PARAFAC-BASED MIMO RADAR MODELING WITH SPACE-TIME-CODING DIVERSITY FOR PARAMETER ESTIMATION

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ABSTRACT

In this paper we propose tensor-based methods for parameter estimations in bistatic MIMO radar. We first present the basic MIMO radar system parameters including the array steering matrix at the reception and the transmission, the fading coefficients matrix, the symbol matrix and the delay propagation matrix and second the useful PARAFAC models. Different approaches of modelisation are detailed considering instant or convolutive channel processing with raw and statistical data. Our proposed algorithm is based on the knowledge of a coding matrix using Khatri-Rao products at the transmission. Using nested-PARAFAC model, two stages of alternating least squares algorithm allow us to estimate with accuracy all of the transmission parameters: DoA, DoD, fading coefficients and transmitted symbols. Several criteria such as RMSE, PSD, NMSE and SER are tested in the simulation section, where we can see that tensor-based methods provide efficiency due to the fact that they can take into account different kinds of diversity simultaneously.

Keywords : *Nested-PARAFAC, Khatri-Rao product, MIMO radar, ALS, estimation.*

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1. INTRODUCTION

Parameter estimation in wireless communication systems has caught great attention recently. Estimating the transmitted information with the right parameter of transmission is crucial in fields of radar. In [1],[2] and [3]-[5], classical methods based on eigenspace algorithms like multiple signal classification (MUSIC) and signal parameters via rotational invariance techniques (ESPRIT) are used to estimate the DoD (direction of departure) and DoA (direction of arrival). In [6], the pionner work of source separation with PARAFAC models [7] has been proposed. Then appeared several papers on channel estimation [8]-[11], spatial signature estimation [12]-[16] and parameter estimation [17]-[20].

The main advantage with tensor modelisation lies in the fact that several diversities can be taken into account simultaneously. Effectively, the basic operations as Kronecker and Khatri-Rao product allows us to estimate the parameters of transmission only up to permutation and scale ambiguity factor under Kruskal condition [21]. Minimum a priori information helps remove those ambiguities.

Throughout this paper, we propose a new PARAFAC-based approach to estimate the information and the parameters of transmission. Nested-PARAFAC decomposition [22] is used to model the received signal. At the transmitter, we assume a known coding matrix applied to the symbol matrix via Khatri-Rao product. At the receiver, the transmitted symbols, the fading coefficients, the DoD and DoA are estimated from the received signals. To deal with, we apply double alternating least squares (ALS) algorithm.

The rest of this paper is organized as follow. Section 2 provides us the basic MIMO radar system parameters including the array steering matrix at the reception and the transmission, the fading coefficients matrix, the sycmbol matrix and the delay propagation matrix. The useful tensor deompositions are discussed in Section 3. Section 4 details us the system model considering instant or convolutive channel processing with raw and statistical data.

Section 5 presents the system model using a coding matrix with Khatri-Rao product and the proposed algorithm. Numercal simulation results are given in Section 6 before we conclude in Section 7.

2. MIMO radar system parameters

Before modeling the received signals and proposing algorithms of estimation of the transmission paramaters, let's first see, in this section, the different matrix systems which are taken into account in the tensorial modeling of a radar MIMO system. Those parameters are the factor matrices to estimate from the received signals. In other words, they are the spatial signature containing the angles of departure and of arrival, the Radar Cross Section (RCS) coefficients and the delay propagation matrix linked with the memory of the transmission channel.

2.1 Receiver array steering matrix

The We suppose narrowband planar waveforms impinging on a sensor array which is used as a receiver array in a radar MIMO system. The number of temporal samples N is grater than the number of elements M_R in the receiver array, which is itself greater than the number of targets :
$N > M_R > L$
Thus, the signals from each target can be written as :

$$s(n) = e^{-j\omega n} \tag{01}$$

For a uniform linear array (ULA), the first element in the array receiver is used as reference for modeling the recieved signal, is illustrated on **Fig. 1**.
One can write :

$$\begin{gathered} x_1 = s(n) = e^{-j\omega n} \\ x_2 = s(n + \tau_l) = e^{-j\omega(n+\tau_l)} = e^{-j\omega n} e^{-j\omega \tau_l} \\ x_3 = s(n + 2\tau_l) = e^{-j\omega(n+2\tau_l)} = e^{-j\omega n} e^{-j\omega 2\tau_l} \\ \vdots \\ x_{M_R} = s(n + (M_R - 1)\tau_l) = e^{-j\omega(n+(M_R-1)\tau_l)} = e^{-j\omega n} e^{-j\omega(M_R-1)\tau_l} \end{gathered} \tag{02}$$

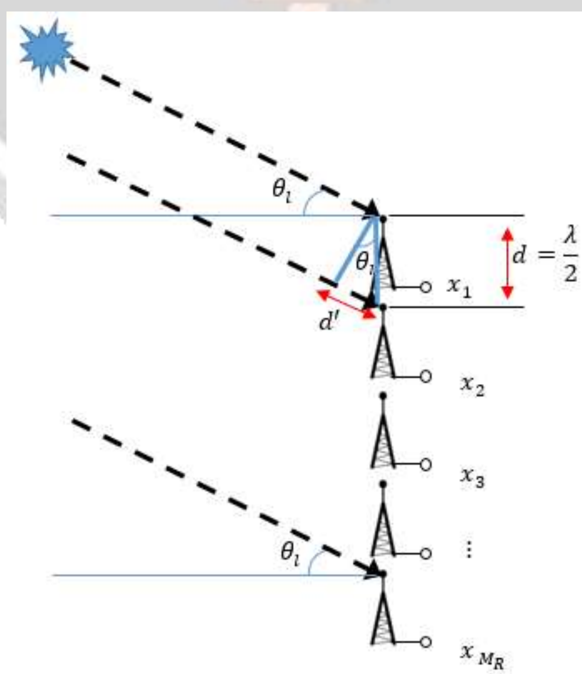

Fig. 1 Illustration of the angle of arrival

So we can obtain the vectorized form of the received signals :

$$\boldsymbol{x}(n) = \boldsymbol{a}^{(R)}(\tau_l)s(n) \tag{03}$$

With
$\boldsymbol{a}^{(R)}(\tau_l) = \left[1, e^{-j\omega\tau_l}, e^{-j\omega 2\tau_l}, \cdots, e^{-j\omega(M_R-1)\tau_l}\right]^T \in \mathbb{C}^{M_R}$. As $\omega\tau_l = \mu_l$ is the spatial frequency, we can deduce the expression of the steering vector at the recption such as :

$$\boldsymbol{a}^{(R)}(\tau_l) = \left[1, e^{-j\mu_l}, e^{-j2\mu_l}, \cdots, e^{-j(M_R-1)\mu_l}\right]^T \tag{04}$$

Where
$\mu_l = \omega\tau_l = 2\pi f \tau_l$ avec $\tau_l = \frac{d'}{c} = \frac{d\,sin(\theta_l)}{c}$. Thus, we have the following relations :

$$\mu_l = 2\pi f \frac{d\,sin(\theta_l)}{c} = 2\pi \frac{1}{\lambda}\frac{\lambda}{2}\,sin(\theta_l)$$
$$\mu_l = \pi\,sin(\theta_l)$$

The expression of the steering vector at the reception for the l-th target according to the angle of arrival θ_l is given by :

$$\boldsymbol{a}^{(R)}(\theta_l) = \left[1, e^{-j\pi\,sin(\theta_l)}, e^{-j2\,sin(\theta_l)}, \cdots, e^{-j(M_R-1)\,sin(\theta_l)}\right]^T \tag{05}$$

And the steering matrix of the L targets in accordance with the receiver array is given by :

$$\boldsymbol{A}^{(R)}(\theta) = \left[\boldsymbol{a}^{(R)}(\theta_1), \boldsymbol{a}^{(R)}(\theta_2), \cdots, \boldsymbol{a}^{(R)}(\theta_L)\right] \in \mathbb{C}^{M_R\times L} \tag{06}$$

2.2 Transmitter array steering matrix

By analogy with the former paragraph, let now φ_l indicates the direction of departure of the l-th target in accordance with the transmit array.
The expression of the steering vector at the transmission for the l-th target according to the angle of departure φ_l is as :

$$\boldsymbol{a}^{(T)}(\varphi_l) = \left[1, e^{-j\pi\,sin(\varphi_l)}, \cdots, e^{-j(M_R-1)\,sin(\varphi_l)}\right]^T \tag{07}$$

And the steering matrix of the L targets in accordance with the transmit array is given by :

$$\boldsymbol{A}^{(T)}(\varphi) = \left[\boldsymbol{a}^{(T)}(\varphi_1), \boldsymbol{a}^{(T)}(\varphi_2), \cdots, \boldsymbol{a}^{(T)}(\varphi_L)\right] \in \mathbb{C}^{M_T\times L} \tag{08}$$

2.3 Fading coefficients matrix

Assuming I transmission blocks at each m_T-th transmit antenna, the fading coefficients from the L targets may be gathered in a matrix such as :

$$\boldsymbol{W} = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,L} \\ \beta_{2,1} & \beta_{2,2} & \vdots & \beta_{2,L} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{I,1} & \beta_{I,2} & \cdots & \beta_{I,L} \end{bmatrix} \in \mathbb{C}^{I\times L} \tag{09}$$

2.4 Delay propagation matrix

In the presence of a convolutive transmission channel, the transission system takes into account the memory of the channel, which are translated by the delay propagation. If the finite support fini of the impulse response of the channe lis equal to K period symbols and that there are L targets in the area of interset, then the propagation delay matrix is given by :

$$\boldsymbol{G}(\tau) = [\boldsymbol{g}(\tau_1)\, \boldsymbol{g}(\tau_2)\, \cdots\, \boldsymbol{g}(\tau_L)] \in \mathbb{C}^{K\times L} \quad (10)$$

With generally
$\boldsymbol{g}(\tau_l) = \left[g(-\tau_l), g(T-\tau_l), \cdots, g\left((K-1)T-\tau_l\right)\right]^T \in \mathbb{C}^K$ and T is the sampling period.

