

# ROV MODELS, HYDRODYNAMICS AND ASSOCIATED SENSORS

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## ABSTRACT

*Underwater vehicles are among the most important tools for exploring the seas and oceans. Examples have shown that ROVs and AUVs are used in many fields, be it military, economic, environmental or academic research. The motion equations are represented using Fossen's Robotic vector model. The main goal of obtaining a hydrodynamic model is to be able to simulate the movements of the ROV and to exploit the knowledge of ROV dynamics in the design of observers, controllers and propeller allocation. The equations of motion for an ROV are presented from two models. A model plant model, which is a detailed mathematical model of ROV dynamics, and a model control plant model, which is a simplified model.*

**Keyword:** ROV, Fossen's, hydrodynamic model, Kinematics, Sensors

## 1. INTRODUCTION

The mathematical models of ROV and hydrodynamics are given in this paper. Notations and motion equations are represented using Fossen's Robotic vector model. This is an efficient way to describe differential equations in 6 degrees of freedom with coupling effects in matrix form. A section describing the sensors and their measurement equations are also included.

This article is mainly based on [1] where the theory has been adapted to fulfill the purpose of the ROV. The sensor section is derived as part of the article, although the matrix notation is adopted from [1].

The main goal of obtaining a hydrodynamic model is to be able to simulate the movements of the ROV and to exploit the knowledge of ROV dynamics in the design of observers, controllers and propeller allocation.

## 2. KINEMATICS

The most important basic tool for understanding the ROV mathematical model is provided in this section. A method for writing matrix-based motion equations is presented and the frames used in this paper are explained. Transformations are also described.

### 2.1 Robotic vector model of Fossen

The motion equations developed for the ROV are written in a vectorial reference adopted from [2]. An example of using coordinates and generalized matrices to describe the 6 differential equations of motion of DOF is observed in (1) and (2).

$$\dot{\eta} = J(\eta)v, \tag{1}$$

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{2}$$

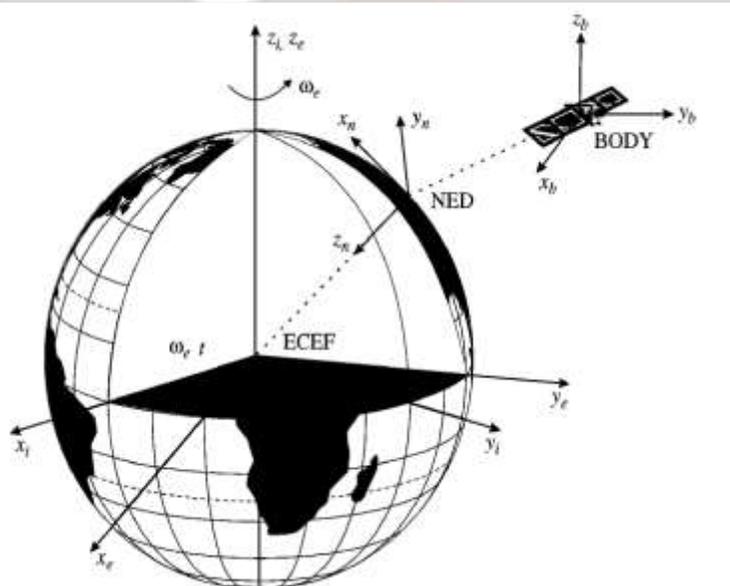
Where  $J \in \mathbb{R}^{6 \times 6}$  is the rotation matrix of the reference associated with the vehicle at the reference mark.  $M \in \mathbb{R}^{6 \times 6}$  is the mass matrix,  $C \in \mathbb{R}^{6 \times 6}$  is the matrix of Coriolis and centripetal forces and  $D \in \mathbb{R}^{6 \times 6}$  is the damping matrix.  $g \in \mathbb{R}^{6 \times 1}$  is a vector with restoring forces. Equation (1.9) is Newton's second law expressed in a mobile coordinate framework, hence the need to compensate for Coriolis and centripetal forces.

**2.2 References and frames used**

Modeling requires the step of defining reference points in relation to which we will describe the evolution of the machine.

