1000.15 °K and 1421.15 °K [8]. So that we can have better optimal wavelengths, we will try to find optimal wavelengths from the temperature set (T_S) at 1073.15 °K, 1173.15 °K, 1223.15 °K, 1273.15 °K and 1373.15 °K.

4.1. Criteria on the spectral range of the pyrometer

Our first criterion for the selection of optimal wavelengths is the spectral range of our pyrometer which operates in the band between 0.4 µm to 3 µm. The emissivity of the steels is very low from the spectrum of length 2 µm. But to have many choices on the wavelengths obtained, we will use the spectral band of 0.4 µm to 2.5 µm (Table-1). Measuring the temperature of a metal requires the use of short wavelength to avoid the relative error due to emissivity. The multi spectral measurement minimizes the error so the choice of short wavelength gives a better precision on the temperature. It is observed that each time a wavelength is added, the standard deviation deteriorates.

Table-1: Optimum wave lengths preselected according to the spectral range of the pyrometer

	CHANNEL 1		CHANNEL 2		CHANNEL 3		CHANNEL 4	
T_S[°K]	λ_{OP1} [µm]	σ_T [°K]	λ_{OP2} [µm]	σ_T [°K]	λ_{OP3} [µm]	σ_T [K]	λ_{OP4} [µm]	σ_T [°K]
1073.15	2.246	0.000959	1.535	0.005373	1.186	0.043606	0.967	0.461628
							1.392	0.471311
							2.000	0.132117
					1.945	0.038898	1.099	0.128315
							1.719	0.484844
							2.122	0.659303
1173.15	2.054	0.000671	1.403	0.003772	1.085	0.030674	0.884	0.323962
							1.273	0.333100
							1.896	0.090029
					1.778	0.027332	1.005	0.090259
							1.572	0.340205
							1.939	0.465602
1223.15	1.970	0.000568	1.346	0.003185	1.041	0.025831	0.848	0.272897
							1.222	0.279541
							1.821	0.075742
					1.707	0.023056	0.965	0.075973
							1.509	0.287155
							1.857	0.392977
1273.15	1.893	0.000684	1.294	0.002712	1	0.022004	0.815	0.232649
							1.174	0.238138
							1.749	0.064520
					1.640	0.019644	0.900	0.065571
							1.449	0.244430
							1.788	0.334194
1373.15	1.755	0.000357	1.199	0.002005	0.927	0.016302	0.756	0.172321
							1.088	0.176781
							1.622	0.047746
					1.520	0.014494	0.855	0.047922
							1.343	0.180200
							1.658	0.245986

4.2. Criteria on the minimum deviation of the two successive wavelengths

To avoid amplifying the measurement error, while remaining as close as possible in order to minimize the measurement error due to the spectral variation of the emissivity, the minimum difference of the two successive wavelengths $\Delta_{M\,in}\lambda$ must be respected.

$$\Delta_{M\,in}\lambda_{ji} = \left|\lambda_j - \lambda_i\right| \succ \left.\frac{T\lambda_j^2}{C_2}\right|_{\lambda_j \succ \lambda_i}$$

The minimum difference between the first and the second wavelength will therefore be $\lambda_{OP1} - \lambda_{OP2} \geq \Delta_{M\,in}\lambda_{1-2}$. So the second maximum wavelength will be selected according to this relation $\lambda_{OP2Max} \leq \lambda_{OP1} - \Delta_{M\,in}\lambda_{1-2}$. The difference between the second and the third wavelength will be $\lambda_{OP2} - \lambda_{OP3} \geq \Delta_{M\,in}\lambda_{2-3}$. The maximum value of the third wavelength is then $\lambda_{OP3Max} \leq \lambda_{OP2} - \Delta_{M\,in}\lambda_{2-3}$. Same principle for the last and fourth optimal wavelength, $\lambda_{OP3} - \lambda_{OP4} \geq \Delta_{M\,in}\lambda_{3-4}$ then $\lambda_{OP4Max} \leq \lambda_{OP3} - \Delta_{M\,in}\lambda_{3-4}$. The four selected optimal wavelengths respecting the criterion of the minimum standard deviation on the temperature and the minimum difference between two successive wavelengths, for a temperature of 1073.15 °K, 1173.15 °K, 1223.15 °K, 1273.15 °K and 1373.15 °K will be represented in table-2.