2.5 Symbols matrix

Let's suppose a transmission composed by I transmission blocks and that each transmit antenna of a radar system has a training sequency formed by N symbols known at the reception :

$$\boldsymbol{s}(i) = \begin{bmatrix} s(i,1) \\ \vdots \\ s(i,N) \end{bmatrix} \in \mathbb{C}^N \quad (11)$$

Thus the training sequency transitted by the M_T transit antennas can be written as :

$$\boldsymbol{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(M_T)] \in \mathbb{C}^{N\times M_T} \quad (12)$$

3. Useful Tensor Decompositions

3.1 PARAFAC decomposition

A third-order tensor $\mathcal{X} \in \mathbb{C}^{I\mathrm{x}J\mathrm{x}K}$ can be represneted under its scalar form as :

$$x_{ijk} = \sum_{r=1}^{R} a_{ir}\, b_{jr}\, c_{kr} \quad (13)$$

Where
$\boldsymbol{A} \in \mathbb{C}^{I\mathrm{x}R}$, $\boldsymbol{B} \in \mathbb{C}^{J\mathrm{x}R}$, $\boldsymbol{C} \in \mathbb{C}^{K\mathrm{x}R}$ are the factor matrices and R is the tensor rank. A PARAFAC decomposition can also be written as a mode-n product such as:

$$\mathcal{X} = \mathcal{I}_{3R} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C}$$

Where $\mathcal{I}_{3R}$ is an identity matrix of ordre-3 and dimension R. The corresponding 1-mode, 2-mode and 3-mode unfoldings representations are given by :

$$\begin{aligned} X_{I\mathrm{x}JK} &= \mathbf{A}\,(\mathbf{B} \diamond \mathbf{C})^T \\ X_{J\mathrm{x}KI} &= \mathbf{B}\,(\mathbf{C} \diamond \mathbf{A})^T \\ X_{K\mathrm{x}IJ} &= \mathbf{C}\,(\mathbf{A} \diamond \mathbf{B})^T \end{aligned} \quad (14)$$

Figure 1 provides an illustration of the PARAFAC decomposition of a third-order tensor with rank R as a sum of outer products involving the columns of the corresponding factor matrices.

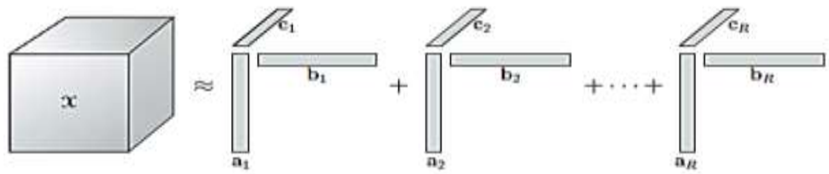

Fig. 1 Illustration of a PARAFAC as a sum of outer products

The unique estimation of the factor matrices by means of Alternating Least Squares (ALS) algorithm, up to permutation and factor scale ambiguity such as:

$$\hat{A} = A\, \Delta_A \Pi \tag{15}$$

Where Δ_A is a diagonal scalar factor and Π is a permutation matrix of order- R.
Each iteration is composed by three steps and in each step, a factor matrix will be estimated although the two others will be fixed at their former values. The ALS algorithm is represented in **Table 1.** The convergence in an ALS algorithm depends on the NMSE criteria, where the error of estimation should be inferior or equal to 10^{-6}.

Table 1. ALS algorithm for a third-order tensor

1 : for t = 0
2 : Initialize $\hat{B}\ (t = 0)$ and $\hat{C}\ (t = 0)$
3 : for t = t + 1
4 : Obtain an estimation of $\hat{A}$ via the expression of X_{IxJK}
$\hat{A}(t) = X_{IxJK}\left(\left(\hat{B}\ (t-1) \diamond \hat{C}\ (t-1)\right)^T\right)^{\dagger}$
5 : Obtain an estimation of $\hat{B}$ via the expression of X_{JxKI}
$\hat{B}(t) = X_{JxKI}\left(\left(\hat{C}\ (t-1) \diamond \hat{A}(t)\right)^T\right)^{\dagger}$
6 : Obtain an estimation of $\hat{C}$ via the expression of X_{KxIJ}
$\hat{C}(t) = X_{KxIJ}\left(\left(\hat{A}(t) \diamond \hat{B}(t)\right)^T\right)^{\dagger}$
7 : Repeat the steps **3-6** until convergence

3.2 Nested-PARAFAC decomposition

The nested – PARAFAC decomposition assumes that the n –th factor matrix $B^{(n)} \in \mathbb{C}^{I_n \times L}$ in a PARAFAC model is an unfolding of an additionnal PARAFAC decomposition. So, if $B^{(1)} \in \mathbb{C}^{I_1 \times L}$ B(1) is the mode-1 unfolding of a tensor of order-3

$$\mathcal{Y} \in \mathbb{C}^{J_1 \times J_2 \times J_3} \tag{16}$$

then,

$$B^{(1)} = [\mathcal{Y}]_{(1)} \tag{17}$$

Where

$$J_1 = I_1$$

and

$$J_2 J_3 = L\ .$$

The nested – PARAFAC decomposition of a tensor of order-3 $\mathcal{X}$ is given by the relations :

$$\mathcal{X} = \mathcal{I}_{3,L} \times_1 [\mathcal{Y}]_{(1)} \times_2 \boldsymbol{B}^{(2)} \times_3 \boldsymbol{B}^{(3)}$$
$$\mathcal{Y} = \mathcal{I}_{3,L} \times_1 \boldsymbol{A}^{(1)} \times_2 \boldsymbol{A}^{(2)} \times_3 \boldsymbol{A}^{(3)} \quad (18)$$

The first equation is the external PARAFAC model and the second one is the internal PARAFAC model with factor matrices $(\boldsymbol{A}^{(1)}, \boldsymbol{A}^{(2)}, \boldsymbol{A}^{(3)})$. The estimation of the factor matrices in a nested – PARAFAC model can be achived through two successive ALS. The first stage consists of estiamting th e factor matrices in the external PARAFAC and the second is performed by means of the estimation of the unfolding of the tensor $\mathcal{Y}$ obtained previously.
The details of an ALS – Nested – PARAFAC are presented in **Table 2**.