- ECI, {i}: Earth-centered inertial fix with axes  $\{i\} = [x_i, y_i, z_i]$ ,
- ECEF, {e}: Earth-centered and fixed landmark with axes  $\{e\} = [x_e, y_e, z_e]$ ,
- NED, {n}: North, East, Bottom with axes  $\{n\} = [x_n, y_n, z_n]$ ,

Where the frame {i} is supposed to be an inertial reference when precision is required. For even greater accuracy, an inertial fixture centered on the sun could be used, but this is not usually necessary. For vehicles that move slowly with a restricted movement zone, the {n} frame can be assumed to be inertial for most applications. The frame {e} is useful for describing motion over longer distances where the FI to Earth approximation of {n} is inaccurate. For example, the GPS coordinates are given in the frame {e}, but it is impossible to display the attitude in this image because the bearing and the step of zero usually mean the level. The measurement frame is also a moving frame, generally moving and rotating with the frame associated with the body. Vector measurements, from a vehicle-mounted instrument, are in the instrument's measurement frame and move with the body frame. Other notations for the measurement frame are used when, for example, more than one instrument with vector measurements is mounted on a vehicle.



**Fig -1 : references and frames**

The reference associated with the body of a ROV is in **Fig -1**. The {b} axes define the surge, sway and heave directions and the rotation directions according to the right-hand rule.

### 3. EQUATIONS OF MOTION

The equations of motion for a ROV are presented in the following. A process plant model, which is a detailed mathematical model of ROV dynamics, and control plant model, which is a simplified model [86]. The process plant model is used in the simulations, and the control plant model is used in the design of the controller and the observer. The equations are expressed in the center of origin (CO), which can be placed anywhere but most convenient on the center line or at the intersections of symmetry planes. The vector of CO at the center of gravity (CG) est  $r_g^b = [x_g y_g z_g]^T$

#### 3.1 Process Plant Model

The *Process Plant Model* given in (3) and (4) is the Newton-Euler equation of motion around CO. This model is based on the Fossen robotic model described in [2]. Newtonian mechanics is expressed in the structure of the body and transformed into a NED frame. The model contains rigid body dynamics terms, hydrodynamic terms, a hydrostatic term, a propulsive force, and external forces.

$$\dot{\eta} = J(\eta)v, \tag{3}$$

$$M_{RB}\dot{v} + C_{RB}(v)v + M_A\dot{v}_r + C_A(v_r)v_r + D(v_r)v_r + g(\eta) = \tau + \tau_{ext} \tag{4}$$

Where  $M_{RB} \in \mathbb{R}^{6 \times 6}$  is the mass matrix of the rigid body in CO and  $C_{RB}(v) \in \mathbb{R}^{6 \times 6}$  is the coriolis matrix with rigid and centripetal body.  $M_A \in \mathbb{R}^{6 \times 6}$  is the added mass matrix in CO,  $C_A(v_r) \in \mathbb{R}^{6 \times 6}$  is the added mass matrix and the centripetal matrix and  $D(v_r) \in \mathbb{R}^{6 \times 6}$  is the damping matrix.  $g(\eta) \in \mathbb{R}^{6 \times 1}$  is the hydrostatic restoring force vector,  $\tau \in \mathbb{R}^{6 \times 6}$  is the vector of the propulsive force and  $\tau_{ext} \in \mathbb{R}^{6 \times 1}$  is a vector of external forces, including umbilical and manipulative forces. Each of the terms and matrices is explained in more detail in the following.

$v_r \in \mathbb{R}^{6 \times 1}$  is the relative speed vector with respect to water and is calculated as

$$v_r = v - v_c \tag{5}$$

$$v_r = v - v_c \tag{6}$$

Where  $v_c \in \mathbb{R}^{6 \times 1}$  is the velocity vector of the ocean current decomposed in the landmark attached to the body. For the irrotational ocean currents

$$v_c = [u_c v_c w_c \ 0 \ 0 \ 0]^T \tag{7}$$

Where  $u_c, v_c$  and  $w_c$  are the speed components of the ocean current.

