Picture Fuzzy BG-Subalgebra in BG-algebra

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ABSTRACT

In this article, we introduce the notion of picture fuzzy BG-subalgebra in BG-algebra with an example. We have discussed some of their characteristics. We also showed that the intersection of two picture fuzzy BG-subalgebras is also a picture fuzzy BG-subalgebra.

Keyword: *BG-algebra, Fuzzy BG-Subalgebra, Picture Fuzzy set, Picture Fuzzy BG-Subalgebra.*

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1. Introduction

In 2014, Cuong [5] introduced the notion of picture fuzzy sets, which extended the ideas of fuzzy sets and intuitionistic fuzzy sets. This new perspective proves particularly useful in scenarios where people's attitudes involve a variety of response types, such as abstain, yes, no, and rejection. Picture fuzzy sets focus on evaluating the positive degree of membership, the neutral degree of membership, and the negative degree of membership. In particular, they represent a wider space than fuzzy sets and intuitionistic fuzzy sets. Kim and Kim proposed the concept of BG-algebras, which is a generalisation of B-algebras. In 2004, Ahn and Lee [3] introduced the notion of fuzzy subalgebra in the context of BG-algebras. In this study, we applied the picture fuzzy set concept to subalgebras of BG-algebra and introduced the concept of picture fuzzy BG-subalgebra with an example. We also discussed some of their properties.

2. Preliminaries

Definition 2.1 [1] A non-empty set $\mathcal{X}$ with a constant 0 and a binary operation $*$ is said to be BG-Algebra if it satisfies the following axioms

1. $x * x = 0$
2. $x * 0 = 0$
3. $(x * y) * (0 * y) = x$ for all $x, y \in \mathcal{X}$.

Definition 2.2 [1] A non-empty subset $\mathcal{I}$ of a BG-algebra $\mathcal{X}$ is called a subalgebra of $\mathcal{X}$ if $x * y \in \mathcal{I}$ for all $x, y \in \mathcal{I}$.

Definition 2.3 [2] Let $\mathcal{X}$ be the collection of objects. Then a fuzzy set $\mathcal{F}$ in $\mathcal{X}$ is defined as $\mathcal{F} = \{(x, \alpha(x)) \mid x \in \mathcal{X}\}$,

where $\alpha(x)$ is called the membership degree of x in $\mathcal{F}$ and $0 \leq \alpha(x) \leq 1.$

Definition 2.5 [3] A fuzzy set $\mathcal{F}$ is said to be a fuzzy subalgebra of $\mathcal{X}$ if $\alpha(x) \geq min\{\alpha(x * y), \alpha(y)\}$ for all $x, y \in \mathcal{X}$.

Definition 2.5 [4] An Intuitionistic fuzzy set $\mathfrak{B}$ in a non-empty set $\mathcal{X}$ is an object having the form $\mathfrak{B} = \{(x, \alpha(x), \beta(x)) | x \in \mathcal{X}\}$ where $\alpha(x), \beta(x)$ are degree of belongingness and degree of non-belongingness of $x \in \mathcal{X}$ respectively and $0 \leq \alpha(x) + \beta(x) \leq 1$ for all $x \in \mathcal{X}$.

Definition 2.6 [5] Let $\mathcal{X}$ be a non-empty and finite set. A Picture fuzzy set in $\mathcal{X}$ is defined by

$$\mathcal{N} = \{\langle x, \alpha(x), \beta(x), \gamma(x)\rangle | x \in \mathcal{X}\}$$

Where $\alpha: \mathcal{X} \to [0,1], \beta: \mathcal{X} \to [0,1]$ and $\gamma: \mathcal{X} \to [0,1]$ positive, neutral, and negative membership functions respectively, and $0 \leq \alpha(x) + \beta(x) + \gamma(x) \leq 1$. Furthermore, $H(x) = 1 - \alpha(x) - \beta(x) - \gamma(x)$ is the refusal membership function.

