REVIEW OF NUMERICAL APPROACHES

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ABSTRACT

The various formulations existent for the finite element analysis differ in the reference coordinates used to describe the motion and the governing equations. The Lagrangian method uses material coordinates (also known as Lagrangian coordinates) as the reference; these coordinates are generally denoted as X. The ALE formulation is a combination of the Lagrangian and Eulerian methods. In this method the reference coordinate is arbitrary and is generally presented as χ. The SPH is a meshless or griddles technique that does not suffer from the normal problems of grid tangling in large deformation problems.

Keyword: - *Lagrangian, ALE, SPH*

1. LAGRANGIAN APPROACH

In the Lagrangian formulation, the mesh nodes are attached to the particles of the material. Therefore, each node represents one particle of the material under examination. The reference coordinate system in Lagrangian formulation is the material coordinate X. The motion of each particle in this formulation is described by

$$X=\varphi(X,t)$$

Where φ(X, t) is the function mapping the initial position of the material to its current position. In this method, since the mesh nodes follow the material deformation, and therefore the boundary nodes remain on the external surfaces of the material, imposing the boundary conditions is simple. The other advantage associated with the Lagrangian method is the simplistic traceability of each material point. However, as described previously, using Lagrangian formulation for materials under very excessive deformations can lead to large distortions of the elements, which in turn increases the computational time. For having accurate results, the time step Δt must be at least smaller than the time required for a shock wave to travel through the smallest dimension of the element, lmin:

$$\Delta t=l\min / C$$

Where C is the speed of the sonic waves in the material. As a result of the excessive distortion in an element, while one of the dimensions of the element increases greatly, its other dimension (lmin) decreases to an unacceptable low value leading to very small time steps (Δt). The other problem that can arise from excessive element distortion is that the volume of some elements becomes non-physically negative, because they fold in on themselves. In the element removing technique, a critical failure strain is defined for the elements. The elements in which the effective strain exceeds the critical strain are removed. This formulation is used mostly to describe solid materials. The imposition of boundary conditions is simplified since the boundary nodes re-main on the material boundary. Another advantage of the Lagrangian method is the ability to easily track history dependant materials. The main disadvantage of the Lagrangian method is the possibility of inaccurate results and the need of remeshing due to mesh deformations. Since in this method the material moves with the mesh, if the material suffers large

deformations as observed in Figure 2, the mesh will also suffer equal deformation and this could leads to inaccurate results.