In the infrared, the standard deviation can not be exceeded by 5%. The highest values were achieved for the shortest wavelength and wavelengths disturbed by atmospheric absorption. In the other infrared spectral ranges, the standard deviation between the first and the last spectrum was less than 2% [4].

Table-2: Optimum wavelengths obtained from 1073.15 °K, 1173.15 °K, 1223.15 °K, 1273.15 °K and 1373.15 °K according to the criterion of minimum deviation of the two successive wavelengths

T_S[°K]	λ_{OP} [µm]	σ_T [°K]	σ_T [%]	$\Delta\lambda$ [µm]	$\Delta_{\min}\lambda$ [µm]
1073.15	λ_{OP1} = 2.246	0.000959	0.000089	λ_{OP1} - λ_{OP2} = 0.711 λ_{OP2} - λ_{OP3} = 0.349 λ_{OP3} - λ_{OP4} = 0.219	λ_{OP1} - λ_{OP2} = 0.411 λ_{OP2} - λ_{OP3} = 0.192 λ_{OP3} - λ_{OP4} = 0.114
	λ_{OP2} = 1.535	0.005373	0.000501		
	λ_{OP3} = 1.186	0.043606	0.004063		
	λ_{OP4} = 0.967	0.461628	0.043016		
1173.15	λ_{OP1} = 2.054	0.000671	0.000057	λ_{OP1} - λ_{OP2} = 0.651 λ_{OP2} - λ_{OP3} = 0.318 λ_{OP3} - λ_{OP4} = 0.201	λ_{OP1} - λ_{OP2} = 0.344 λ_{OP2} - λ_{OP3} = 0.160 λ_{OP3} - λ_{OP4} = 0.095
	λ_{OP2} = 1.403	0.003772	0.000321		
	λ_{OP3} = 1.085	0.030674	0.002614		
	λ_{OP4} = 0.884	0.323962	0.027614		
1223.15	λ_{OP1} = 1.970	0.000568	0.000046	λ_{OP1} - λ_{OP2} = 0.624 λ_{OP2} - λ_{OP3} = 0.305 λ_{OP3} - λ_{OP4} = 0.193	λ_{OP1} - λ_{OP2} = 0.329 λ_{OP2} - λ_{OP3} = 0.154 λ_{OP3} - λ_{OP4} = 0.092
	λ_{OP2} = 1.346	0.003185	0.000260		
	λ_{OP3} = 1.041	0.025831	0.002112		
	λ_{OP4} = 0.848	0.272897	0.022311		
1273.15	λ_{OP1} = 1.893	0.000684	0.000054	λ_{OP1} - λ_{OP2} = 0.599 λ_{OP2} - λ_{OP3} = 0.294 λ_{OP3} - λ_{OP4} = 0.185	λ_{OP1} - λ_{OP2} = 0.317 λ_{OP2} - λ_{OP3} = 0.148 λ_{OP3} - λ_{OP4} = 0.088
	λ_{OP2} = 1.294	0.002712	0.000213		
	λ_{OP3} = 1.000	0.022004	0.001728		
	λ_{OP4} = 0.815	0.232649	0.018273		
1373.15	λ_{OP1} = 1.755	0.000357	0.000026	λ_{OP1} - λ_{OP2} = 0.556 λ_{OP2} - λ_{OP3} = 0.272 λ_{OP3} - λ_{OP4} = 0.171	λ_{OP1} - λ_{OP2} = 0.293 λ_{OP2} - λ_{OP3} = 0.119 λ_{OP3} - λ_{OP4} = 0.082
	λ_{OP2} = 1.199	0.002005	0.000146		
	λ_{OP3} = 0.927	0.016302	0.001187		
	λ_{OP4} = 0.756	0.172321	0.012549		

Observation of Table-2:

- First, it is observed that only one group of optimal wavelength which is obtained from each temperature set respects the criterion of minimum distance between two successive optimal wavelengths.
- Second, in addition, the minimum distance required between two wavelengths is proportionally with the largest wavelength among the two successive ones.
- Third, we also note that all groups of optimal wavelengths that have the lowest standard deviation do not meet the criterion of minimum distance between two successive wavelengths.