3. Picture Fuzzy BG-Subalgebra

Definition 3.1. A Picture fuzzy set $\mathcal{N} = (\alpha, \beta, \gamma)$ in $\mathcal{X}$ is called a Picture Fuzzy BG-Subalgebra if it satisfies the following conditions

(PFBG-SA 1) $\alpha(x * y) \geq min\{\alpha(x), \alpha(y)\}$

(PFBG-SA 2) $\beta(x * y) \geq min\{\beta(x), \beta(y)\}$

(PFBG-SA 3) $\gamma(x * y) \leq max\{\gamma(x), \gamma(y)\}$ for all $x, y \in \mathcal{X}$.

Definition 3.2. Consider a set $\mathcal{X} = \{0,1,2,3,4,5\}$ with the binary operation '$*$' which is given in table.1

Table -1 BG-algebra

$*$	0	1	2	3	4	5
0	0	5	4	3	2	1
1	1	0	5	4	3	2
2	2	1	0	5	4	3
3	3	2	1	0	5	4
4	4	3	2	1	0	5

Then $(\mathcal{X}; *, 0)$ is a BG-algebra. Let $\mathcal{N} = (\alpha, \beta, \gamma)$ be a Picture fuzzy set in $\mathcal{X}$ defined by table.2

$\mathcal{X}$	0	1	2	3	4	5
$\alpha(x)$	0.51	0.50	0.51	0.50	0.51	0.50
$\beta(x)$	0.39	0.30	0.39	0.30	0.39	0.30
$\gamma(x)$	0.10	0.20	0.10	0.20	0.10	0.20

Table:2

It is routine to verify that $\mathcal{N} = (\alpha, \beta, \gamma)$ is a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$.

Proposition 3.3. If $\mathcal{N} = (\alpha, \beta, \gamma)$ in $\mathcal{X}$ is a Picture Fuzzy BG-Subalgebra in $\mathcal{X}$, then for all $x \in \mathcal{X}$ $\alpha(0) \geq \alpha(x), \beta(0) \geq \beta(x)$ and $\gamma(0) \leq \gamma(x)$.

Proof: Let $x \in \mathcal{X}$. Then

$$\alpha(0) = \alpha(x * x) \geq min\{\alpha(x), \alpha(x)\} = \alpha(x) \Rightarrow \alpha(0) \geq \alpha(x),$$

$$\beta(0) = \beta(x * x) \geq min\{\beta(x), \beta(x)\} = \beta(x) \Rightarrow \beta(0) \geq \beta(x),$$

$$\gamma(0) = \gamma(x * x) \leq max\{\gamma(x), \gamma(x)\} = \gamma(x) \Rightarrow \gamma(0) \leq \gamma(x).$$

Theorem 3.4. Let $\mathcal{N} = (\alpha, \beta, \gamma)$ be a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$. If there exists a sequence $\{x_n\}$ in $\mathcal{X}$ such that $\lim_{n\to\infty} \alpha(x_n) = 1$, $\lim_{n\to\infty} \beta(x_n) = 1$ and $\lim_{n\to\infty} \gamma(x_n) = 0$, then $\alpha(0) = 1$, $\beta(0) = 1$ and $\gamma(0) = 0$.

Proof: Using the proposition 3.3, we know that $\alpha(0) \geq \alpha(x_n)$ $\beta(0) \geq \beta(x_n)$ and $\gamma(0) \leq \gamma(x_n)$ for every positive integer n. Note that

$$1 \geq \alpha(0) \geq \lim_{n\to\infty} \alpha(x_n) = 1$$

$$1 \geq \beta(0) \geq \lim_{n\to\infty} \beta(x_n) = 1$$

$$0 \leq \gamma(0) \leq \lim_{n\to\infty} \gamma(x_n) = 0$$

Therefore $\alpha(0) = 1, \beta(0) = 1$ and $\gamma(0) = 0$.