#### a. Rigid body dynamics

The rigid body mass matrix in CG is

$$M_{RB}^{CG} = \begin{bmatrix} mI_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_g \end{bmatrix} \tag{8}$$

Where  $m$  is the mass of the vehicle and  $I_g \in \mathbb{R}^{3 \times 3}$  is the inertia matrix around CG given by

$$I_g = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{9}$$

Where  $I_x, I_y, I_z$  are the moments of inertia around the axes {b}, and  $I_{xy} = I_{yx}, I_{xz} = I_{zx}$  and  $I_{yz} = I_{zy}$  are the products of inertia. The inertia matrix  $I_g$  is approximated as if the ROV is a uniformly distributed mass box.

Coriolis matrix with a rigid and centripetal body in CG is

$$C_{RB}^{CG} = \begin{bmatrix} mS(\omega_{b/n}^b) & 0_{3 \times 3} \\ 0_{3 \times 3} & -S(I_g \omega_{b/n}^b) \end{bmatrix} \tag{10}$$

And is calculated directly from the mass, the inertia matrix and the rotational speed vector. The transformation is performed using the matrix (12)

$$H(r_g^b) = \begin{bmatrix} I_{3 \times 3} & S^T(r_g^b) \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \tag{11}$$

To have

$$M_{RB} = H^T(r_g^b)M_{RB}^{CG}H(r_g^b) \tag{12}$$

$$C_{RB} = H^T(r_g^b)C_{RB}^{CG}H(r_g^b) \tag{13}$$

Which are respectively the mass matrix and Coriolis in CO.

**b. Hydrodynamic**

**Mass added :**

The added mass of a body (vehicle) in a fluid is given by the added mass matrix  $M_A$  with components such as

$$M_A = \begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \tag{14}$$

Where the hydrodynamic derivative  $X_{\dot{r}}$  is for example the coefficient of the added mass force due to the acceleration in the yaw. Note that  $M_A$  is symmetric,  $M_A = M_A^T$ . When a body moves or oscillates in the water, part of the surrounding body of water also moves. This means that there is a pressure field in the water around the body. This pressure, not to mention the hydrostatic pressure, can be integrated on the surface of the body to find added mass forces. Thus, the added mass is not a specific quantity of water that moves with the body, but it must be understood in terms of hydrodynamic pressure.

The pressure field in the water, set up by the movements of the body, depends on the boundary conditions of the surrounding water, such as the free surface and the bottom of the sea. The movements of a body in or near surface will produce waves, and the pressure field will depend on the frequency. For submarine vehicles, it is assumed that no wave is generated and that the added mass is constant.

An underwater vehicle has a restoring force only in roll and pitch, so there are no clean frequencies for other DOFs. The constant added mass is calculated as zero frequency added mass for overvoltage, sway, lift and yaw. For roll and height, the added mass corresponding to the natural frequency is used. The natural frequencies of roll and height are calculated by

$$w_{roll} = \sqrt{\frac{C_{44}}{I_x + K_{\dot{p}}(w_{roll})}}, w_{pitch} = \sqrt{\frac{C_{55}}{I_x + K_{\dot{p}}(w_{roll})}} \tag{15}$$

Where  $C_{44}$  and  $C_{55}$  are the hydrostatic booster forces to be defined later. We note that it is a recursive equation.

It can be difficult to find the 36 terms, including the diagonal cross-coupling terms. As  $M_A$  is symmetrical, it is enough to find the 6 diagonal elements and the 15 elements above the diagonal. This can be further reduced by exploiting the symmetry properties of the vehicle itself. For simplicity, the ROV is supposed to have a symmetry around the  $xz$ ,  $yz$  and  $xy$  planes (port / starboard, forward / backward, down / up). Thus, there are no mass terms added by cross-coupling and the added mass matrix is reduced to

$$M_A = -diag\{X_{\dot{u}}(0), Y_{\dot{v}}(0), Z_{\dot{w}}(0), K_{\dot{p}}(w_{roll}), M_{\dot{q}}(w_{pitch}), N_{\dot{r}}(0)\} \tag{16}$$

Where the values of each element are calculated from the hydrodynamic tables. Alternatively, the 36-element mass matrix may be calculated by a computer program such as WADAM or WAMIT if CAD drawings are available. This was done for ROV 30k in [3].