4.3. Standard deviation on the temperature at the temperature range of the fluxes obtained from the optimal wavelengths

Verification of the standard deviation of the optimal wavelengths is almost necessary to know the errors on the temperature throughout the temperature range from 975.15 °K to 1473.15 °K of the heat treatment of the steels (Table-3).

Observation of Table-3:

- First, we have four (04) optimal lengths that respect the minimum difference between two (2) successive wavelengths for a temperature set. More the temperatures sets increases to 1373.15 °K, the wavelengths decrease to at least 0.140 µm.
- Second, we also note that the standard deviation on temperature improves as well as the temperature to be measured increases.
- Third, more the wavelength decreases, the standard deviation deteriorates, that is to say increases (T_S = 1223.15 °K: λ_{OP1} = 1.970 µm, λ_{OP2} = 1.346 µm, λ_{OP3} = 1.041 µm, λ_{OP4} = 0.848 µm If we apply these wavelengths at T = 1073.15 °K, we have respectively a standard deviation of temperature: 0.001011 °K, 0.002417 °K, 0.001023 °K, 0.052467 °K).

- Fourth, the standard deviation deteriorates rapidly if the wavelength exceeds the lower limit of the near-infrared range (λ_{OP4} = 0.756 μm calculated from the temperature set at T_S = 1373.15 °K, the standard deviations on the temperature of this wavelength for the temperature range of our pyrometer are, at T = 975.15 °K gives σ_T = 0.917752 °K and at T = 1473.15 °K gives σ_T = 0.002754 °K).
- Fifth, the temperature range of our pyrometer is 975.15 °K up to 1473.15 °K, we see that the optimal lengths obtained at T_S = 1073.15 °K have a better standard deviation compared to those obtained at T_S = 1373.15 °K. The worst standard deviation is 0.056348 °K at T = 975.15 °K for T_S = 1073.15 °K, against 0.917752 °K at T = 975.15 °K for T_S = 1373.15 °K.

Table-3: Standard deviation on the temperature σ_T [°K] at the spectral range of the pyrometer of the optimal wavelengths obtained

T_S [°K]	**λ_{OP} [μm]**	**Pyrometer temperature range [°K]**						
		975.15	**1073.15**	**1173.15**	**1223.15**	**1273.15**	**1373.15**	**1473.15**
1073.15	λ_{OP1} = 2.246	0.001459	0.000959	0.000686	0.000595	0.000524	0.000420	0.000349
	λ_{OP2} = 1.535	0.003146	0.001559	0.000884	0.000693	0.000555	0.000378	0.000273
	λ_{OP3} = 1.186	0.011394	0.004336	0.001977	0.001408	0.001033	0.000600	0.000379
	λ_{OP4} = 0.967	0.056348	0.016485	0.006042	0.003911	0.002627	0.001305	0.000719
1173.15	λ_{OP1} = 2.054	0.001582	0.000982	0.000671	0.000571	0.000493	0.000382	0.000310
	λ_{OP2} = 1.403	0.004541	0.002068	0.001094	0.000832	0.000648	0.000419	0.000290
	λ_{OP3} = 1.085	0.021317	0.007281	0.003035	0.002078	0.001471	0.000801	0.000478
	λ_{OP4} = 0.884	0.138194	0.035367	0.011602	0.007152	0.004595	0.002106	0.001084
1223.15	λ_{OP1} = 1.970	0.001674	0.001011	0.000675	0.000568	0.000486	0.000371	0.000296
	λ_{OP2} = 1.346	0.005532	0.002417	0.001235	0.000925	0.000711	0.000448	0.000304
	λ_{OP3} = 1.041	0.029580	0.009576	0.003817	0.002564	0.001782	0.000940	0.000546
	λ_{OP4} = 0.848	0.219035	0.052467	0.016292	0.009805	0.006161	0.002715	0.001351
1273.15	λ_{OP1} = 1.893	0.001789	0.001050	0.000685	0.000571	0.000484	0.000343	0.000286
	λ_{OP2} = 1.294	0.006791	0.002847	0.001406	0.001038	0.000786	0.000484	0.000321
	λ_{OP3} = 1.000	0.041610	0.012759	0.004862	0.003202	0.002185	0.001116	0.000631
	λ_{OP4} = 0.815	0.349686	0.078430	0.023061	0.013551	0.008329	0.003529	0.001697
1373.15	λ_{OP1} = 1.755	0.002101	0.001164	0.000725	0.000592	0.000492	0.000357	0.000274
	λ_{OP2} = 1.199	0.010626	0.004095	0.001886	0.001350	0.000995	0.000582	0.000370
	λ_{OP3} = 0.927	0.084590	0.023272	0.008105	0.005130	0.003377	0.001616	0.000863
	λ_{OP4} = 0.756	0.917752	0.180401	0.047544	0.026628	0.015659	0.006132	0.002754