Proposition 3.5. If a Picture fuzzy set $\mathcal{N} = (\alpha, \beta, \gamma)$ in $\mathcal{X}$ be a Picture Fuzzy BG-Subalgebra, then for all $x \in \mathcal{X}$ $\alpha(0 * x) \geq \alpha(x), \beta(0 * x) \geq \beta(x)$ and $\gamma(0 * x) \leq \gamma(x)$.

Proof: For all $x \in \mathcal{X}$, we have

$$\alpha(0 * x) \geq min\{\alpha(0), \alpha(x)\} = min\{\alpha(x * x), \alpha(x)\} \geq$$
$$min\{min\{\alpha(x), \alpha(x)\}, \alpha(x)\} = \alpha(x) \Rightarrow \alpha(0 * x) \geq \alpha(x)$$

$$\beta(0 * x) \geq min\{\beta(0), \beta(x)\} = min\{\beta(x * x), \beta(x)\} \geq$$
$$min\{min\{\beta(x), \beta(x)\}, \beta(x)\} = \beta(x) \Rightarrow \beta(0 * x) \geq \beta(x)$$

$$\gamma(0 * x) \leq max\{\gamma(0), \gamma(x)\} = max\{\gamma(x * x), \gamma(x)\} \leq$$
$$max\{max\{\gamma(x), \gamma(x)\}, \gamma(x)\} = \gamma(x) \Rightarrow \gamma(0 * x) \leq \gamma(x)$$

Definition 3.6.[5] Let $\mathcal{N}_1 = (\alpha_1, \beta_1, \gamma_1)$ *and* $\mathcal{N}_2 = (\alpha_2, \beta_2, \gamma_2)$ be two Picture Fuzzy Sets, then the intersection is defined as

$$\mathcal{N}_1 \cap \mathcal{N}_2 =$$
$$\left\{\left(x, min\left(\alpha_1(x), \alpha_2(x)\right), min\left(\beta_1(x), \beta_2(x)\right), max\left(\gamma_1(x), \gamma_2(x)\right)\right) : x \in \mathcal{X}\right\}$$

Theorem 3.7. Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be two Picture Fuzzy BG-Subalgebras of $\mathcal{X}$, then $\mathcal{N}_1 \cap \mathcal{N}_2$ is a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$.

Proof: Let $x, y \in \mathcal{N}_1 \cap \mathcal{N}_2$, then $x, y \in \mathcal{N}_1$ and $x, y \in \mathcal{N}_2$.

$$\alpha_{\mathcal{N}_1 \cap \mathcal{N}_2}(x * y) = min\{\alpha_{\mathcal{N}_1}(x * y), \alpha_{\mathcal{N}_2}(x * y)\}$$
$$\geq min\left\{min\{\alpha_{\mathcal{N}_1}(x), \alpha_{\mathcal{N}_1}(y)\}, min\{\alpha_{\mathcal{N}_2}(x), \alpha_{\mathcal{N}_2}(y)\}\right\}$$
$$= min\left\{min\{\alpha_{\mathcal{N}_1}(x), \alpha_{\mathcal{N}_2}(x)\}, min\{\alpha_{\mathcal{N}_1}(y), \alpha_{\mathcal{N}_2}(y)\}\right\}$$
$$= min\{\alpha_{\mathcal{N}_1 \cap \mathcal{N}_2}(x), \alpha_{\mathcal{N}_1 \cap \mathcal{N}_2}(y)\}$$

$$\beta_{\mathcal{N}_1 \cap \mathcal{N}_2}(x * y) = min\{\beta_{\mathcal{N}_1}(x * y), \beta_{\mathcal{N}_2}(x * y)\}$$
$$\geq min\left\{min\{\beta_{\mathcal{N}_1}(x), \beta_{\mathcal{N}_1}(y)\}, min\{\beta_{\mathcal{N}_2}(x), \beta_{\mathcal{N}_2}(y)\}\right\}$$
$$= min\left\{min\{\beta_{\mathcal{N}_1}(x), \beta_{\mathcal{N}_2}(x)\}, min\{\beta_{\mathcal{N}_1}(y), \beta_{\mathcal{N}_2}(y)\}\right\}$$