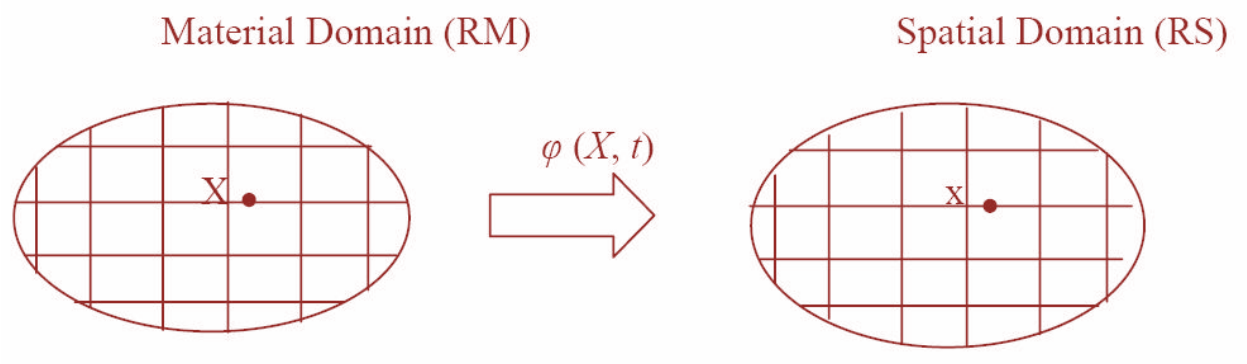

Figure 1 Description of motion formulation

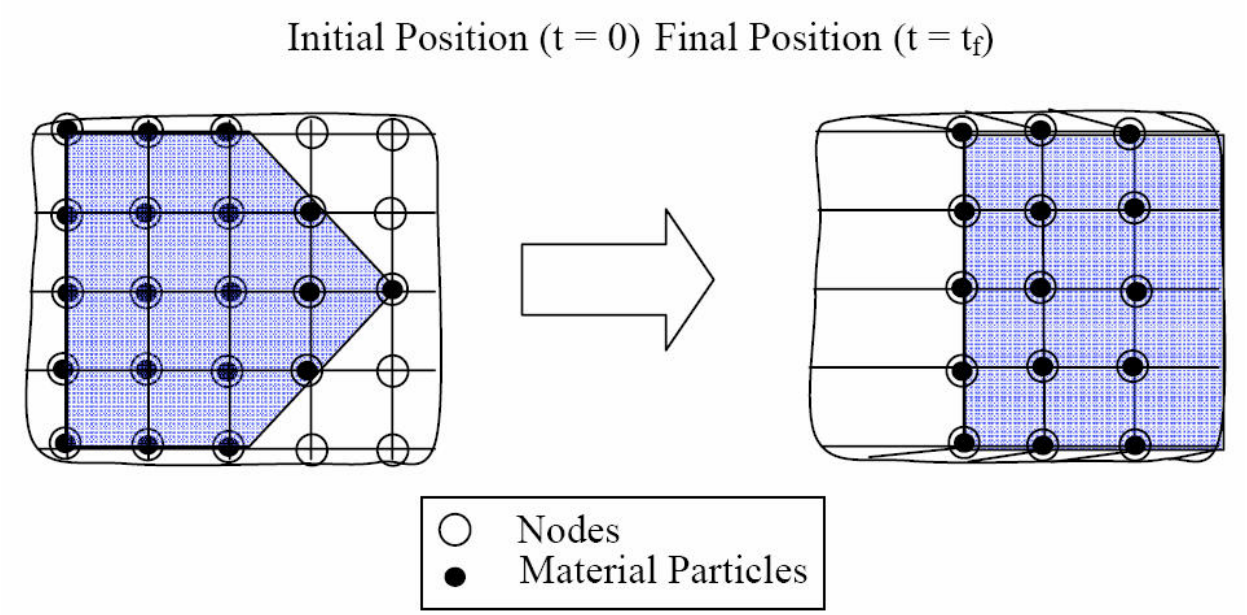

Figure 2 Lagrangian mesh.

The major disadvantage of this method is that large deformations of the material lead to large distortions and possible entanglement of the mesh. Since in the Lagrangian formulation the material moves with the mesh, if the material suffers large defor-mations, the mesh will also suffer equal deformation and this leads to inaccurate results. These mesh deformations cause inaccuracy in the simulation results. To correct this problem, re-meshing must be performed which requires extra time. In the adaptive remeshing/rezoning technique, the regions that include several distorted elements are remeshed. This procedure increases the solution time and is a very complex task. In fact, if it is desired to do the process automatically, a very advanced remapping algorithm is required. In practice, the available remapping algorithms are not very successful in accurately mapping the original distorted mesh to the new well shaped mesh and cause numerical errors, especially for complex geometries. Three main advantages of this method are: Easy tracking of the time-history properties of each particle of material, Simpler imposing of boundary conditions on the material interfaces, Low computational cost, simpler modeling and low number of analysis parameters that have to be dealt with by user. Three main disadvantages of this method are: Severe element distortion can increase the number of required time steps, Element distortion can cause element tangling.

2. EULERIAN APPROACH

In the Eulerian method, instead of material, the space is discretized. A mesh consists of several stacked cells fixed in the space. Some of the cells of the mesh are initially filled by fluid and some are left hollow. As the fluid flows in the space, it leaves some cells and fills some initially hollow cells. Since the mesh does not follow fluid deformation, severe mesh distortions do not exist. Unlike the Lagrangian method, tracking the time history of material points as well as material interfaces is very difficult to be performed in the Eulerian method. The description of the Eulerian motion is the opposite of that in the Lagrangian formulation. This method uses the spatial coordinate system to express the material coordinates. Since the Eulerian mesh is fixed in the space, it must include not only the locations where the fluid initially exists, but also all the locations where the fluid might be present at a later time. This fact does not pose a big difficulty in prevalent fluid dynamics problems in which the fluid usually flows around fixed objects such as pipe walls, skyscrapers, or airfoils. However, if the Eulerian mesh is going to be used for following the path of high velocity objects, such as a bird, the required meshed volume is much larger than the volume of the bird itself. The other problem which accompanies the Eulerian approach for bird strike is that tracking the history of material particles in the domain is not easy at all. However, in order to do this, the stress and strain tensors can be moved from one cell to another. This measure does not usually lead to accurate results. The main advantage of this method is that there is No element distortion.