4.4. Sensitivity of the flux at temperature and wavelength

The model called *TNL.Tabc* consists in making temperature measurements without mastering all the influencing factors. However, it is necessary to take certain precautions to minimize the measurement error on the temperature. However, our field of work is on the increasing part of the Planck curve because the reduced sensitivities of the flux at the temperature χ_T and at the wavelength χ_λ are all the better that we work at short wavelengths. The wavelengths obtained should give better sensitivity to temperature (Table-4) and wavelength (Table-5).

$$\chi_T = \frac{1}{L_\lambda(T)}\frac{dL_\lambda(T)}{dT} \quad \text{and} \quad \chi_\lambda = \frac{1}{L_\lambda(T)}\frac{dL_\lambda(T)}{d\lambda}$$

Observations of table-4:

- First, the sensitivity of the flux to the temperature increases as the wavelength decreases.
- Second, in the spectral band of our pyrometer, the sensitivity of the flux to the temperature applied to a wavelength decreases if the temperature increases.
- Third, in the spectral band between 0.4 μm and 2.5 μm, the sensitivity of the flux to the temperature is more and better at 975.15 °K than at 1473.15 °K upper limit of our temperature to be measured.

Table-4: Sensitivity of the flux obtained from the optimal wavelengths at the temperature

T_S [°K]	λ_{OP} [µm]	Pyrometer temperature range [°K]						
		975.15	**1073.15**	**1173.15**	**1223.15**	**1273.15**	**1373.15**	**1473.15**
1073.15	λ_{OP1}=2.246	0.006773	0.005576	0.004674	0.004304	0.003978	0.003429	0.002990
	λ_{OP2}=1.535	0.009898	0.008140	0.006813	0.006268	0.005786	0.004976	0.004326
	λ_{OP3}=1.186	0.012810	0.010534	0.008815	0.008109	0.007485	0.006435	0.005591
	λ_{OP4}=0.967	0.015711	0.012919	0.010811	0.009945	0.009179	0.007891	0.006856
1173.15	λ_{OP1}=2.054	0.007402	0.006091	0.005102	0.004697	0.004339	0.003737	0.003255
	λ_{OP2}=1.403	0.010829	0.008905	0.007452	0.006856	0.006328	0.005442	0.004729
	λ_{OP3}=1.085	0.014002	0.115147	0.009635	0.008863	0.008181	0.007033	0.006111
	λ_{OP4}=0.884	0.017186	0.014132	0.011826	0.010879	0.010041	0.008632	0.007500
1223.15	λ_{OP1}=1.970	0.007716	0.006348	0.005317	0.004894	0.004520	0.003892	0.003389
	λ_{OP2}=1.346	0.011287	0.009282	0.007767	0.007146	0.006596	0.005671	0.004929
	λ_{OP3}=1.041	0.014594	0.012001	0.010042	0.009238	0.008527	0.007330	0.006369
	λ_{OP4}=0.848	0.017916	0.014732	0.012328	0.011341	0.010467	0.008998	0.007818
1273.15	λ_{OP1}=1.893	0.008029	0.006605	0.005531	0.005090	0.004701	0.004047	0.003522
	λ_{OP2}=1.294	0.011741	0.009655	0.008079	0.007432	0.006861	0.005898	0.005126
	λ_{OP3}=1.000	0.015193	0.012493	0.010454	0.009617	0.008876	0.007631	0.006630
	λ_{OP4}=0.815	0.018641	0.015329	0.012827	0.011800	0.010891	0.009363	0.008135
1373.15	λ_{OP1}=1.755	0.008658	0.007122	0.005962	0.005486	0.005066	0.004359	0.003792
	λ_{OP2}=1.199	0.012671	0.010420	0.008719	0.008021	0.007403	0.006365	0.005531
	λ_{OP3}=0.927	0.016389	0.013477	0.011277	0.010374	0.009575	0.008231	0.007152
	λ_{OP4}=0.756	0.020096	0.016525	0.013828	0.012721	0.011741	0.010093	0.008769

