Probabilistic inventory model based Hazardous Substance Storage for decaying Items and Inflation using Particle Swarm Optimization

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Abstract
Hazardous Substance Storage inventory model is developed for decaying items with ramp type demand and the effects of inflation using Particle Swarm Optimization. The Hazardous Substance Storage has unlimited capacity. Here, we assumed that the inventory holding cost in Hazardous Substance Storage is higher. Shortages in inventory are allowed and partially backlogged and it is assumed that the inventory deteriorates over time at a variable deterioration rate. Cost minimization technique is used to get the expressions for total cost using Particle Swarm Optimization and numerical example is also used to study the behavior of the model.

Keywords: - Inventory model, Deterioration rate, Ramp type demand, Inflation, PSO algorithm & Probabilistic tools.

1. Introduction
In the current market, the explosion of choice due to fierce competition means that no company can resist inventory because there are a variety of alternative products with additional features. In addition, there is no dry cut recipe, with which the need can be determined exactly. Therefore, when an enterprise requires an inventory, it must be preserved so that the physical attributes of inventory elements can be preserved and protected. How an organization stores its stocks depends critically on its ability to achieve the ultimate goal of inventory management. An organization collects the same stocks in different ways to achieve different results, even generating profits and losses. Materials storage ultimately turns out to be the core of the overall inventory management practice. Hazardous Substance storage containers are stores for the storage and storage of goods and the provision of other related services to encourage distributors and / or manufacturers to maintain products in a scientific and systematic manner, in order to that they regain their original value, quality and utility preserve. It is an integral part of an industrial unit. It acts as the custodian of all materials required by the industrial unit and supplier materials as needed. Sometimes the total requirement for an item is such that the supplier is more likely to buy than he can store in his Hazardous Substance store. You could have been influenced by the offer of a reduced price for the stock if you bought at least a certain amount, Or you can expect a season of strong sales and you want to be prepared in advance so you do not lose the chance to make big profits. Another may be an impending strike, subcontracting or lockout that may threaten a recovery period. He may want to prepare for this period by buying more than he can actually store in his own Hazardous Substance store. In busy markets such as supermarkets, community markets, etc., storage space for items is limited. If an attractive discount is available for bulk purchases, or if the purchase cost of the goods is greater than other costs related to inventory or demand for very large items or if there are problems with frequent purchases, management decides to buy a lot of items. Save time. These items
cannot be stored in the existing warehouse in the booming market. In this case, an additional storage of Hazardous Substances is rented on a rental basis for the storage of surplus items.

The storage of Hazardous Substances is the storage of controlled Hazardous Substances or hazardous substances in Hazardous Substance storage facilities, Hazardous Substance storage cabinets or similar equipment. Improper storage of Hazardous Substances can compromise workplace safety, including heat, fire, explosions and leaking toxic gases. Hazardous Substance storage cabinets are typically used to safely store small amounts of Hazardous Substances at a workplace or laboratory for regular use. These cabinets are usually made of Hazardous Substance resistant materials that are stored in them and sometimes contain a packed tray to recover spilled material. Hazardous Substance stores are warehouses commonly used by Hazardous Substance or pharmaceutical companies for the storage of bulk Hazardous Substances. In the United States, the storage and handling of potentially hazardous products must be disclosed to occupants in accordance with the applicable Occupational Safety and Health Administration (OSHA) legislation. Hazardous Substance storage devices are typically found in workplaces that require the use of non-hazardous and / or hazardous Substances. Proper storage is essential for the safety and access of laboratory technicians.

2. Particle Swarm Optimization
The PSO algorithm is based on the social behavior of birds. This algorithm first creates a random population. Each individual called particle is given a speed and a small social network. For all particles, the values of the fitness or objective function are evaluated. On the basis of physical condition in relation to GA, the PSO has no cross / mutation, but the personal optimum for each individual, the overall optimum in the total population and the neighborhood optimum found by the neighbors of each individual are stored for speed and position update each. This process is repeated until the maximum generations or convergence is reached.