Table 2. ALS – Nested – PARAFAC algorithm

First stage
1 : for t = 0
2 : Initialize $\hat{\boldsymbol{B}}^{(2)}(t=0)$ and $\hat{\boldsymbol{B}}^{(3)}(t=0)$
3 : for t = t + 1
4 : Obtain an estimation of $\hat{\boldsymbol{B}}^{(1)}$ via the expression of $[\mathcal{X}]_{(1)}$
$\hat{\boldsymbol{B}}^{(1)T}(t) = \left(\hat{\boldsymbol{B}}^{(3)}(t-1) \diamond \hat{\boldsymbol{B}}^{(2)}(t-1)\right)^{\dagger} [\mathcal{X}]_{(1)}$
5 : Obtain an estimation of $\hat{\boldsymbol{B}}^{(2)}$ via the expression of $[\mathcal{X}]_{(2)}$
$\hat{\boldsymbol{B}}^{(2)T}(t) = \left(\hat{\boldsymbol{B}}^{(3)}(t-1) \diamond \hat{\boldsymbol{B}}^{(1)}(t)\right)^{\dagger} [\mathcal{X}]_{(2)}$
6 : Obtain an estimation of $\hat{\boldsymbol{B}}^{(2)}$ via the expression of $[\mathcal{X}]_{(3)}$
$\hat{\boldsymbol{B}}^{(3)T}(t) = \left(\hat{\boldsymbol{B}}^{(2)}(t) \diamond \hat{\boldsymbol{B}}^{(1)}(t)\right)^{\dagger} [\mathcal{X}]_{(3)}$
7 : Repeat the steps **3-6** until convergence
Second stage
8 : Reconstruct the tensor $\mathcal{Y}$ from $\hat{\boldsymbol{B}}^{(1)} = [\mathcal{Y}]_{(1)}$
9 : for t = 0
10 : Initialize $\hat{\boldsymbol{A}}^{(2)}(t=0)$ and $\hat{\boldsymbol{A}}^{(3)}(t=0)$
11 : for t = t + 1
12 : Obtain an estimation of $\hat{\boldsymbol{A}}^{(1)}$ via the expression of $[\mathcal{Y}]_{(1)}$
$\hat{\boldsymbol{A}}^{(1)T}(t) = \left(\hat{\boldsymbol{A}}^{(3)}(t-1) \diamond \hat{\boldsymbol{A}}^{(2)}(t-1)\right)^{\dagger} [\mathcal{Y}]_{(1)}$
13 : Obtain an estimation of $\hat{\boldsymbol{A}}^{(2)}$ via the expression of $[\mathcal{Y}]_{(2)}$
$\hat{\boldsymbol{A}}^{(2)T}(t) = \left(\hat{\boldsymbol{A}}^{(3)}(t-1) \diamond \hat{\boldsymbol{A}}^{(1)}(t)\right)^{\dagger} [\mathcal{Y}]_{(2)}$
14 : Obtain an estimation of $\hat{\boldsymbol{A}}^{(2)}$ via the expression of $[\mathcal{Y}]_{(3)}$
$\hat{\boldsymbol{A}}^{(3)T}(t) = \left(\hat{\boldsymbol{A}}^{(2)}(t) \diamond \hat{\boldsymbol{A}}^{(1)}(t)\right)^{\dagger} [\mathcal{Y}]_{(3)}$
15 : Repeat the steps **9-14** until convergence

4. System Model

Suppose a system with M_T antennas at the transmission and M_R antennas at the reception. We assume that the channel between the transmitter and the receiver may be regarded as the superposition of L paths. At one path is associated a scatterer, which defines an angle of arrival θ_l in accordance with the receive array and an angle of departure ϕ_l in accordance with the transmit array. One transmission is composed by I blocs and the amplitude fading is assumed to be constant over a transmission data bloc but vary between two blocs. The long-term parameters θ_l and ϕ_l are constant over an entire transmission. The transmitter antenna has a training sequence of N symbols known at the reception, such as:

$$\boldsymbol{s}_i = \begin{bmatrix} s_{i,1} \\ \vdots \\ s_{i,N} \end{bmatrix} \in \mathbb{C}^N \quad (19)$$

4.1 MIMO radar with instant channel

The training sequency is used through the I transmission blocks de transmissions, i.e :

$$\boldsymbol{s}_i = \boldsymbol{s} \; \forall \; i = 1, \cdots, I$$

The training sequency matrix transitted by the M_T antennas is given by:

$$\boldsymbol{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(M_T)] \in \mathbb{C}^{N \times M_T} \quad (20)$$

where $N \geq M_T$. The M_T vectors of the training sequency are linearly independent.

4.1.1 Processing through raw data

The impulse response of the channel between the m_T–th transmit antenna and the m_R–th receive antenna for the i –th transmission bloc is given by:

$$h_{i,m_R,m_T} = \sum_{l=1}^{L} b_{i,l} a_{m_R,l}^{(R)} a_{m_T,l}^{(T)} \quad (21)$$

Which can be viewed as the scalar component of the PARAFAC model of the channel tensor of order-3 $\mathcal{H} \in \mathbb{C}^{I \times M_R \times M_T}$ with the parameters $(\boldsymbol{B}, \boldsymbol{A}^{(R)}, \boldsymbol{A}^{(T)}; L)$.

The n -th signal received by the m_R–th antenna during the i –th transmission bloc is given by:

$$x_{i,m_R,n} = \sum_{m_T=1}^{M_T} \sum_{l=1}^{L} b_{i,l} a_{m_R,l}^{(R)} a_{m_T,l}^{(T)} s_{n,m_T} \quad (22)$$

The matrix model of the channel is given by:

$$\boldsymbol{H}_{i..} = \left[\boldsymbol{A}^{(R)} diag(\boldsymbol{B}_i)\right] \boldsymbol{A}^{(T)T} \in \mathbb{C}^{M_R \times M_T} \quad (23)$$

And the matrix model of the received can be written as:

$$\boldsymbol{X}_{i..} = \boldsymbol{H}_{i..} \boldsymbol{S}^T = \left[\boldsymbol{A}^{(R)} diag(\boldsymbol{B}_i)\right] \boldsymbol{A}^{(T)T} \boldsymbol{S}^T \in \mathbb{C}^{M_R \times N}$$

$$\boldsymbol{X}_{i..} = \left[\boldsymbol{A}^{(R)} diag(\boldsymbol{B}_i)\right] \boldsymbol{C}^T \quad (24)$$

with

$\boldsymbol{C} = \boldsymbol{S}\boldsymbol{A}^{(T)} \in \mathbb{C}^{N \times L}$.

Thus, the matrices empiling I blocks of $H_{i..}$ and I blocks of $X_{i..}$ can be respectively interpreted as the i –th matrix slices of the channel and received signal tensors. We then have :

$$\begin{bmatrix} H_{1..} \\ \vdots \\ H_{I..} \end{bmatrix} = \begin{bmatrix} A^{(R)} diag(B_1) A^{(T)T} \\ \vdots \\ A^{(R)} diag(B_I) A^{(T)T} \end{bmatrix} = \begin{bmatrix} A^{(R)} diag(B_1) \\ \vdots \\ A^{(R)} diag(B_I) \end{bmatrix} A^{(T)T} \quad (25)$$

$$H_{IM_R \times M_T} = \left(B \diamond A^{(R)}\right) A^{(T)T} \quad (26)$$

And

$$X_{IM_R \times N} = \left(B \diamond A^{(R)}\right) C^T \quad (27)$$

$X_{IM_R \times N}$ corresponds to the unfolding of the received signal tensor $\mathcal{X} \in \mathbb{C}^{I \times M_R \times N}$ of order-3 which follows a PARAFAC decomposition with the parameters $(B, A^{(R)}, C; L)$.
The two other unfolding representations are :

$$X_{M_R N \times I} = \left(A^{(R)} \diamond C\right) B^T$$
$$X_{NI \times M_R} = (C \diamond B) A^{(R)T} \quad (28)$$

We can use the ALS to estimate the matrices B, $A^{(R)}$ and C. Then from the expression $\hat{C} = SA^{(T)}$, one can obtain the estimation of the matrix $\hat{A}^{(T)}$ by the least square criteria if S is full column rank, which means $N \geq M_T$. :
$\hat{A}^{(T)} = S^{\dagger}\hat{C}$

Uniqueness and identifiability conditions. As the matrix $A^{(R)}$ has a Vandermonde structure and B and C are random matrices, they are full rank. The Kruskal condition is defined by $k_{A^{(R)}} + k_B + k_C \geq 2L + 2$ or $min(M_R, L) + min(I, L) + min(K, L) \geq 2L + 2$. In fact, as $= SA^{(T)}$, then $r(C) = r(A^{(T)})$ because S is also full rank. The Vandermonde structure in $A^{(R)}$ allows us to suppress the scale factor in $\hat{A}^{(R)}$. And we can deduce the matrix permutation Π.
As we estimate the matrices $C \in \mathbb{C}^{N \times L}$, $B \in \mathbb{C}^{I \times L}$ and $A^{(R)} \in \mathbb{C}^{M_R \times L}$ by means of an ALS algorithm, we should have $IM_R \geq L$; $M_R N \geq L$ and $IN \geq L$ so that we can perform the pseudo-inverses. The main cost of the algorithm corresponds to the cost of the calculus of the SVD to find the pseudo-inverses, given by $O\left(L^2(NI + NM_R + IM_R)\right)$ per iteration, plus tthe complexity of an additional SVD of the matrix $\hat{A}^{(T)} \in \mathbb{C}^{M_T \times L}$ of order $O(M_T L\, min(M_T, L))$ where we should have $N \geq M_T$ to perform $S^{\dagger}\hat{C}$.