Table-5: Flux sensitivity at the wavelength in the temperature range of the pyrometer

T_S [°K]	λ_{OP} [µm]	Pyrometer temperature range [°K]						
		975.15	**1073.15**	**1173.15**	**1223.15**	**1273.15**	**1373.15**	**1473.15**
1073.15	λ_{OP1}=2.246	708783	438419	215437	118132	28815.5	-129307	-264699
	λ_{OP2}=1.535	3017940	2433730	1949550	1737360	1541990	1194490	894948
	λ_{OP3}=1.186	6295360	5315970	4503650	4147350	3819100	3234490	2729580
	λ_{OP4}=0.967	10640700	9167330	7945180	7409060	6915060	6035040	5274610
1173.15	λ_{OP1}=2.054	1072800	748273	480169	363010	255370	64539.6	-99164.8
	λ_{OP2}=1.403	3947540	3247910	2667830	2413500	2179270	1762380	1402700
	λ_{OP3}=1.085	7950900	6780620	5809910	5384100	4991770	4292950	3689210
	λ_{OP4}=0.884	13263700	11500700	10038200	9396680	8805530	7752420	6842330
1223.15	λ_{OP1}=1.970	1273700	920437	628392	500698	383333	175134	-3618.14
	λ_{OP2}=1.346	4446190	3685950	3055540	2779110	2524490	2071220	1680020
	λ_{OP3}=1.041	8840230	7568920	6514380	6051800	5624470	4866320	4210320
	λ_{OP4}=0.848	14664100	12748200	11158900	10461700	9819330	8674880	7685850
1273.15	λ_{OP1}=1.893	1486270	1103270	786471	647884	520461	294298	99976.3
	λ_{OP2}=1.294	4965930	4143300	3461080	3161900	2886300	2395610	1971990
	λ_{OP3}=1.000	9784980	8407280	7264470	6763160	6301240	5478390	4767390
	λ_{OP4}=0.815	16124000	14049800	12329300	11574500	10879000	9639990	8569220
1373.15	λ_{OP1}=1.755	1952330	1506060	1136600	974840	826027	561657	334207
	λ_{OP2}=1.199	6114380	5156120	4361330	4012740	3691580	3119630	2625670
	λ_{OP3}=0.927	11811500	10208300	8878370	8294970	7757400	6799760	5972200
	λ_{OP4}=0.756	19255100	16844500	14844900	13967800	13159500	11719500	10475000

Observations of table-5:

- First, the sensitivity of the flux to the wavelength increases when the wavelength decreases.
- Second, the sensitivity of the flux at the wavelength of the wavelengths obtained from the higher temperature is more and better than that obtained at the lower limit in the temperature range to be measured.
- Third, the existence of negative values justifies that the use of these wavelengths obtained at T_S between 1073.15 °K and 1223.15 °K can give serious errors if the temperature to be measured is greater than 1373.15 °K. Therefore these wavelengths do not cover, in terms of sensitivity on flow, the temperatures necessary for the heat treatment of steels.

4.5. Synthesis of the criteria result

In our case, according to the pyrometer spectral range, the high irradiation range of steels in visible and near infrared region, and all the criteria to select optimal wavelengths for heat treatment of steels, two groups of four wavelengths have been selected.