3. Literature Review

4. Assumptions and Notations

In developing the mathematical model of the inventory system the following assumptions are being made:

1. A single item is considered over a prescribed period T units of time.
2. The demand rate $D(t)$ at time $t$ is deterministic and taken as a ramp type function of time i.e.
   $$D(t) = u_0 e^{(\theta + 1)(t - \theta)}$$
   $$u_0 > 0, \theta > 0$$
3. The replenishment rate is infinite and lead-time is zero.
4. When the demand for goods is more than the supply. Shortages will occur. Customers encountering shortages will either wait for the vender to reorder (backlogging cost involved) or go to other vendors (lost sales cost involved). In this model shortages are allowed and the backlogging rate is exp $[-(\delta + 1)t]$, when inventory is in shortage. The backlogging parameter $(\delta + 1)$ is a positive constant.
5. The variable rate of deterioration in Hazardous Substance Storage is taken as $(\theta + 1) = (\theta + 1)t$.
6. No replacement or repair of deteriorated items is made during a given cycle.
7. The Hazardous Substance Storage has unlimited capacity.

In addition, the following notations are used throughout this paper:

$I_{HSS}(t)$ The inventory level in Hazardous Substance Storage at any time $t$.
$T$ Planning horizon.
$(r_0 + 1)$ Inflation rate.
$A_{HC}$ The holding cost per unit per unit time in Hazardous Substance Storage.
$A_{DC}$ The deterioration cost per unit.
5. Formulation and Solution of The Model

The inventory level at Hazardous Substance Storage is governed by the following differential equations:

\[
\frac{dI(t)}{dt} + (\theta + 1)I(t) = -u_0e^{-(\lambda_0 + 1)t}, \quad 0 \leq t < t_1
\]

(1)

With the boundary condition \(I(0) = 0\), the solution of the equation (1) is

\[
I(t) = u_0 \left( (t_1 - t) \left( \frac{\lambda_0 - 1}{2} \right) (t_1^2 - t^2) + \frac{\theta + 1}{6} (t_1^3 - t^3) \right) e^{-(\theta + 1)t^2/2}, \quad t_1 \leq t \leq t_2
\]

(2)

The total average cost consists of following elements:

1. Ordering cost per cycle in Hazardous Substance Storage

\[
CS_{oc} = A_{oc}
\]

(3)

2. Holding cost per cycle in Hazardous Substance Storage

\[
CS_{HC} = A_{HC} \left[ \int_0^{t_1} I(t) e^{-(\theta_0 + 1)t} dt \right]
\]

(4)

3. Cost of deteriorated units per cycle in Hazardous Substance Storage

\[
HSS_{DC} = A_{DC} \left[ \int_0^{t_1} (\theta + 1)I(t) dt + \int_{t_1}^{t_2} (\theta + 1)I(t) dt \right]
\]

(5)

4. Shortage cost per cycle in Hazardous Substance Storage

\[
HSS_{SC} = A_{SC} \left[ \int_{t_2}^{T} -I(t) e^{-(\theta_0 + 1)t} dt \right]
\]

(6)

5. Opportunity cost due to lost sales per cycle in Hazardous Substance Storage

\[
HSS_{LS} = A_{LS} \left[ \int_{t_2}^{T} u_0(1 - e^{-(\delta_0 + 1)u}) dt \right]
\]

(7)

Therefore, the total average cost per unit time of our model is obtained as follows

\[
HSSTC(t_1, T) = \frac{1}{T} \left[ CS_{oc} + CS_{HC} + HSS_{DC} + HSS_{SC} + HSS_{LS} \right]
\]

(8)

To minimize the total cost per unit time, the optimal values of \(t_1\) and \(T\) can be obtained by solving the following equations simultaneously.

Therefore, numerical solution of these equations is obtained by using the software MATLAB 7.0.1.
6. Continuous Random Variable and Probability Density Function

Continuous Random Variable

Definition:- A random variable X with \( F_X(.) \) as Continuous Random Variable is called Continuous if there exists a function \( f_X(.) : \mathbb{R} \rightarrow [0, \infty) \) such that

\[
F_X(x) = \int_{-\infty}^{x} f_X(t) \, dt \quad \text{for all} \quad x \in \mathbb{R}
\]

The function \( f_X(.) \) is called the Probability density function of X

Probability Density Function

Definition:- Any function \( f_X(.) : \mathbb{R} \rightarrow [0, \infty) \) is said to be a Probability Density Function if

(i) \( f(x) \geq 0 \) for all \( x \in \mathbb{R} \)

\[
\int_{-\infty}^{\infty} f(x) \, dx = 1
\]

Theorem:—If X is a continuous random variable with Probability Density Function \( f_X(x) \) Show that

\[
E(X) = \int_{0}^{\infty} [1 - F_X(x)] \, dx - \int_{-\infty}^{0} F_X(x) \, dx
\]