Table 3. Algorithm for MIMO radar with instant channel processing through raw data

1 : $X_{IM_R \times N} = \left(B \diamond A^{(R)}\right) C^T$
2 : By the ALS algorithm, obtain $\hat{C}$, $\hat{B}$ and $\hat{A}^{(R)}$ from the expressions of $X_{IM_R \times N}$, $X_{M_R N \times I}$ et $X_{NI \times M_R}$
3 : By the least sqaures criteria $\hat{A}^{(R)}$

4.1.2 Processing through statistical data

The covariance matrix of the received signals can be written as:

$$\boldsymbol{R}_{M_R \times M_R} = \boldsymbol{X}_{M_R \times N} \boldsymbol{X}_{M_R \times N}{}^{H}$$

$$\boldsymbol{R}_{IM_R \times IM_R} = \left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)\boldsymbol{C}_S\left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)^{H} \quad (29)$$

Where $\boldsymbol{C}_S = \boldsymbol{C}^T\boldsymbol{C}^* = \boldsymbol{G}^T\boldsymbol{S}^T\boldsymbol{S}^*\boldsymbol{G}^* \in \mathbb{C}^{L\times L}$

Let's define the matrix $\boldsymbol{Z}$ as $\boldsymbol{Z} = \left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)\boldsymbol{C}_S \in \mathbb{C}^{IM_R \times L}$. Then one can have :

$$\boldsymbol{R}_{IM_R \times IM_R} = \boldsymbol{Z}\left(\boldsymbol{B}^* \diamond \boldsymbol{A}^{(R)*}\right)^{T} \quad (30)$$

Which corresponds to the unfolding representation of a third-order tensor $\mathcal{R} \in \mathbb{C}^{IM_R \times I \times M_R}$ with the parameters $(\boldsymbol{Z}, \boldsymbol{B}^*, \boldsymbol{A}^{(R)*}; L)$. The two others unfolding representations are:

$$\boldsymbol{R}_{I \times M_R IM_R} = \boldsymbol{B}^*\left(\boldsymbol{A}^{(R)*} \diamond \boldsymbol{Z}\right)^{T}$$

$$\boldsymbol{R}_{M_R \times II M_R} = \boldsymbol{A}^{(R)*}\left(\boldsymbol{B}^* \diamond \boldsymbol{Z}\right)^{T} \quad (31)$$

By means of the ALS algorithm, it is possible to estimate the factor matrices $\hat{\boldsymbol{Z}}, \hat{\boldsymbol{A}}^{(R)*}, \hat{\boldsymbol{B}}^*$.

<u>Uniqueness and identifiability conditions.</u> The condition of Kruskal defined by $k_{A^{(R)}} + k_B + k_C \geq 2L + 2$ is now equivalent to $min(M_R, L) + min(I, L) + min(I + M_R - 1, L) \geq 2L + 2$ as the matrices $\boldsymbol{A}^{(R)}$ (with Vandermonde structure), $\boldsymbol{B}$ (random) et $\boldsymbol{Z}$ are full rank. In fact, for $\boldsymbol{Z} = \left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)\boldsymbol{C}_S$, so $r(\boldsymbol{Z}) = r\left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)$ because $\boldsymbol{C}_S$ is also full rank. The Vandermonde structure in the matrix $\boldsymbol{A}^{(R)}$ helps delete the scale factor in $\hat{\boldsymbol{A}}^{(R)}$. Thus we can deduce the permutation matrix $\boldsymbol{\Pi}$.

Through the expression $\hat{\boldsymbol{Z}} = \left(\hat{\boldsymbol{B}} \diamond \hat{\boldsymbol{A}}^{(R)}\right)\boldsymbol{C}_S$, we can obtain the estimation of the matrix $\hat{\boldsymbol{C}}_S$ by the least squares criteria, so we can write:

$$\hat{\boldsymbol{C}}_S = \left(\hat{\boldsymbol{B}} \diamond \hat{\boldsymbol{A}}^{(R)}\right)^{\dagger}\hat{\boldsymbol{Z}} \quad (32)$$

if $IM_R \geq L$.

After obtaining an estimation of the matrix $\hat{\boldsymbol{C}}_S$, we shall estimate the matrix $\boldsymbol{G}$ as follow. Define $\boldsymbol{C}_S = \boldsymbol{G}^T\boldsymbol{R}_S\boldsymbol{G}^*$ where $\boldsymbol{R}_S = \boldsymbol{S}^T\boldsymbol{S}^* \in \mathbb{C}^{K\times K}$ is the transmitted signals covariance matrix. Thus we have the relation :

$\boldsymbol{C}_S = \boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^H$

With $\boldsymbol{D} = \boldsymbol{A}^{(T)T} \in \mathbb{C}^{L\times M_T}$.

We can have an estimation of the matrix $\boldsymbol{D} = \boldsymbol{A}^{(T)T}$. The elements v_{i,m_T} of the matrix $\boldsymbol{P} \in \mathbb{C}^{I\times M_T}$ will be the values of the power of the m_T –th source in the i-th tramsission bloc.

<u>Uniqueness and identifiability conditions.</u> We estimate the matrices $\boldsymbol{Z} \in \mathbb{C}^{IM_R\times L}$, $\boldsymbol{B} \in \mathbb{C}^{I\times L}$ and $\boldsymbol{A}^{(R)} \in \mathbb{C}^{M_R\times L}$ by the ALS algorithm, where the condition $IM_R \geq L$ should be satisfied to process with the pseudo – inverses. So the main cost of the corresponding algorithm is equal to the cost of the calculus of the SVD for the three pseudo – inverses, given by $\mathcal{O}\left(L^2(I^2M_R + IM_R^2 + IM_R)\right)$ per iteration, plus the complexity of an additional SVD of the matrix $\hat{\boldsymbol{C}}_S \in \mathbb{C}^{L\times L}$ of order $\mathcal{O}(L^3)$.

The resolution of the equation $\boldsymbol{C}_S = \boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^H$ includes an estimation of the matrices $\boldsymbol{D} \in \mathbb{C}^{L\times M_T}, \boldsymbol{P} \in \mathbb{C}^{I\times M_T}$ by means of an ALS where $IL \geq M_T$ et $L^2 \geq M_T$. Thus we should also take into account the cost of the calculus of the SVD to find the other three pseudo – inverses in the second ALS, given by $\mathcal{O}M_T{}^2(2IL + L^2))$ per iteration.

Table 4. Algorithm for MIMO radar with instant channel processing through statistical data

1 : $\boldsymbol{R}_{M_R \times M_R} = \boldsymbol{X}_{IM_R \times N} \boldsymbol{X}_{IM_R \times N}{}^H$
2 : Define $\boldsymbol{Z} = (\boldsymbol{B} \diamond \boldsymbol{A}^{(R)})\boldsymbol{C}_S$
3 : By the ALS algorithm, obtain $\hat{\boldsymbol{Z}}$, $\hat{\boldsymbol{B}}^*$ et $\hat{\boldsymbol{A}}^{(R)*}$ from the expressions of $\boldsymbol{R}_{IM_R \times IM_R}$, $\boldsymbol{R}_{I \times M_R IM_R}$ and $\boldsymbol{R}_{M_R \times IIM_R}$
4 : Obtain $\hat{\boldsymbol{C}}_S = (\hat{\boldsymbol{B}} \diamond \hat{\boldsymbol{A}}^{(R)})^{\dagger}\hat{\boldsymbol{Z}}$ by the LS criteria
5 : By the resolution of the equation $\boldsymbol{C}_S = \boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^H$, obtain $\hat{\boldsymbol{D}}$
6 : Obtain $\hat{\boldsymbol{D}}_f$ from the two conjugate expressions
7 : Obtenir $\hat{\boldsymbol{A}}^{(T)}$ par la relation $\boldsymbol{D} = \boldsymbol{A}^{(T)T}$

4.2 MIMO radar with convolutive channel

We suppose now the presence of a relative delay propagation τ_l for the l -th way, which is constant over I transmission blocks. The finite support of the channel impulse response is equal to K period symbols. The impulse response matrix with delay is given by :

$$\boldsymbol{G}(\tau) = [\boldsymbol{g}(\tau_1) \cdots \boldsymbol{g}(\tau_L)] \in \mathbb{C}^{K \times L} \tag{33}$$

The matrix of the training sequence transmitted by the tranmsit antenna is such as:

$$\boldsymbol{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(K)] \in \mathbb{C}^{N \times KM_T} \tag{34}$$

where $N \geq KM_T$.

4.2.1 Processing through raw data

The impulse response of the k –th tap of the channel between the m_T-th transmit antenna and the m_R-th receive antenna for the i-th transmission bloc is given by:

$$h_{i,m_R,m_T,k} = \sum_{l=1}^{L} b_{i,l} a^{(R)}_{m_R,l} a^{(T)}_{m_T,l} g_{k,l} \tag{35}$$

Which can be viewed as the scalar component of the PARAFAC model of the channel tensor of order-4 $\mathcal{H} \in \mathbb{C}^{I \times M_R \times M_T}$ with parameters $(\boldsymbol{B}, \boldsymbol{A}^{(R)}, \boldsymbol{A}^{(T)}, \boldsymbol{G}; L)$.