The first group is obtained from T_S =1273.15 °K. Those wavelengths are λ_{OP1}=1.893 µm, λ_{OP2}=1.294 µm, λ_{OP3}=1.000 µm, λ_{OP4}=0.815 µm. They found in the near infrared region. So those optimal wavelengths do not recover totally our spectral range which is in the visible and near infrared.

The second group of optimal wavelengths was obtained by using T_S =1373.15 °K. They are λ_{OP1}=1.755 µm, λ_{OP2}=1.199 µm, λ_{OP3}=0.927 µm, and λ_{OP4}=0.756 µm. In comparison of the first group, they have better standard deviation and cover in the visible and near infrared range that steels have best irradiation with emissivity is higher than 0.4.

5. Graphical verification of the four optimal wavelengths in the visible and near infrared range

5.1. First optimal wavelength λ_{OP1}=1.755 µm

A single wavelength is obtained for the first selection, it is λ_{OP1} = 1.755 µm. It minimizes the standard deviation on the temperature or the error on the temperature. It is in the spectral range of our detector. It is far from the area where the noise equivalent power and the sensitivity of the flux to the temperature are low. This wavelength fully respects all the selection criteria (Chart-1).

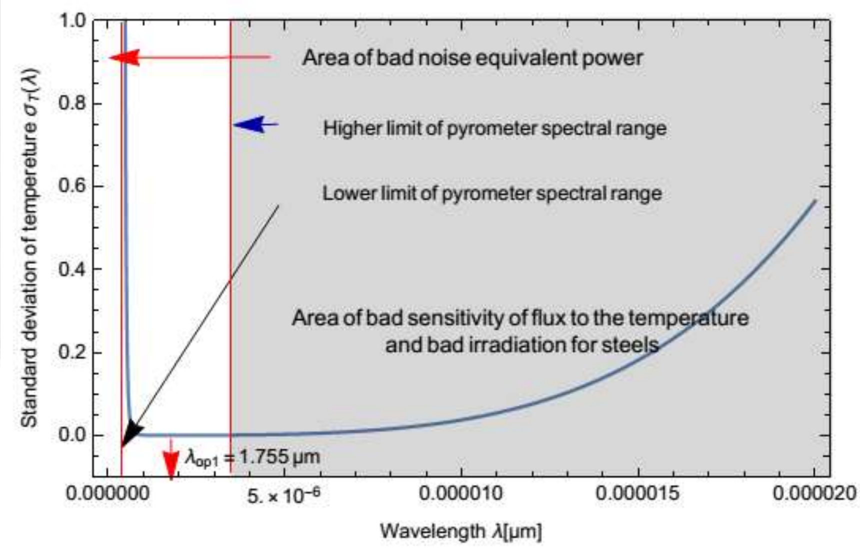

Chart-1: First optimal wavelength minimizing standard deviation of temperature

5.2. Second optimal wavelength λ_{OP2}=1.199 µm

Two (2) optimal wavelengths are available for the second filter. They minimize the temperature error. One is λ_{OP21} = 1.199 µm which has a standard deviation on the temperature of 0.002005 ° K and the other is λ_{OP22} = 3.281 µm which gives a standard deviation on the temperature of 0.001365 ° K. The optimum wavelength λ_{OP22} greatly exceeds the spectral band of our detector. It is also in the area where the sensitivity of the flux of the temperature is low. We know that the emission of steels is very low for spectra longer than 2 µm. However, the optimal wavelength λ_{OP21} = 1.199 µm lies in the spectral range of our detector which is from 0.4 µm to 2.5 µm. It is located neither in the area where the sensitivity of the flux to the temperature is weak, nor in the area of low noise equivalent power. The second optimal wavelength will therefore be λ_{OP2} = 1.199 µm with (Chart-2).