**Coriolis and centripetal forces :**

The hydrodynamic and centripetal Coriolis matrix  $C_A$ , for a rigid body moving through an ideal fluid, can always be parameterized to be asymmetric

$$C_A(v) = -C_A^T(v), \forall v \in \mathbb{R}^{6 \times 1} \tag{17}$$

$C_A$  is calculated from the added mass matrix and the generalized velocity vector. The parametrization of  $C_A$  used for the ROV is given in (18) as suggested by [2].

$$C_A(v) = \begin{bmatrix} 0_{3 \times 3} & -S(A_{11}v + A_{12}w) \\ -S(A_{11}v + A_{12}w) & -S(A_{21}v + A_{22}w) \end{bmatrix} \tag{18}$$

Where  $A_{ij} \in \mathbb{R}^{3 \times 3}$  is given by

$$M_A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{19}$$

**Damping :**

For underwater vehicles, such as ROV, potential damping and other wave damping effects are neglected. The damping of an ROV is mainly caused by vortex excretion and skin friction. The easiest way to determine the damping properties of an ROV is to approximate its box geometry and use coefficients from hydrodynamic tables. However, the box approximation will underestimate the damping forces since the ROV has many cavities, exposed cables, manipulators, and other accessories such as lights and instruments. It is difficult to calculate the damping of each of these appendages and to evaluate the effects of the flux interactions in the ROV frame caused by the motion and thrust of the thrusters. For simplicity, depreciation is approximated by a linear and quadratic term given by

$$D(v_r) = D + D_n(v_r) \tag{20}$$

Where  $D$  is the linear damping matrix due to the friction of the wall and  $D_n(v_r)$  is the quadratic damping, mainly caused by the formation of vortices. The damping matrix  $D(v_r)$  is strictly positive because the energy is dissipated by damping.

An alternative damping model, particularly suitable for low speeds, is given by

$$D(v_r) = D e^{-\alpha \|v_r\|} + D_n(v_r) \tag{21}$$

Where  $\alpha$  is a tuning parameter for linear damping decreasing exponentially with vehicle speed,  $\|v_r\|$ .

Diagonal damping matrices are used in ROV modeling because of the difficulty of finding values for non-diagonal damping terms, from calculations or experiments, and because diagonal terms are dominant. The linear and nonlinear damping matrices are given by

$$DD = -diag\{X_u, Y_v, Z_w, K_p, M_q, N_r\}, \quad (22)$$

$$D_n(v_r) = -diag\{X_{|u|u}|u_r|, Y_{|v|v}|v_r|, Z_{|w|w}|w_r|, K_{|p|p}|p_r|, M_{|q|q}|q_r|, N_{|r|r}|r_r|\} \quad (23)$$

Where the elements of  $D$  and  $D_n(v_r)$  are determined from experiments. The damping coefficients can also be analytically approximated.

### Hydrostatic forces :

The gravitational and buoyancy forces, as well as the corresponding restoring moments are calculated in CG by

$$g^{CG}(\eta) = \begin{bmatrix} (W - B) \sin \theta \\ -(W - B) \cos \theta \sin \phi \\ -(W - B) \cos \theta \cos \phi \\ y_b B \cos \theta \cos \phi - z_b B \cos \theta \sin \phi \\ -z_b B \sin \theta - x_b B \cos \theta \cos \phi \\ x_b B \cos \theta \sin \phi + y_b B \sin \theta \end{bmatrix} \quad (24)$$

Where  $W$  is the weight of the vehicle and  $B$  the buoyancy, calculated by

$$W = mg, B = \rho g \nabla \quad (25)$$

Where  $\rho$  is the density of the water,  $g$  is the acceleration of the gravity and  $\nabla$  is the total volume (displacement) of the ROV. The CG vector at the center of buoyancy (CB) is

$$r^{CB} = [x_b, y_b, z_b]^T$$

$g^{CG}(\eta)$  is converted to CO by

$$g(\eta) = H^T r_g^b g^{CG}(\eta) \quad (26)$$

Note that for most ROVs, CB is directly above CG, that is,  $r^{CB} = [0 \ 0 \ z_b]^T$ . Thus, the linearized coil coefficients for roll and pitch are  $C_{44} = C_{55} = z_b$ , which is used in the calculation of eigenfrequencies for roll and pitch.