The n –th received signal by the m_R-th receiver antenna de for the i-th transmission bloc is given by:

$$x_{i,m_R,n} = \sum_{m_T=1}^{M_T} h_{i,m_R,m_T,k} s_{n,km_T}$$

$$x_{i,m_R,n} = \sum_{m_T=1}^{M_T} \sum_{l=1}^{L} b_{i,l} a^{(R)}_{m_R,l} a^{(T)}_{m_T,l} g_{k,l}\, s_{n,km_T} \tag{36}$$

The matrix model of the channel, by fixing the fisrt and the last dimension of the channel tensor may be written as :

$$\boldsymbol{H}_{i..k} = b_{i,l} g_{k,l} \boldsymbol{A}^{(R)} \boldsymbol{A}^{(T)T} \in \mathbb{C}^{M_R \times M_T} \tag{37}$$

$$H_{i\_k} = A^{(R)} diag(B_i) diag(G_k) A^{(T)T} \quad (38)$$

Empiling the blocks of H_{i_k}, one can obtain :

$$H_{IM_R \times KM_T} = (B \diamond A^{(R)}) W^T \quad (39)$$

Where $W = G \diamond A^{(T)} \in \mathbb{C}^{KM_T \times L}$

The corresponding matrix representation of the received signal is :

$$X_{IM_R \times N} = H_{IM_R \times KM_T} S^T = (B \diamond A^{(R)}) C^T \quad (40)$$

with $C = SW \in \mathbb{C}^{N \times L}$.

Which corresponds to the unfolding representation of a third-order tensor $\mathcal{X} \in \mathbb{C}^{I \times M_R \times N}$ with the parameters $(B, A^{(R)}, C; L)$. The two others unfoldings representation are:

$$X_{M_R N \times I} = (A^{(R)} \diamond C) B^T \quad (41)$$

$$X_{NI \times M_R} = (C \diamond B) A^{(R)T}$$

By means of the ALS algorithm, it is possible to estimate the factor matrices $\hat{B}, \hat{A}^{(R)}, \hat{C}$. At last, as $W = G \diamond A^{(T)}$, we can use the LS-KRP algorithm described in section ... in order to estimate the factor matrices $\hat{G}$ and $\hat{A}^{(T)}$.

<u>Uniqueness and identifiability conditions.</u> Knowing that the matrices $A^{(R)}$ (with Vandermonde structure), B and C (random) are full rank, the condition of Kruskal defined by $k_{A^{(R)}} + k_B + k_C \geq 2L + 2$ is equivalent to $min(M_R, L) + min(I, L) + min(K, L) \geq 2L + 2$. In fact, as $C = SW$, thus $r(C) = r(W)$ because S is also full rank. The Vandermonde structure in the matrice $A^{(R)}$ helps delete the scale factor in $\hat{A}^{(R)}$. So, one can deduce the permutation matrix Π in the estimations. Then through the espression of $\hat{C}$, we can obtain the estimation of the matrix by the least squares criteria: $\hat{W} = S^{\dagger}\hat{C}$ where $N \geq K$.

As we estimate the matrices $C \in \mathbb{C}^{N \times L}$, $B \in \mathbb{C}^{I \times L}$ and $A^{(R)} \in \mathbb{C}^{M_R \times L}$ by means of an ALS algorithm, we should have $IM_R \geq L$; $M_R N \geq L$ and $IN \geq L$ so that we can perform the pseudo-inverses. The main cost of the algorithm corresponds to the cost of the calculus of the SVD to find the pseudo-inverses, given by $O(L^2(NI + NM_R + IM_R))$ per iteration, plus tthe complexity of an additional SVD of the matrix $\hat{W} \in \mathbb{C}^{KM_T \times L}$ of order $O(KM_T L\, min(KM_T, L))$ where we should have $N \geq KM_T$ to perform $S^{\dagger}\hat{C}$. And the cost of the LS KRP algorithm to estimate $\hat{G}$ and $\hat{A}^{(T)}$ is given by $O(KM_T L)$.

Table 5. Algorithm for MIMO radar with convolutive channel processing through raw data

1 : $X_{IM_R \times N} = (B \diamond A^{(R)}) C^T$
2 : By the ALS algorithm, obtain $\hat{C}$, $\hat{B}$ and $\hat{A}^{(R)}$ through the expressions of $X_{IM_R \times N}$, $X_{M_R N \times I}$ and $X_{NI \times M_R}$
3 : Otain $\hat{W}$ by the least squares criteria
4 : By the LS-KRP algorithm, obtain $\hat{G}$ and $\hat{A}^{(T)}$ from relation $W = G \diamond A^{(T)}$

4.2.2 Processing through statistical data

The covariance matrix of the received signals can be written as:

$$R_{IM_R \times IM_R} = X_{IM_R \times N} X_{IM_R \times N}{}^H$$
$$R_{IM_R \times IM_R} = \left(B \diamond A^{(R)}\right) C_S \left(B \diamond A^{(R)}\right)^H \quad (42)$$

Where $C_S = C^T C^* = \left(G \diamond A^{(T)}\right)^T S^T S^* \left(G \diamond A^{(T)}\right)^* \in \mathbb{C}^{L \times L}$. Let's define $Z^T = C_S \left(B \diamond A^{(R)}\right)^H \in \mathbb{C}^{L \times IM_R}$, then we can write:

$$R_{IM_R \times IM_R} = \left(B \diamond A^{(R)}\right) Z^T \quad (43)$$

Which corresponds to the unfolding representation of a third-order tensor $\mathcal{R} \in \mathbb{C}^{I \times M_R \times IM_R}$ with the parameters $(B, A^{(R)}, Z; L)$. The two others unfoldings representation are:

$$R_{M_R IM_R \times I} = \left(A^{(R)} \diamond Z\right) B^T$$
$$R_{IM_R I \times M_R} = (Z \diamond B)\, A^{(R)T} \quad (44)$$

By means of the ALS algorithm, it is possible to estimate the factor matrices $\hat{B}, \hat{A}^{(R)}, \hat{Z}$.

<u>Uniqueness and identifiability conditions.</u> The condition of Kruskal defined by $k_{A^{(R)}} + k_B + k_Z \geq 2L + 2$ is now equivalent to $min(M_R, L) + min(I, L) + min(I + M_R - 1, L) \geq 2L + 2$ as the matrices $A^{(R)}$ (with Vandermonde structure), B (random) et Z are full rank. In fact, for $Z^T = C_S \left(B \diamond A^{(R)}\right)^H$, so $r(Z) = r\left(B \diamond A^{(R)}\right)^H$ because C_S is also full rank. The Vandermonde structure in the matrix $A^{(R)}$ helps delete the scale factor in $\hat{A}^{(R)}$. Thus we can deduce the permutation matrix Π.

Through the expression $\hat{Z}^T = C_S \left(B \diamond A^{(R)}\right)^H$, we can obtain the estimation of the matrix $\hat{C}_S$ by the least squares criteria, so we can write:

$$\hat{C}_S = \hat{Z}^T \left[\left(\hat{B}^* \diamond \hat{A}^{(R)*}\right)^T\right]^\dagger \quad (45)$$

if $L \geq IM_R$.

After obtaining an estimation of the matrix $\hat{C}_S$, we shall estimate the matrix W as follow. Define $C_S = W^T R_S W^*$ where $R_S = S^T S^* \in \mathbb{C}^{KM_T \times KM_T}$ is the transmitted signals covariance matrix. Thus we have the relation :

$$C_S = D R_S D^H \quad (46)$$

With $D = W^T \in \mathbb{C}^{L \times KM_T}$.

We then can have an estimation of the matrix $D = W^T$. The elements v_{i,km_T} of the matrix $P \in \mathbb{C}^{I \times KM_T}$ will be the values of the power of the k –th channel coefficient in the i-th tramsission block. At last, as $W = G \diamond A^{(T)}$, we can use the LS-KRP algorithm described in section ... in order to estimate the factor matrices $\hat{G}$and $\hat{A}^{(T)}$.