Proof:—By definition we have

\[
E(X) = \int_{-\infty}^{\infty} xf_X(x) \, dx
\]

\[
E(X) = \int_{0}^{\infty} xf_X(x) \, dx + \int_{-\infty}^{0} xf_X(x) \, dx
\]

(12)

\[
E(X) = \int_{0}^{\infty} [1 - F_X(x)] \, dx - \int_{-\infty}^{0} F_X(x) \, dx
\]

\[
F_X(x) = P(X \leq x)
\]

and

\[
1 - F_X(x) = P(X > x)
\]

\[
\int_{0}^{\infty} [1 - F_X(x)] \, dx = \int_{0}^{\infty} P(X > x) \, dx
\]

\[
\int_{0}^{\infty} [1 - F_X(x)] \, dx = \int_{0}^{\infty} \int_{x}^{\infty} f_X(y) \, dy \, dx
\]

\[
\int_{0}^{\infty} [1 - F_X(x)] \, dx = \int_{0}^{\infty} f_X(y) \left[ \int_{0}^{y} dx \right] \, dy
\]

By change of order of integration in the region A

\[
\int_{0}^{\infty} [1 - F_X(x)] \, dx = \int_{0}^{\infty} xf_X(y) \, dy
\]
\[
\int_0^\infty [1 - F_X(x)] dx = \int_0^\infty xf_X(x) dx
\]

(13)

Consider
\[
\int_0^\infty [F_X(x)] dx = \int P(X \leq x) dx
\]
\[
\int_0^\infty [F_X(x)] dx = \int \int f_X(y) dy dx
\]
\[
\int_0^\infty [F_X(x)] dx = \int f_X(y) \left[ \int_0^x dx \right] dy
\]

By change of order of integration in the region B
\[
\int_0^\infty [F_X(x)] dx = - \int_0^\infty yf_X(y) dy
\]
\[
\int_0^\infty [F_X(x)] dx = - \int_0^\infty xf_X(x) dx
\]
\[
- \int_0^\infty [F_X(x)] dx = \int_0^\infty xf_X(x) dx
\]

(14)

From (12), (13), (14); we obtain
\[
E(X) = \int_0^\infty [1 - F_X(x)] dx - \int_{-\infty}^0 F_X(x) dx
\]

Theorem: If X is a continuous random variable with Probability Density Function \( f_X(x) \) Show that
\[
\text{var}[X] = \int_0^\infty 2x[1 - F_X(x) + F_X(-x)] dx - \mu_X^2
\]

Solution: By definition
\[
\text{var}[X] = E[X^2] - \{E[X]\}^2
\]
\[
\text{var}[X] = E[X^2] - \mu_X^2
\]

(15)

\[
E[X^2] = \int_{-\infty}^\infty x^2 f_X(x) dx
\]

(16)

\[
\int_0^\infty 2x[1 - F_X(x)] dx = \int_0^\infty 2xP(X > x) dx
\]
\[
\int_0^\infty 2x[1 - F_X(x)] dx = \int_0^\infty 2x \left[ \int_0^\infty f_X(y) dy \right] dx
\]
\[
\int_0^\infty 2x[1 - F_X(x)] dx = \int_0^\infty f_X(y) \left[ \int_0^y 2x dx \right] dy
\]

By change of order of integration
\[
\int_0^\infty 2x[1 - F_X(x)] dx = \int_0^\infty y^2 f_X(y) dy
\]

(17)
Similarly
\[ \int_{0}^{\infty} 2x \left[ 1 - F_x(x) \right] dx = \int_{0}^{\infty} x^2 f_x(x) dx \]
\[ \int_{0}^{\infty} 2x \left[ F_x(-x) \right] dx = \int_{0}^{\infty} 2x \left[ \int_{-\infty}^{x} f_x(y) dy \right] dx \]
\[ \int_{0}^{\infty} 2x \left[ F_x(-x) \right] dx = \int_{0}^{\infty} \left[ \int_{-\infty}^{y} f_x(y) \right] \left[ \int_{0}^{y} 2x dx \right] dy \]

By change of order of integration
\[ \int_{0}^{\infty} 2x \left[ F_x(-x) \right] dx = \int_{0}^{\infty} y^2 f_x(y) dy \]
\[ \int_{0}^{\infty} 2x \left[ 1 - F_x(x) \right] dx = \int_{0}^{\infty} x^2 f_x(x) dx \] (18)