$$M\dot{v}_r + C(v_r)v_r + D(v_r)v_r + g(\eta) = \tau + \tau_{ext} \quad (27)$$

If the current of the ocean is irrotational, the Coriolis and centripetal matrix is parametrized independently of the linear velocity [2],  $v_c = [u_c v_c w_c \ 0 \ 0 \ 0]^T$ . The matrices of effective mass,  $M = M_{RB} + M_A$  and of Coriolis and centripetal,  $C(v_r) = C_{RB}(v_r) + C_A(v_r)$ , are the sum of the respective rigid and hydrodynamic matrices.

### 3.2 Control Plant Model

The Control Plant Model, given by (28), (29) and (30), is used for the analysis and design of observers and controllers. This is a simplified version of the Process Plant Model. We suppose that

- Vehicle speeds are low,  $<1$  m / s. Thus, centripetal Coriolis forces and nonlinear damping are neglected
- The ocean current velocities are constant or vary slowly. Thus, equations are given in terms of vehicle velocity,  $v$ , and additional forces from ocean currents are included in the bias estimate,  $b$ .
- Roll and pitch motions are low,  $<10$  degrees, and the ROV has neutral buoyancy with CB directly above CG. Thus, the restoring forces are linearized using  $G$ . Control Plant Model is a simplified model, it still reflects the main dynamics of the ROV.

$$\dot{\eta} = J(\eta)v, \tag{28}$$

$$M\dot{v} + Dv + G\eta = \tau + J^T(\eta)b, \tag{29}$$

$$\dot{b} = -T_b^{-1}b + w_b \tag{30}$$

Where  $D \in \mathbb{R}^{6 \times 6}$  is the linear damping matrix.  $b \in \mathbb{R}^{6 \times 1}$  is the bias that accounts for unmodeled dynamics and slowly varying loads, which is modeled as an order Markov process 1.  $T_b \in \mathbb{R}^{6 \times 6}$  is a diagonal matrix with positive polarization time constants.  $w_b \in \mathbb{R}^{6 \times 1}$  is a moderately neutral neutral Gaussian noise process [87].  $G$  is a linearized restoration matrix given by

$$G^{CG} = \text{diag}\{0, 0, 0, -z_b B, -z_b B, 0\} \tag{31}$$

$$G = H^T r_g^b G^{CG} H(r_g^b) \tag{32}$$

Bias estimation takes into account slowly varying forces, such as ocean currents, and errors in modeling.

## 4. GENERALIZED FORCES

### 4.1 Ocean Current Forces

Ocean current forces are included in the Process Plant Model through the velocity of the ocean current in the body. In simulations, this is generated using a model of the speed and direction of the ocean current in the geographic reference, e.g. the NED frame.

The velocity of the ocean current is  $V_c$  and it has a vertical direction and horizontal  $\alpha_c$  and  $\beta_c$  in the reference  $\{n\}$ . The velocity vector of the ocean current in  $\{n\}$  is

$$v_c^n = R_{y,\alpha_c}^T R_{z,-\beta_c}^T \begin{bmatrix} V_c \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

Where the rotation matrices are found from (1.16bis) with  $\alpha_c$  and  $-\beta_c$  as arguments.

The dynamics for  $V_c$ ,  $\alpha_c$  and  $\beta_c$  can be added to obtain a slowly varying ocean current. A model suggested in [2] is to use a first order Gauss-Markov process given by

$$\dot{V}_c + \mu V_c = w, \tag{34}$$

$$\dot{\alpha}_c + \mu_\alpha \alpha_c = w_\alpha, \tag{35}$$

$$\dot{\beta}_c + \mu_\beta \beta_c = w_\beta, \tag{36}$$

Où  $\mu_i \geq 0$  est une constante et  $w_\alpha$  est un bruit blanc gaussien.

Where  $\mu_i \geq 0$  is a constant and  $w_\alpha$  is a white Gaussian noise.