Table 6. Algorithm for MIMO radar with convolutive channel processing through statistical data

1 : $\boldsymbol{R}_{IM_R \times IM_R} = \boldsymbol{X}_{IM_R \times N} \boldsymbol{X}_{IM_R \times N}{}^H$

2 : Define $\boldsymbol{Z}^T = \boldsymbol{C}_S \left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)} \right)^H$

3 : By the ALS algorithm, obtain $\widehat{\boldsymbol{Z}}$, $\widehat{\boldsymbol{B}}$ and $\widehat{\boldsymbol{A}}^{(R)}$ from the expressions of $\boldsymbol{R}_{IM_R \times IM_R}$, $\boldsymbol{R}_{M_R IM_R \times I}$ and $\boldsymbol{R}_{IM_R I \times M_R}$

4 : Obtain $\widehat{\boldsymbol{C}}_S = \widehat{\boldsymbol{Z}}^T \left[\left(\widehat{\boldsymbol{B}}^* \diamond \widehat{\boldsymbol{A}}^{(R)*} \right)^T \right]^{\dagger}$ by the least squares criteria

5 : By the resolution of the equation $\boldsymbol{C}_S = \boldsymbol{D} \boldsymbol{R}_S \boldsymbol{D}^H$, obtain $\widehat{\boldsymbol{D}}$

6: Obtain $\widehat{\boldsymbol{D}}_f$ by the two conjugate expressions

7 : Obtain $\widehat{\boldsymbol{G}}$ via the relation $\boldsymbol{D} = \boldsymbol{W}^T$

8 : By the LS-KRP algorithm, obtain $\widehat{\boldsymbol{G}}$ and $\widehat{\boldsymbol{A}}^{(T)}$ from relation $\boldsymbol{W} = \boldsymbol{G} \diamond \boldsymbol{A}^{(T)}$

Uniqueness and identifiability conditions. We estimate the matrices $\boldsymbol{Z} \in \mathbb{C}^{IM_R \times L}$, $\boldsymbol{B} \in \mathbb{C}^{I \times L}$ and $\boldsymbol{A}^{(R)} \in \mathbb{C}^{M_R \times L}$ by the ALS algorithm, where the condition $IM_R \geq L$ should be satisfied to process with the pseudo – inverses. So the main cost of the corresponding algorithm is equal to the cost of the calculus of the SVD for the three pseudo – inverses, given by $O\left(L^2 (I^2 M_R + IM_R^2 + IM_R)\right)$ per iteration, plus the complexity of an additional SVD of the matrix $\widehat{\boldsymbol{C}}_S \in \mathbb{C}^{L \times L}$ of order $O(L^3)$.

The resolution of the equation $\boldsymbol{C}_S = \boldsymbol{D} \boldsymbol{R}_S \boldsymbol{D}^H$ includes an estimation of the matrices $\boldsymbol{D} \in \mathbb{C}^{L \times KM_T}$, $\boldsymbol{P} \in \mathbb{C}^{I \times KM_T}$ by means of an ALS where $IL \geq KM_T$ et $L^2 \geq KM_T$. Thus we should also take into account the cost of the calculus of the SVD to find the other three pseudo – inverses in the second ALS, given by $O(K^2 M_T^2 (2IL + L^2))$ per iteration. And the cost of the LS KRP algorithm to estimate $\widehat{\boldsymbol{G}}$ and $\widehat{\boldsymbol{A}}^{(T)}$ is given by $O(KM_T L)$.

5. System model with coding matrix

Assume a radar MIMO system with respectively M_T and M_R elements at the transmission and the reception. $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{M_T}] \in \mathbb{C}^{R \times M_T}$ is th ematrix of R packets of M_T symbols multiplexed on the M_T transmit antennas. Assuming that the system is multipath, we suppose that L is the number of scatterers which corresponds to the number of paths or targets between each transit and receive antenna.

The receiver array matrix is given by :

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_L)] \in \mathbb{C}^{M_R \times L} \quad (47)$$

with

$$\boldsymbol{a}(\theta_l) = \left[1, e^{-j\pi \sin(\theta_l)}, \ldots, e^{-j\pi (M_R - 1) \sin(\theta_l)}\right]^T \in \mathbb{C}^{M_R} \quad (48)$$

And the transmit array matrix is

(5.11)

$$\boldsymbol{B} = [\boldsymbol{b}(\varphi_1), \ldots, \boldsymbol{b}(\varphi_L)] \in \mathbb{C}^{M_T \times L} \quad (49)$$

with

$$\boldsymbol{b}(\varphi_l) = \left[1, e^{-j\pi \sin(\varphi_l)}, \ldots, e^{-j\pi (M_T - 1) \sin(\varphi_l)}\right]^T \in \mathbb{C}^{M_T} \quad (50)$$

Thus, φ_l and θ_l are respectively the direction of departure and of arrival of the l-th path. We always assume that a transmission is composed by I blocks. If the vector of fading coefficients of the signal is such as $\boldsymbol{w}_i = [\beta_1^{(i)}, \dots, \beta_L^{(i)}]^T \in \mathbb{C}^L$, then the signal fading matrix la matrice is given by :

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1^T \\ \vdots \\ \boldsymbol{w}_I^T \end{bmatrix} \in \mathbb{C}^{I \times L} \quad (51)$$

Where $\beta_l^{(i)}$ is the fading coefficient of the signal in the i –th block for the l –th target..

5.1 Coding matrix

A space-time coding system based on Khatri-Rao product is applied through the source coding matrix $\boldsymbol{C} \in \mathbb{C}^{N \times M_T}$ where N is the number of repetitions of each R paqckets of M_T symbols. The coded signals are the results of the Khatri-Rao product which spread symbols over N symbol periods.

5.2 Received signal model

The signal transmitted by the M_T transmit antennas de through the l-th path is given by :

$$d_{n,r,l} = \sum_{m_T=1}^{M_T} c_{n,m_T} s_{r,m_T} b_{m_T,l} \quad (52)$$

Which corresponds to the PARAFAC decomposition of a tensor $\mathcal{D} \in \mathbb{C}^{N \times R \times L}$ with parameters $(\boldsymbol{C}, \boldsymbol{S}, \boldsymbol{B}^T; M_T)$, having the matrix represntation as :

$$\boldsymbol{D}_{NR \times L} = (\boldsymbol{C} \diamond \boldsymbol{S})\boldsymbol{B} \quad (53)$$

The two other unfoldings can be written as :

$$\boldsymbol{D}_{RL \times N} = (\boldsymbol{S} \diamond \boldsymbol{B}^T)\boldsymbol{C}^T$$
$$\boldsymbol{D}_{LN \times R} = (\boldsymbol{B}^T \diamond \boldsymbol{C})\boldsymbol{S}^T \quad (54)$$

If $NR = P$, then $d_{n,r,l} = d_{p,l}$.

For the i-th transmission block, the signal received by the m_R-th receive antenna is :

$$x_{m_R,p,i} = \sum_{l=1}^{L} a_{m_R,l} d_{p,l} w_{i,l} \quad (55)$$

Which can be viewed as the scalar representation of th received signals tensor of order -3 $\mathcal{X} \in \mathbb{C}^{M_R \times P \times I}$ which follows a PARAFAC model with parameters $(\boldsymbol{A}, \boldsymbol{D}, \boldsymbol{W}; L)$. Considering the noiseless term and collecting the slices $\boldsymbol{X}_{..i}$ through the third dimension, one can obtain the unfolding form of the received signal as :

$$\boldsymbol{X}_{M_R NR \times I} = (\boldsymbol{A} \diamond \boldsymbol{D})\boldsymbol{W}^T \quad (56)$$

The two other unfoldings can be written as :

$$X_{IM_R \times NR} = (W \diamond A)D^T$$
$$X_{NRI \times M_R} = (D \diamond W)A^T \quad (57)$$

Finally, for the whole system,we obtain a nested – PARAFAC model such that :

$$\mathcal{X} = \mathcal{I}_{3,L} \times_1 A \times_2 D_{NR \times L} \times_3 W$$
$$\mathcal{D} = \mathcal{I}_{3,M_T} \times_1 C \times_2 S \times_3 B^T \quad (58)$$

By means of an ALS nested-PARAFAC algorithm, one could estimate the factor A, W, S, B^T.

Uniqueness and identifiability conditions. ccording to Kruskal conditions, it is possible to estimate up to a permutation and a scale factor the matrices in the first stage of our ALS – nested - PARAFAC algorithm if :

$$k_A + k_D + k_W \geq 2L + 2$$
$$\Leftrightarrow min(M_R, L) + min(NR, L) + min(I, L) \geq 2L + 2 \quad (59)$$

Based on the least squares criteria, we can obtain the following estimations :

$$\widehat{W}^T = (A \diamond D)^{\dagger} X_{M_R NR \times I}$$
$$\widehat{D}^T = (W \diamond A)^{\dagger} X_{IM_R \times NR} \quad (60)$$
$$\widehat{A}^T = (D \diamond W)^{\dagger} X_{NRI \times M_R}$$

As we perform right pseudo-inverses, the identifiability conditions $M_R NR \geq L$, $NRI \geq L$ and $IM_R \geq L$ should be satisfied.
For the second stage of the ALS – nested – PARAFAC,we can obtain essential unique estimations of the factor matrices if we have :

$$k_C + k_S + k_B \geq 2M_T + 2$$
$$\Leftrightarrow min(N, M_T) + min(R, M_T) + min(L, M_T) \geq 2M_T + 2 \quad (61)$$

And the identifiability conditions $NR \geq M_T$, $RL \geq M_T$ and $LN \geq M_T$ should be satisfied to have :

$$\widehat{B} = (C \diamond S)^{\dagger} D_{NR \times L}$$
$$\widehat{S}^T = (B^T \diamond C)^{\dagger} D_{LN \times R} \quad (62)$$

5.3 Proposed algorithm

The matrices B and A have the Vandermonde structure and we will simulate the coding matrix C with a Fourier matrix. As for S, its first row is filled with ones and the others are filled with symbols taken randomly from QAM alphabet. We define the matrix W by the repetition of I rows containing random weight. Then we collect the slices $X_{..i}$ through the third dimension to obtain $\widetilde{X}_{M_R NR \times I}$.