From (17) and (18) we get
\[ \int_{0}^{\infty} 2x \left[ 1 - F_x(x) \right] dx + \int_{0}^{\infty} 2x \left[ F_x(-x) \right] dx = \int_{0}^{\infty} x^2 f_x(x) dx + \int_{-\infty}^{0} x^2 f_x(x) dx \]
\[ \int_{0}^{\infty} 2x \left[ 1 - F_x(x) \right] + \left[ F_x(-x) \right] dx = \int_{-\infty}^{\infty} x^2 f_x(x) dx \]
\[ \int_{0}^{\infty} 2x \left[ 1 - F_x(x) \right] + \left[ F_x(-x) \right] dx = E \left[ X^2 \right] \]

Putting in (15) we get
\[ \text{var} \left[ X \right] = \int_{0}^{\infty} 2x \left[ 1 - F_x(x) + F_x(-x) \right] dx - \mu_x^2 \]

7. Numerical Illustration

To illustrate the model numerically the following parameter values are considered.
\[ u_0 = 100 \text{ units}, \quad A_{OC} = \text{Rs. 200 per order}, \quad r_0 = 1.05 \text{ unit}, \quad \lambda_0 = 0.4 \text{ unit}, \quad A_{HC} = \text{Rs. 20.0 per unit}, \quad \theta = 0.004 \text{ unit}, \quad t_i = 0.4 \text{ year}, \quad A_{LS} = \text{Rs. 8.0 per unit}, \quad \delta_0 = 0.2 \text{ unit}, \quad T = 1 \text{ year}, \]

Then for the minimization of total average cost and with help of software, the optimal policy can be obtained such as:
\[ t_i = 1.993344 \text{ year}, \quad S = 76.9877225 \text{ units} \quad \text{and C.S.T.C.} = \text{Rs.316.22608 per year}. \]

Comparison of the optimization methods PSO to enhance Hazardous Substance Storage Inventory in presence of FACTS device is presented in this section. The control parameter values for all the optimization algorithms are given below
- PSO: population=60, generations=600, cognitive learning factor=4, cooperative factor=4, social learning factor=1.0, inertial constant=1.0 and number of neighbors=10.

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<th>AVG</th>
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</table>
8. Conclusion
This study incorporates some realistic features that are likely to be associated with the Hazardous Substance Storage inventory of any material using Particle Swarm Optimization. Decay (deterioration) overtime for any material product and occurrence of shortages in inventory are natural phenomenon in real situations using Particle Swarm Optimization. Hazardous Substance Storage inventory shortages are allowed in the model using Particle Swarm Optimization. In many cases customers are conditioned to a shipping delay, and may be willing to wait for a short time in order to get their first choice using Particle Swarm Optimization. Generally speaking, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not using Particle Swarm Optimization. The willingness of a customer to wait for backlogging during a shortage period declines with the length of the waiting time using Particle Swarm Optimization. Thus, Hazardous Substance Storage inventory shortages are allowed and partially backordered in the present chapter and the backlogging rate is considered as a decreasing function of the waiting time for the next replenishment using Particle Swarm Optimization. Demand rate is taken as exponential ramp type function of time, in which demand decreases exponentially for the some initial period and becomes steady later on using Particle Swarm Optimization. Since most decision makers think that the inflation does not have significant influence on the inventory policy, the effects of inflation are not considered in some inventory models. However, from a financial point of view, an inventory represents a capital investment and must complete with other assets for a firm’s limited capital funds using Particle Swarm Optimization. Thus, it is necessary to consider the effects of inflation on the inventory system. Therefore, this concept is also taken in this model. From the numerical illustration of the model, it is observed that the period in which inventory holds increases with the increment in backlogging and ramp parameters while inventory period decreases with the increment in deterioration and inflation parameters. Initial inventory level decreases with the increment in deterioration, inflation and ramp parameters while inventory level increases with the increment in backlogging parameter using Particle Swarm Optimization. The total average cost of the system goes on increasing with the increment in the backlogging and deterioration parameters while it decreases with the increment in inflation and ramp parameters. The proposed model can be further extended in several ways. For example, we could extend this deterministic model into stochastic model. Also, we could extend the model to incorporate some more realistic features, such as quantity discount or the unit purchase cost, the inventory holding cost and others can also taken fluctuating with time using Particle Swarm Optimization.

References