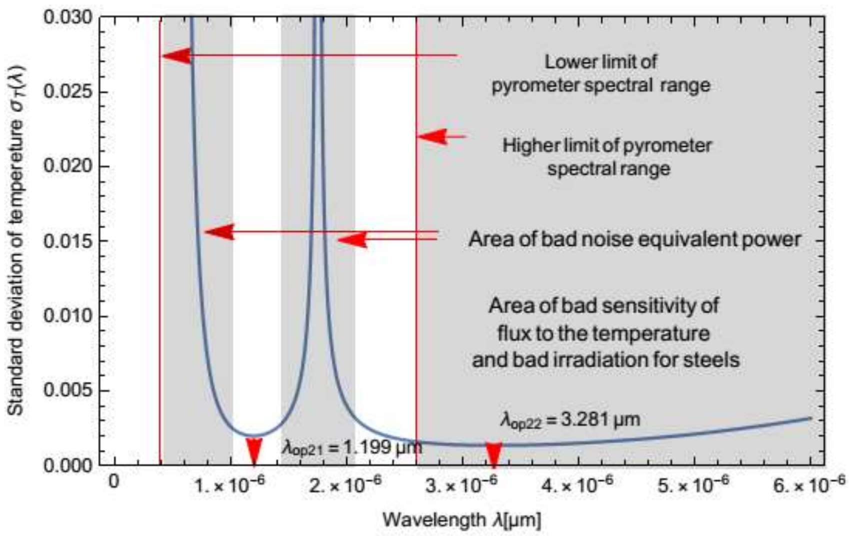

Chart-2: Second optimal wavelength minimizing standard deviation of temperature

5.3. Fird optimal wavelength λ_{OP3}=0.927 µm

In the selection of the third wavelength, three wavelengths were selected and minimized the standard deviation on the temperature. Two wavelengths λ_{OP31} = 0.927 µm, λ_{OP32} = 1.521 µm are in the spectral range of our detector which are between 0.4 µm and 2.5 µm. These two wavelengths are neither the zone where noise equivalent power is low, nor the area of low sensitivity of the flux to temperature. Their temperature errors do not exceed 5%. The third length λ_{OP33} = 3.281 µm far exceeds the upper limit of the spectral range of the pyrometer. It is in the part of the spectrum where the steels emit weakly so it can give an unreliable measurement for the temperature. By calculating the minimum difference between λ_{OP2} = 1.199 µm and λ_{OP3}, only λ_{OP31} = 0.927 µm which may be the third optimal wavelength for the third filter because it respects the minimum distance required by the second optimal wavelength. This wavelength has a standard deviation on the temperature of 0.016302 ° K (Chart-3).