In the ALS – nested – PARAFAC, we perform $MC = 1000$ Monte Carlo runs, where we process with $F = 10$ initializations of the matrices $(\boldsymbol{B}, \boldsymbol{A}, \boldsymbol{W}, \boldsymbol{S})$ and we keep the one with the least error for each run, such that for the external ALS we calculate the error of reconstruction on the received signal unfolding :

$$err = \frac{\left\| \boldsymbol{X}_{M_R NR \times I} - (\widehat{\boldsymbol{A}} \diamond \widehat{\boldsymbol{D}})\widehat{\boldsymbol{W}}^T \right\|_F^2}{\left\| \boldsymbol{X}_{M_R NR \times I} \right\|_F^2} \quad (63)$$

Table 7. Algorithm for MIMO radar with convolutive channel processing through statistical data

1 : $\boldsymbol{R}_{IM_R \times IM_R} = \boldsymbol{X}_{IM_R \times N} \boldsymbol{X}_{IM_R \times N}{}^H$
2 : Define $\boldsymbol{Z}^T = \boldsymbol{C}_S \left(\boldsymbol{B} \diamond \boldsymbol{A}^{(R)}\right)^H$
3 : By the ALS algorithm, obtain $\widehat{\boldsymbol{Z}}$, $\widehat{\boldsymbol{B}}$ and $\widehat{\boldsymbol{A}}^{(R)}$ from the expressions of $\boldsymbol{R}_{IM_R \times IM_R}$, $\boldsymbol{R}_{M_R IM_R \times I}$ and $\boldsymbol{R}_{IM_R I \times M_R}$
4 : Obtain $\widehat{\boldsymbol{C}}_S = \widehat{\boldsymbol{Z}}^T \left[\left(\widehat{\boldsymbol{B}}^* \diamond \widehat{\boldsymbol{A}}^{(R)*}\right)^T\right]^\dagger$ by the least squares criteria
5 : By the resolution of the equation $\boldsymbol{C}_S = \boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^H$, obtain $\widehat{\boldsymbol{D}}$
6: Obtain $\widehat{\boldsymbol{D}}_f$ by the two conjugate expressions
7 : Obtain $\widehat{\boldsymbol{G}}$ via the relation $\boldsymbol{D} = \boldsymbol{W}^T$
8 : By the LS-KRP algorithm, obtain $\widehat{\boldsymbol{G}}$ and $\widehat{\boldsymbol{A}}^{(T)}$ from relation $\boldsymbol{W} = \boldsymbol{G} \diamond \boldsymbol{A}^{(T)}$

And for the internal ALS, we evaluate the error :

$$err = \frac{\left\| \boldsymbol{D}_{RN \times L} - (\widehat{\boldsymbol{C}} \diamond \widehat{\boldsymbol{S}})\widehat{\boldsymbol{B}} \right\|_F^2}{\left\| \boldsymbol{D}_{RN \times L} \right\|_F^2} \quad (64)$$

As for the convergence criteria during the iterations, the loop is broken when we find an error inferior or equal $\varepsilon = 10^{-6}$ or if the maximal number of iterations $T = 1000$ has been reached.
The complexity of an ALS algorithm lies mainly on the cost of the calculus of the SVD during the factor matrices estimations. The cost of the calculus of $\widehat{\boldsymbol{W}}^T$ is given by the cost of the pseudo-inverse $(\boldsymbol{A} \diamond \boldsymbol{D})^\dagger \in \mathbb{C}^{M_R NR \times L}$ which is equal to $\mathcal{O}(M_R NRL\, min\,(M_R NR; L)) = \mathcal{O}(M_R NRLL) = \mathcal{O}(M_R NRL^2)$. And the complexity of the ALS – nested – PARAFAC is given by : $\mathcal{O}\left(L^2(M_R NR + NRI + IM_R)\right)$ per itération, which corresponds to the complexity of the first stage, plus this of the second stage, which is of order : $\mathcal{O}\left(M_T{}^2(NR + RL + LN)\right)$ per itération.

6. Simulations and Results

6.1. Comparison with other methods

The results are the average of 1000 Monte Carlo simulations, in order to verify the efficiency of the proposed method. The simulation conditions are such as $L =$ 3 targets, between the uniform linear array (ULA) at the transmission and the reception, having respectively the followind DoA and DoD ayant $(\theta_1, \varphi_1) = (25°, 60°)$, $(\theta_2, \varphi_2) = (70°, 35°)$ and $(\theta_3, \varphi_3) = (-10°, -20°)$. The scatterer coefficients satisfy SwerlingI model and note that the bistatic MIMO radar is equipped by M_T transmit antennas and M_R receive antennas.
We admit the SNR (Signal to Noise Ratio) to be the rate between the signal power and the noise power. To evaluate the system performance, we perform the test for two criteria: the first one consits of the mean quadratic error on the estimation of the DoA and DoD, i.e. the RMSE (Root Mean Square Error) defined by the relation :

$$RMSE = \frac{1}{L}\sum_{l=1}^{L}\sqrt{\frac{1}{1000}\sum_{mc=1}^{1000}\left(\hat{\theta}_{mc,l} - \theta_l\right)^2 + \left(\hat{\varphi}_{mc,l} - \varphi_l\right)^2} \quad (65)$$

Where $\hat{\theta}_{mc,l}$ is the DoA estimation for the mc – th Monte Carlo simulation and the l – th target.
$\hat{\varphi}_{mc,l}$: is the DoD estimation for the mc – th Monte Carlo simulation and the l – th target.
In fact we have the relations :

$$\hat{\theta}_{mc,l} = arcsin(\hat{\boldsymbol{c}}_R^l) \quad (66)$$

$$\hat{\varphi}_{mc,l} = arcsin(\hat{\boldsymbol{c}}_T^l)$$

With

$$\hat{c}_R^l = \boldsymbol{P}_R^{\dagger}\hat{\boldsymbol{h}}_R^l \quad (67)$$

$$\hat{c}_T^l = \boldsymbol{P}_T^{\dagger}\hat{\boldsymbol{h}}_T^l$$

Where we define $\boldsymbol{P}_R$ and $\boldsymbol{P}_T$ respectively by :

$$\boldsymbol{P}_R = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & \pi & \cdots & (M_R - 1)\pi \end{bmatrix} \in \mathbb{C}^{M_R \times 2} \quad (68)$$

$$\boldsymbol{P}_T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & \pi & \cdots & (M_T - 1)\pi \end{bmatrix} \in \mathbb{C}^{M_T \times 2}$$

With $\boldsymbol{h}_R^l = -phase\{\boldsymbol{a}(\theta_l)\}$ and $\boldsymbol{h}_T^l = -phase\{\boldsymbol{b}(\varphi_l)\}$.

The second measurement guves the PSD (Probability of Successful Detection), where a good detection of the target is accepted if the absolute error of all of the estimated angles are less than $0.3°$.
We assume $M_T = 6$ and $M_R = 4$. To compare, we represent on **Fig. 2** the RMSE of classical angle estimation methods ESPRIT, MUSIC and HOSVD, but also those of the tensorial approaches PARAFAC, QALS with the result of our proposed algorithm. As expected, the tensor-based mothods have better performance in terms of direction estimation in contrast with classical methods for low SNR. This is essentially due to the tensor gain. An other observation is on the fact that the PARAFAC-based algorithm excels the other estmators since least squares strategies rely on the array degrees of freedom.
Fig. 3 illustrates the probability of successful detection curves. We can see that all of the methods provide a PSD of 100% for high SNR. When the le SNR decreases, the PSD of each approach begins to decrease en certain points, which is the SNR threshold. The PARAFAC-based estimators have SNR thresholds smaller than others. Note that the proposed method based on nested-PARAFAC model has the same performance as thePARAFAC algorithm in the presence of white Gaussian noise.

6.2. System performance

In this subsection we present the performances of our SB receiver (semi-blind) modeled with nested-PARAFAC decomposition where we use Fourier matrix to simulate the coding matrix $\boldsymbol{C}$. We assume a MIMO radar system with $M_T = 6$ and $M_R = 4$ with a 4-QAM modulation for the symbol matrix. The results are still the average of 1000 Monte Carlo simulations, where each simulation corresponds to an independent realisation of the channel, symbols and noise. **Fig. 4** presents the resluts for different configurations of the system :

- space-time diversity with matrix method (STM)

- space-time diversity with tensor method (STT)
- space-time with coding diversity with matrix method (STCM)
- space-time with coding diversity with tensor method (STCT)

On **Fig. 4**, we can notice that tensor-based methods with spatio-temporal and coding diversity reach better performance compared to matrix-based methods. Which justify the efficiency of tensor-based approach within multidimensional system. The more the SNR increases, the more the tensor-based algorithm improves.