3. ARBITRARY LAGRANGIAN METHOD (ALE)

The ALE formulation is a combination of the Lagrange and Eulerian formulations in which the reference is set arbitrarily by the user, in order to capture the advantages of the two methods whilst minimizing the disadvantages. In this method the reference coordinate is arbitrary and is generally presented as χ. Depending on the motion, the calculations are Lagrangian based (nodes move with the material) or Eulerian (nodes fixed and the material moves through the mesh). The user must specify the optimal mesh motion, which is the major disadvantage of the ALE method. In the ALE method, the referential domain is denoted as RR and the reference coordinates are denoted as χ. The relationship between material coordinates and ALE coordinates, is given by

$$\chi = \hat{\varphi}^{-1}\Big(\varphi(X,t),t\Big) = \Psi(X,t)$$

For the Lagrange mesh, the nodes are assigned to material particles; therefore the mesh motion is equal to the material motion. On the other hand, the nodes in the Eulerian mesh are fixed and the material flows through the mesh. The ALE formulation is a combination of the Lagrange and Eulerian, therefore the nodes can be fixed (as in the Eulerian mesh) or moving with the material (Lagrangian mesh). In the ALE method, the mesh is arranged to be independent of the fluid motion; therefore, problems such as element distortion and tangling would not easily happen. The ALE method can better capture the material boundaries due to the deformations of the background mesh. Since in the ALE method, the background mesh is allowed to deform in accordance to the deformation of the fluid, the transport of fluid particles between the different cells is not as necessary as in the Eulerian method.