The ocean current is transformed into a body before use in the Process Plant Model as

$$v_c = J^T(\eta)v_c^n \tag{37}$$

Where the elements of  $v_c$  are given by (4).

### 4.2 Propulsive forces

The forces produced by each thruster constitute the total control force. However, the actual thrust is not easy to measure and must be estimated from a propeller model. Propeller dynamics are complex and errors in the propeller model affect the performance of the higher level control because it is uncertain if the desired thrust vector  $\tau$

is produced. Thus, the actual control input is the speed of rotation of the thrusters, and a mapping connecting the speed of rotation to the thrust is necessary.

A basic model of the thrust production of a single propeller is given by (38) as

$$f = K_T(J)\rho D^4 n^2 \quad (38)$$

Where  $f$  is the thrust,  $K_T(J)$  is the thrust coefficient,  $\rho$  is the density of the water,  $D$  is the diameter of the helix and  $n$  is the rotational speed of the helix in revolutions per second (r / s).

The thrust coefficient depends on the number of advances  $J$  that is given by

$$J = \frac{V_a}{nD} \quad (39)$$

Where  $V_a$  is the speed of the flow water of the propeller. The thrust coefficient can be given in a quadrant diagram to show the performance of the thruster under all operating conditions [4]. An open water test was conducted in 2005 in the Marintek Cavitation Tunnel, Trondheim by Martin Ludvigsen for a propellant used on the Minerva ROV. The results were reported in [5] and the polynomials of (40) and (41) were found from curve-fitting experimental data for the first and third quadrants, respectively

$$K_T(J)_1 = 0.5J^3 - 0.66J^2 - 0.25J + 0.24 \quad (40)$$

$$K_T(J)_3 = 0.025J^3 - 0.28J^2 - 0.17J - 0.15 \quad (41)$$

For any ocean current, the 1st quadrant is the forward thrust, and the third quadrant is the reverse thrust while advancing. The second quadrant is the forward pushback and the 4th quadrant is reversed backward.

## 5. SENSORS

Underwater navigation is one of the main challenges in the development of a motion control system because no global positioning system (GPS) is available below the surface. The need for an observer who functions during manipulative work and other cases with variable and uncertain disturbances and dynamics, inspired research on sensor-based state estimation. Specifically, a new type of attitude estimator known as the explicit supplementary filter has been adopted and modified for the use of ROV. This attitude estimator is also used in an integration filter to estimate translation positions. The main contribution here is a new method for including velocity measurements from a Doppler velocity register (DVL) or velocity estimates, to approximate the correct acceleration of the vehicle. This makes it possible to improve the attitude estimation for the accelerated vehicles, and therefore the estimated positions when they are used in cascade with an integration filter.

The orientation of the ROV is necessary to perform automatic tasks such as trajectory tracking and terrain monitoring. Guiding modes require different levels of operator interaction.

### 5.1 Description

There are four common ROV sensors in **Fig -2**. It is a transponder, which is mounted on the vehicle in a part of the Acoustic Positioning System (APS), a DVL, an Inertial Measurement Unit (IMU) and a pressure gauge.

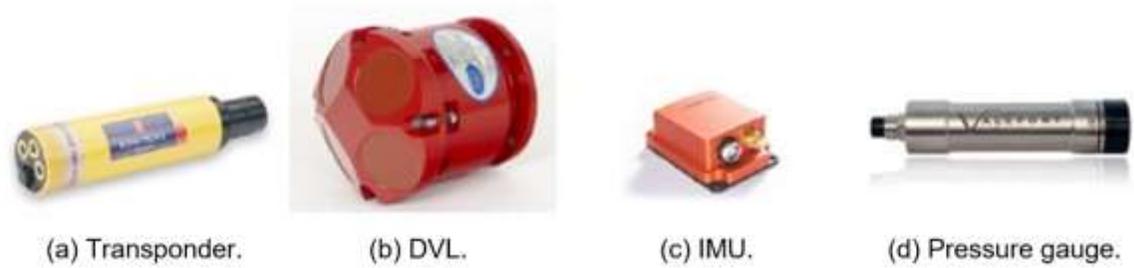


Fig -2 : Sensors of a ROV

## 5.2 Measurement systems

The sensors are placed and aligned in different positions on the ROV. Thus, all measures must be transformed into CO or another origin of common interest in order to provide additional information. The equations describing these translations and transformations are given below. Measurement equations are also used to simulate sensor measurements in computer simulations.