$$= min\{\beta_{\mathcal{N}_1 \cap \mathcal{N}_2}(x), \beta_{\mathcal{N}_1 \cap \mathcal{N}_2}(y)\},$$

$$\gamma_{\mathcal{N}_1 \cap \mathcal{N}_2}(x * y) = max\{\gamma_{\mathcal{N}_1}(x * y), \gamma_{\mathcal{N}_2}(x * y)\}$$

$$\geq max\left\{max\{\gamma_{\mathcal{N}_1}(x), \gamma_{\mathcal{N}_1}(y)\}, max\{\gamma_{\mathcal{N}_2}(x), \gamma_{\mathcal{N}_2}(y)\}\right\}$$

$$= max\left\{max\{\gamma_{\mathcal{N}_1}(x), \gamma_{\mathcal{N}_2}(x)\}, max\{\gamma_{\mathcal{N}_1}(y), \gamma_{\mathcal{N}_2}(y)\}\right\}$$

$$= max\{\gamma_{\mathcal{N}_1 \cap \mathcal{N}_2}(x), \gamma_{\mathcal{N}_1 \cap \mathcal{N}_2}(y)\}.$$

Hence $\mathcal{N}_1 \cap \mathcal{N}_2$ is a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$. Theorem 3.7. can be generalizes as follows.

Theorem 3.8. Let $\{\mathcal{N}_i : i = 1,2,3 \ldots \ldots\}$ be a family of Picture Fuzzy BG-Subalgebra of $\mathcal{X}$. Then $\bigcap \mathcal{N}_i$ is also a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$.

Theorem 3.9. A Picture Fuzzy Set $\mathcal{N} = (\alpha, \beta, \gamma)$ is a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$ if and only if the fuzzy sets α, β and γ^c are fuzzy subalgebra of $\mathcal{X}$.

Proof: Let $\mathcal{N} = (\alpha, \beta, \gamma)$ be a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$ then we have $\alpha(x * y) \geq rmin\{\alpha(x), \alpha(y)\}, \beta(x * y) \geq min\{\beta(x), \beta(y)\}$ and $\gamma(x * y) \leq max\{\gamma(x), \gamma(y)\}$ for all $x, y \in \mathcal{X}$. Clearly α, β are fuzzy subalgebra of $\mathcal{X}$.Now $1 - \gamma(x * y) \geq 1 - max\{\gamma(x), \gamma(y)\} \Rightarrow \gamma^c(x * y) \geq min\{1 - \gamma(x), 1 - \gamma(y)\} = min\{\gamma^c(x), \gamma^c(y)\}$
. Hence γ^c is a fuzzy subalgebra of $\mathcal{X}$.

Conversely, assume that α, β and γ^c are fuzzy subalgebra of $\mathcal{X}$. For every $x, y \in \mathcal{X}$ we have $\alpha(x * y) \geq rmin\{\alpha(x), \alpha(y)\}, \beta(x * y) \geq min\{\beta(x), \beta(y)\}$ and $\gamma^c(x * y) \geq min\{\gamma^c(x), \gamma^c(y)\} \Rightarrow 1 - \gamma^c(x * y) \leq 1 - min\{\gamma^c(x), \gamma^c(y)\} = max\{1 - \gamma^c(x), 1 - \gamma^c(y)\} \Rightarrow \gamma(x * y) \leq max\{\gamma(x), \gamma(y)\}$
. Hence $\mathcal{N} = (\alpha, \beta, \gamma)$ is a Picture Fuzzy BG-Subalgebra of $\mathcal{X}$.

4. CONCLUSIONS

In this manuscript we introduced the concept of Picture Fuzzy BG-Subalgebra in BG-algebra, along with examples. We analysed some key characteristics of this concept. Additionally, we proved that the intersection of two Picture Fuzzy BG-Subalgebras results in another Picture Fuzzy BG-Subalgebra.

5. REFERENCES

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