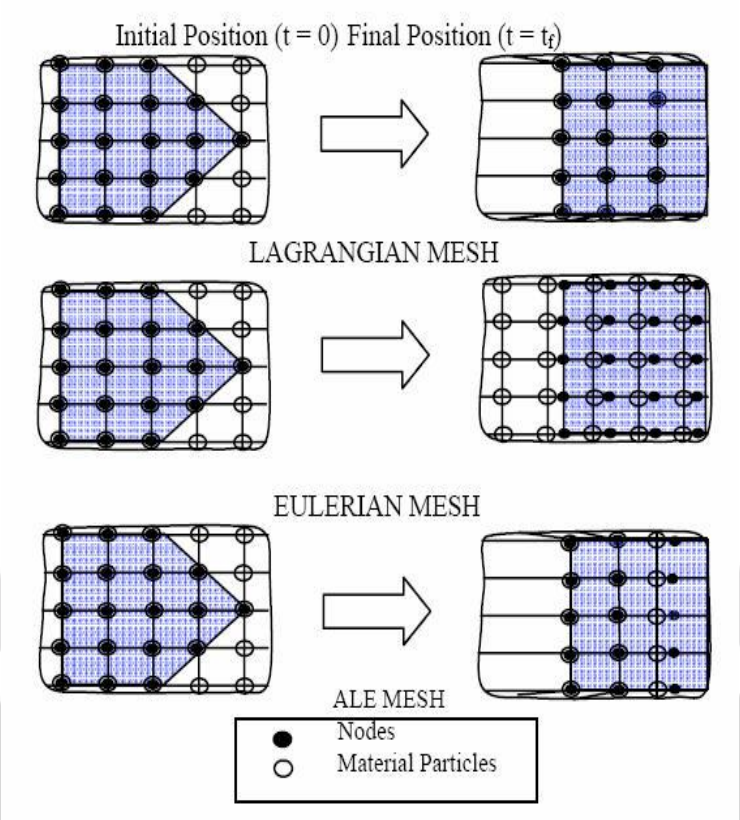

Figure 3 Lagrange, Eulerian, ALE Methods

4 SMOOTHED PARTICLES HYDRODYNAMICS (SPH)

The SPH is a meshless or griddles technique that does not suffer from the normal problems of grid tangling in large deformation problems. The major advantages of the SPH technique is that it does not require a numerical grid and, since it is a Lagrangian method by nature, it allows efficient tracking of material deformations and history-dependent behavior. Because the SPH method has not been fully developed there remain some issues in the areas of stability, consistency, and conservation. Lagrangian motions of mass points or particles are really interpolation points, which are approximated by a cubic B-spline function. Unlike finite element representations for a structure, this finite element model does not exhibit a fixed connectivity between adjacent elements during the impact event. The determination of which elements are nearest neighbors is limited by the search radius. The search radius defines the maximum distance from the center node that an element may search for nearest neighbors and becomes in effect a measure of the fluid cohesive strength. A fluid can be represented by several separate particles (but in interaction with each other) provided that the size of particles are large enough to include sufficient number of molecules so that the fluid properties can be considered uniform in each particle, and small enough to be able to show the gradual change of macroscopic fluid properties. Each SPH element is given a mass, the amount of which is determined by dividing the total mass of the fluid by the total number of the SPH elements. In addition to mass, each SPH element carries hydrodynamic and thermodynamic information of the fluid at that point. In the SPH method, the particles are described by:

$$(x_i(t), m_i(t))_{i \epsilon P}$$

Where P is the set of moving particles and xi(t) and mi(t) are, respectively, the position and mass of the moving particle i.

In initial SPH studies, the smoothing length was considered identical for all the particles and was unvaried throughout the fluid deformation. However, later studies revealed that it is better to consider different Smoothing lengths for different particles depending on the number of particles close to them. The main concept of this idea is that it is necessary to keep enough but not too many numbers of particles in interaction with a particle. Therefore in very sparse locations, it is better to increase the smoothing length so that the change of variables in the fluid can be considered continuous. On the other hand, in highly populated regions, the interaction of a very large number of particles does not provide much additional accuracy compared to a reasonable lower number of interacting particles. In summary, the smoothing length is variable in time and space to avoid the numerical fracture caused by material expansion and the lengthy run times caused by material compression.

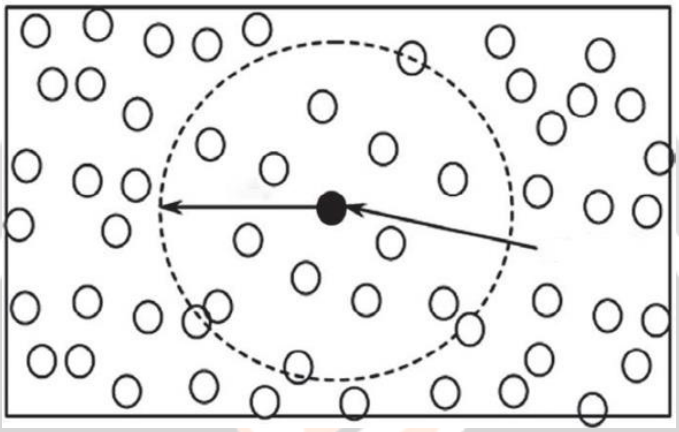

Figure 4 Active domain around a particle in the SPH

The neighbor search is a primary step in SPH simulations and requires high attention. It is important to determine what particles are in interaction with a particular particle at each time step. All the particles are given an influence domain which is a sphere with radius 2h. In the neighbor search step, all the neighbor particles that lie inside the influence domain of a particle at the time step are listed. In the search for finding the influencing particles for a set of N particles, the distance between the particle and (N-1) other particles must be checked. For the total number of N particles, a total number of N(N-1) distance calculations are needed, which can take a very long time for large models. The neighbor particle search for each particle only takes place inside the bucket containing the particle and also its neighbor buckets. This algorithm is called bucket sort. After the hypothetical neighbors of a particle from the main bucket and its neighbor buckets are completely listed, the distances between the corresponding particle and the other particles are checked to see if the distances are lower than twice the smoothing length.

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