### a. IMU

The measurement equations for IMU are given by (42), (43) and (44) for accelerometers, gyroscopes and magnetometers, respectively. These equations are given in terms of states in the IMU frame.

$$\alpha_{imu}^m = \dot{v}_{m/e}^m + w_{m/i}^m \times v_{m/e}^m + R_n^m (w_{e/i}^n + w_{n/e}^n) \times v_{m/e}^m - R_n^m g_l^n + b_{acc}^m + w_{acc}^m \quad (42)$$

$$\alpha_{imu}^m = \alpha_{m/n}^m + R_n^m (w_{e/i}^n + w_{n/e}^n) + b_{gyro}^m + w_{gyro}^m \quad (43)$$

$$m_{imu}^m = R_n^m R_e^n m^e + b_{mag}^m + w_{mag}^m \quad (44)$$

Where  $\alpha_{imu}^m \in \mathbb{R}^3$  is the acceleration vector measured in  $\{m\}$ .  $b_{acc}^m$  is the polarization vector of the accelerometer, and  $w_{acc}^m$  is the noise vector of the accelerometer.

### b. DVL

The measurement equations for the DVL are given by (45)

$$v_{d/e}^d = R_b^d (\Theta_{bd}) (v_{b/n}^b + w_{b/n}^b \times r_{dvl/b}^b) + w_{dvl}^d \quad (45)$$

$v_{d/e}^d \in \mathbb{R}^3$  is the speed measured in  $\{d\}$ , and  $w_{dvl}^d$  is the noise emitted by the DVL. The equation. (1.83) is used as is for sensor simulation and must be solved for  $v_{b/n}^b$  for use in the control system

### c. Transponder

The measurement equation for the transponder is given by (46)

$$p_{tp/n}^n = p_{b/n}^n + R_b^n (\Theta_{nb}) r_{tp/b}^b + w_{tp}^n \quad (46)$$

Where  $p_{tp/n}^n \in \mathbb{R}^3$  is the transponder position measured in  $\{n\}$ ,  $p_{b/n}^n$  is the position of the ROVet  $w_{tp}^n$  is the APS noise. Equation (1.84) is used as is for sensor simulation and must be solved for  $p_{b/n}^n$  for use in the control system.

### d. Pressure gauge

The measurement equations for the manometer are given by (47) and (48).

$$p_{pg} = p_{atm} + \rho g z_{pg/n}^n + w_{pg}, \quad (47)$$

$$z_{pg/n}^n = z_{b/n}^n + [0 \ 0 \ 1] R_b^n(\Theta_{nb}) r_{pg/b}^b \quad (48)$$

Where  $p_{pg}$  is the measured pressure,  $p_{atm}$  is the atmospheric pressure at the surface,  $\rho$  is the density of water,  $g$  is the acceleration of gravity,  $z_{pg/n}^n$  is the depth of the pressure gauge and  $w_{pg}$  is the sound of the pressure gauge.  $z_{pg/n}^n$  is the depth of the ROV. Equation (47) is only valid for a constant water density in the whole water. For real implementations, finer measurement equations to convert the pressure to depth should be used, eg. the depth conversion formula in [6]. Equation (48) is used as is for sensor simulation and must be solved for  $z_{b/n}^n$  for use in the control system.

## 6. CONCLUSION

This paper allowed us to study the different kinematic models, notably the Fossen Robotic model and the different transformations associated with each of the reference points used. The models for the equations of motion are given by the "Process Plant Model" and the "Control Plant Model". The generalized forces acting on the ROV come mainly from the ocean current, the umbilical, the manipulator and the thrusters. The most important sensors used for navigation of the ROV are also described in this first article.

Thus, a method to include velocity measurements from a Doppler velocity register or velocity estimates, to approximate the correct acceleration of the vehicle, has been investigated.

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