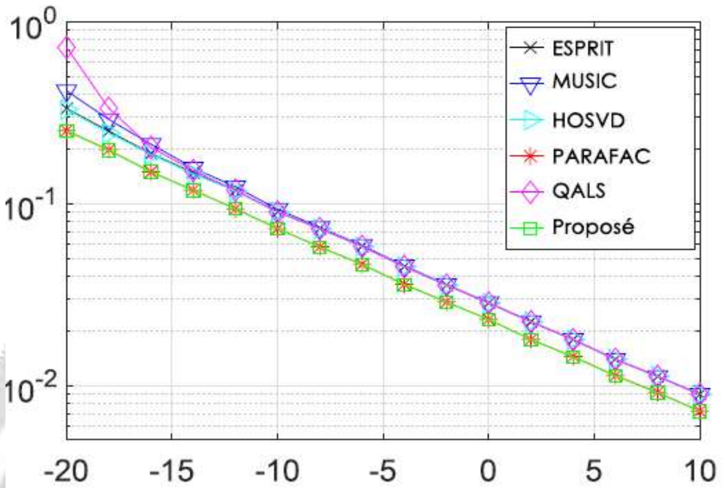

Fig. 2 RMSE vs SNR

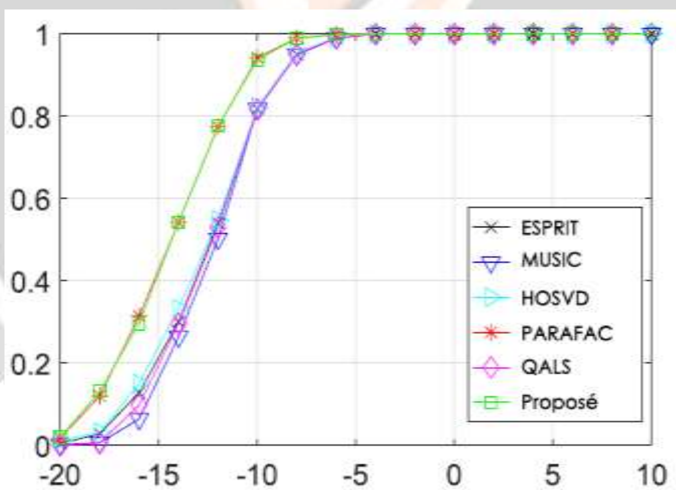

Fig. 3 PSD vs SNR

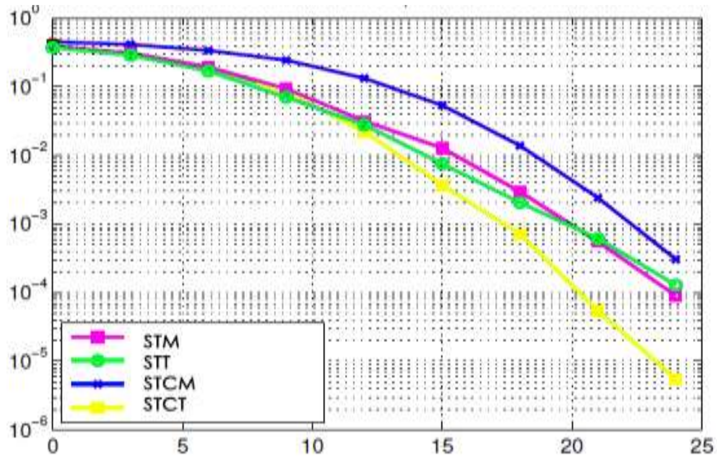

Fig. 4 SER vs SNR

6.3. Parameter design influence

Finally we evaluate the system performance under parameters influence.
First, spatial diversity is translated by the increas of number of elements at the transmit and/or receive sides. **Fig. 5** and **Fig. 6** sohw effectively that this diversity improves the system performance. Meanwhile, note that the reception diversity is more considerable than this at the transmission as the gad between the results on **Fig.5** are greater.
Second, the number of block transmissions allows us to introduce time diversity. The more we repeat the coded signals, the more the system has a better performance in terms of estimation of the transmitted information and the parameters of transmission.
On **Fig. 7**, SB indicates our semi-blind receiver and ZF indicates zero forcing in the case where the channe lis perfectly known. Note that the proposed approach in this work provides results close to the ZF receiver.

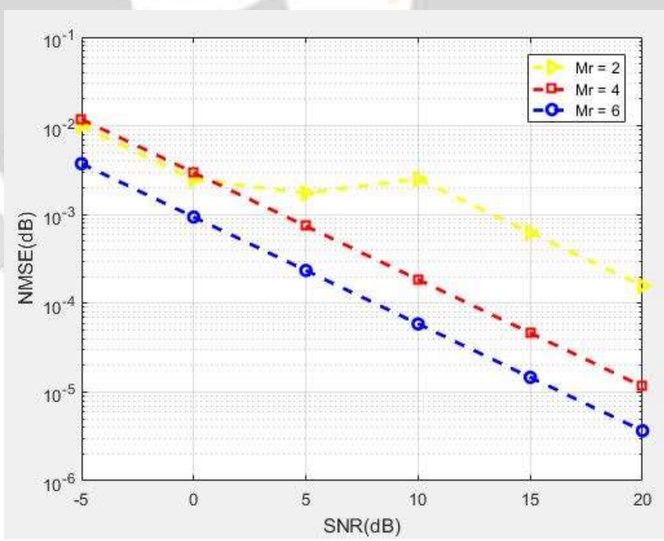

Fig. 5 NMSE vs SNR, Mr changing

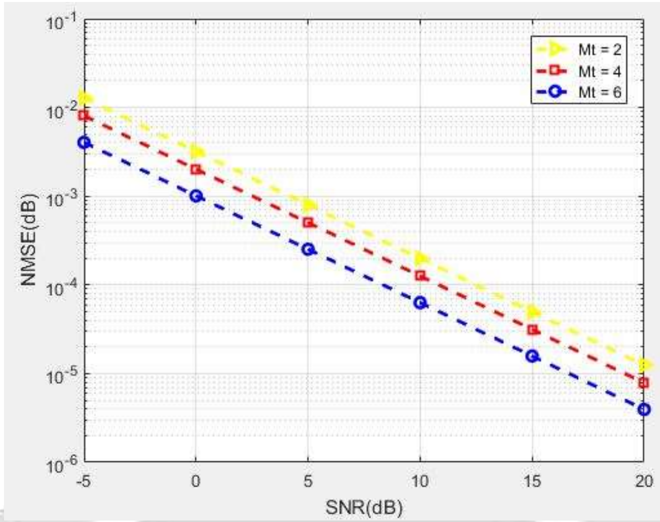

Fig. 6 NMSE vs SNR, Mt changing

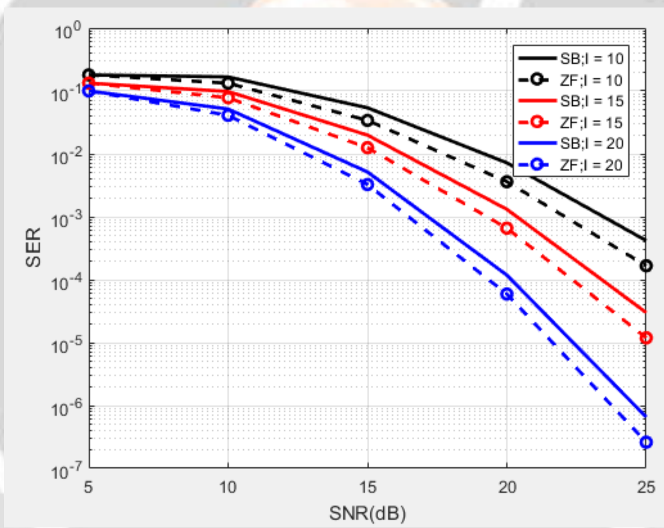

Fig. 7 SER vs SNR, I changing

7. Conclusion

To conclude, we have seen in this work the modelisation of the received signals in MIMO radar using instant and convolutive channel. We have exploited two kinds of data considering raw and statistical cases. We have proposed our semi-blind receiver based on the use of a known coding matrix at the transmitter. The nested-PARAFAC model of the received signal has led to the conception of our proposed algorithm based on a double ALS to estimate first the external parameters and then the internal ones. Simulations were divided into three categories: comparison with classical method for angle estimation, comparison between matrix-based approach and our semi-blind receiver, influence of parameter design. Numerical results has proved that tensor-based method has better performance compared with classical method; the semi-blind receiver is more efficient than the matrix-based methods; the increase of the number of elements at the transmitter and/or at the receiver with the variation of the number of transmission blocks affects the system performance. Space-time and coding diversity applied to the MIMO radar system with tensor approach allowed performance increasing.

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