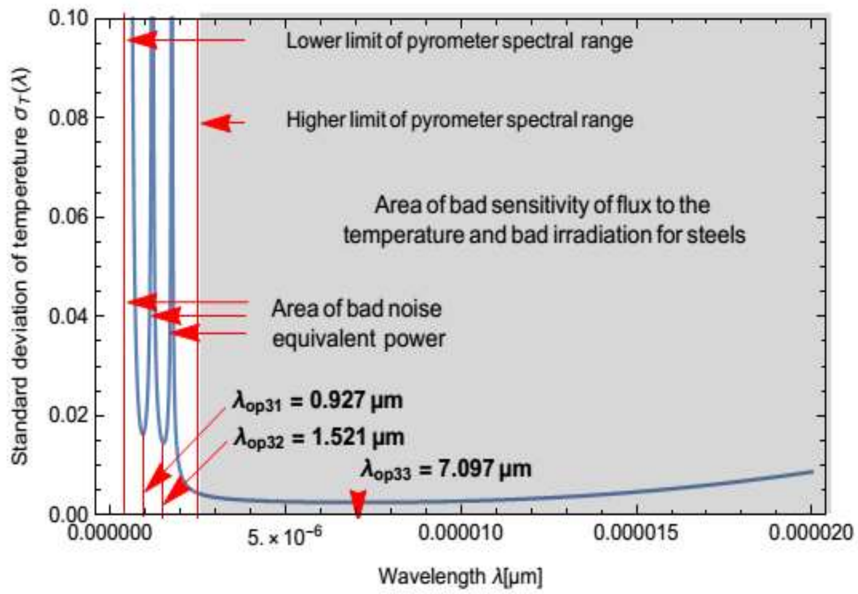

Chart-3: Third optimal wavelength minimizing standard deviation of temperature

5.4. Fourth optimal wavelength λ_{OP3}=0.756 µm

Four optimal wavelengths were obtained for the selection of the fourth length. Three λ_{OP41} = 0.756 µm, λ_{OP42} = 1.088 µm, λ_{OP43} = 1.622 µm are in the visible and near-infrared spectra area. They are also in the spectral range of our detector. These three wavelengths are separated by areas where the signal-to-noise ratio is low. Their standard deviations are all less than 5%. The wavelength λ_{OP44} = 47.890 µm which is largely far from the spectral range of our pyrometer. This wavelength is obviously not optimal for our case. Steels have a very low spectral emission in the far infrared region. The minimum difference required by the third optimum wavelength λ_{OP3} = 0.927 µm is 0.171 µm. So only λ_{OP41} = 0.756 µm which could be the fourth optimal wavelength. It has an error on the temperature of 0.172321 ° K.

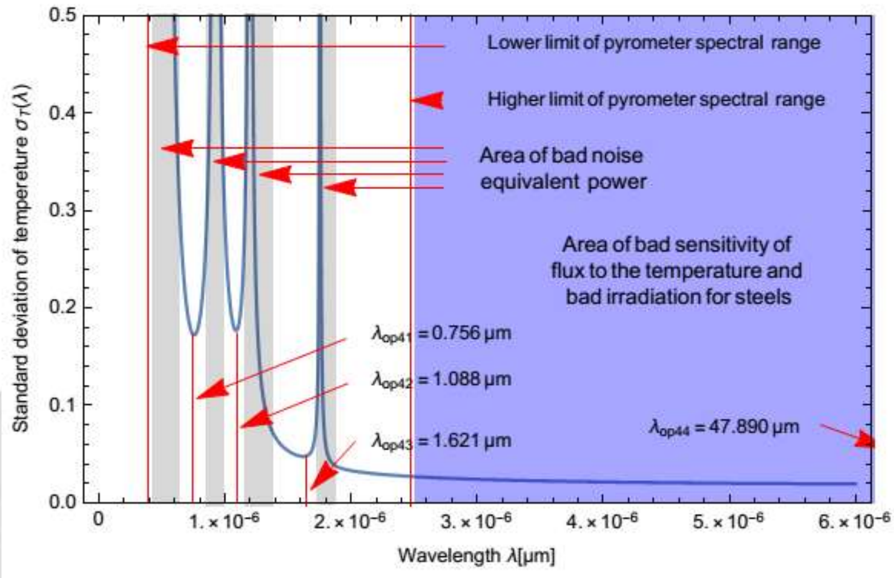

Chart-4: Forth optimal wavelength minimizing standard deviation of temperature

6. CONCLUSIONS

The TNL.Tabc model based on Planck's law gives the possibility of finding temperature and spectral emissivity at the same time. This model allowed us to sequentially select optimal wavelengths for a multi spectral pyrometer for the heat treatment of steels.

Among the optimal wavelengths obtained from 1073.15 °K, 1173.15 °K, 1223.15 °K, 1273.15 °K and 1373.15 °K, only a group of four wavelengths respects the criterion of minimum distance between two successive wavelengths. The choice of these wavelengths starting from the minimum standard deviation is then an insufficient criterion.

All optimal wavelengths have good temperature sensitivity between 975.15 °K and 1473.15 °K. But the fluxes at these optimal wavelengths are not at all sensitive if the temperature is between 1373.15 °K and 1473.15 °K. In addition, their standard deviation is very far from exceeding the 5% error limit.

To conclude, only the optimal wavelengths obtained from the temperature set at 1273.15 °K meet all the criteria for selecting the four optimal wavelengths for a quadri-spectral pyrometer in the visible and near infrared range for the heat treatment of steels. Therefore, with the method of sequential selection of the optimal wavelengths of the model *TNL.Tabc* by the ordinary least square method, it is better to use the temperature towards the upper limit of the temperature range (975.15 °K and 1473.15 °K) to be measured than the temperature towards the lower limit. And the closer you get to the higher limit, the longer the wavelengths reach the visible part. In the visible and near infrared range, just one group of four optimal wavelength meet all criteria. Those wavelengths are λ_{OP1}=1.755 µm, λ_{OP2}=1.199 µm, λ_{OP3}=0.927 µm, and λ_{OP4}=0.756 µm.

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