SOME NEW RESULTS ON EQUITABLE AND END EQUITABLE DOMINATION NUMBERS

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ABSTRACT

A subset D of $V(G)$ is called an *equitable dominating set* of a graph G if for every $u \in (V-D)$, there exists a vertex $v \in D$ such that $uv \in E(G)$ and $|\deg(u)-\deg(v)| \le 1$. An equitable dominating set D is said to be an *end equitable dominating set* of G if D contains all the end vertices of graph G. In this paper, we discuss the domination and equitable domination numbers of the graphs such as square of a cycle C_n^2, complete-path-complete $K_m P_{n+2} K_m$, cycle-path-cycle $C_m P_{n+2} C_m$, line graph $L(C_m P_{n+2} C_m)$ and prism $Y_{n,2}$ and end equitable domination number of graph in subdivision of jellyfish $S(J(m,n))$.

Keywords: Domination number; equitable domination number; end equitable domination number.

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1. INTRODUCTION

All the graphs considered here are finite, undirected with no loops and multiple edges. A *graph* G consists of a pair (V,E) where V is a non-empty finite set whose elements are called *vertices or nodes* and E is a set of unordered pairs of distinct elements of V. The elements of E are called *edges* of the graph G. The *degree* of a vertex of a graph is the number of edges incident to the vertex, with loops counted twice. The degree of a vertex v is denoted by $\deg(v)$. The maximum and minimum degree of a graph is denoted by $\Delta(G)$ and $\delta(G)$ respectively. $N(v)$ and $N[v]$ denote the open and closed neighborhoods of a vertex v respectively. A vertex $v \in G$ is called *pendant vertex* or end vertex of G if $\deg(v)=1$. An edge of a graph is said to be *pendant* if one of its vertices is a pendant vertex. For graph theoretic terminology we refer to Chartrand and Lesniak [4].

The rigorous study of dominating sets in Graph theory began around 1960. According to Hedetneimi and Laskar (1990) [8], the domination problems were studied from the 1950's onwards, but the rate of research on domination significantly increased in the mid 1970's. In 1958, Berge [1] defined the concept of the domination number of a graph called as "coefficient of external stability". In 1962, Ore [15] coined the name "dominating set" and "domination number" for the same concept. In 1977 Cockayane and Hedetneimi [5] made an interesting and extensive survey of the results known at the time about dominating sets in graphs [7], [14]. They have used the notation $\gamma(G)$ for the domination number of a graph, which has become very popular since then [3]. The survey paper of Cockayane and Hedetniemi [5] has generated lot of interest in the study of dominating sets in graphs. Recent books on domination, have stimulated a sufficient inspiration leading to the expansive growth of this field to study. The domination number for the helm graph H_n and web graph W_n were found by Ayhan A. Khalil [10]. The domination and equitable domination number for the friendship graph F_n and windmill graph $Wd(m,n)$ were proved by Dr. C S Nagabhushana et al. [13]. The domination and equitable domination number for the book graph B_n and stacked book graph $B_{3,n}$ were proved by Kavitha B N and Indrani Kelkar [9].

The concept of equitable domination number in graphs was introduced by Swaminathan et al. [16] by considering the following real world problems such as network nodes with nearly equal capacity may interact with each other in a better way, in our society persons with nearly equal status, tend to be friendly, in an industry, employees with nearly equal powers form association and move closely, equitability among citizens in terms of wealth, health, status etc is the goal of a democratic nation. The end equitable domination number in graph has introduced by J.H.Hattingh and M.H.Henning [6]. The end equitable domination results are proved by K. B. Murthy and Puttaswamy [11],[12].

2. DEFINITIONS AND NOTATIONS

In this section we recall some of basic definitions in literature which will be useful for our present work.

Definition 2.1: [2], [17] A set D of vertices in a graph $G=(V,E)$ is called a *dominating set* of G, if every vertex in $V-D$ is adjacent to some vertex in D. The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of the dominating set in G.

Definition 2.2: A subset D of $V(G)$ is called an *equitable dominating set* of a graph G if for every $u\in(V-D)$; there exists a vertex $v\in D$ such that $uv\in E(G)$ and $|\deg(u)-\deg(v)|\leq 1$. The minimum cardinality of such a *dominating set* is denoted by $\gamma_e(G)$ and is called *equitable domination number* of G.

Definition 2.3: If a vertex $u\in V$ be such that $|\deg(u)-\deg(v)|\geq 2$ for all $v\in N(u)$, then u is called an *equitable isolate vertex*.

Definition 2.4: An equitable dominating set D is said to be an *end equitable dominating set* of G if D contains all the end vertices of graph G. The minimum cardinality of an end equitable dominating set is called the *end equitable domination number* of G and is denoted by $\gamma_{ee}(G)$.

Definition 2.5: The *line graph* of G, written $L(G)$, is the simple graph whose vertices are the edges of G, with $uv\in E(L(G))$ when u and v have a common end vertex in G.A *path* is a trail in which all vertices (except possibly the first and last) are distinct. A trail is a walk in which all edges are distinct. A walk of length k in a graph is an alternating sequences of vertices and edges such as $v_0,e_0,v_1,e_1,...,e_{k-1},v_k$ which begins and ends with vertices. The path on n vertices is denoted by P_n. P_n is a path of length $n-1$.

Definition 2.6: For a simple connected graph G the *square of a cycle* graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G. A *cycle-path-cycle* graph is the graph that can be obtained from a cycle C_m of two copies with each of these cycles are attached with initial and final vertices of path P_n, the resulting graph is denoted by $C_mP_{n+2}C_m$.

Definition 2.7: A *complete* graph is a graph in which every vertex is adjacent to each of the other vertices in the given graph. We represent a complete graph with m vertices using the symbol K_m. Note that every complete graph is a regular. However the converse is need not be true. A *complete-path-complete* graph is the graph that can be obtained from a complete graph K_m of two copies with each of these complete graphs are attached with initial and final vertices of path P_n, the resulting graph is denoted by $K_mP_{n+2}K_m$.

Definition 2.8: A *prism* graph $Y_{n,2}$ is defined as the graph of Cartesian product $K_2\times C_n$, where K_2 is the complete graph on two nodes and C_n is the cycle graph on n nodes. As a result, it is a unit distance graph.

Definition 2.9: A G is a graph that can be obtained from G by subdivision of each edge of G is called the *subdivision* of a graph G and it is denoted by $S(G)$.

Definition 2.10: The *jellyfish* graph $J(m,n)$ is obtained from a 4-cycle v_1, v_2, v_3, v_4 by joining v_1 and v_3 with an edge and appending m pendant edges to v_2 and n pendant edges to v_4.

Definition 2.11: The *floor function* of a real number x is the greatest integer less than or equal to x and it is denoted by $\lfloor x \rfloor$. Suppose that $n \le x < n+1$, where n is an integer, then $\lfloor x \rfloor = n$.

Definition 2.12: The *ceiling function* of a real number x is the lowest integer greater than or equal to x and it is denoted by $\lceil x \rceil$. Suppose that $n-1 < x \le n$, where n is an integer, then $\lceil x \rceil = n$.

3. INTRODUCTION ON NEW RESULTS

In this one formally we prove the graphs such as square of a cycle C_n^2, complete-path-complete $K_m P_{n+2} K_m$, cycle-path-cycle $C_m P_{n+2} C_m$, line graph $L(C_m P_{n+2} C_m)$ and prism $Y_{n,2}$ are domination and equitable domination number of graphs and end equitable domination number of graphs in subdivision of jellyfish $S(J(m,n))$.

4. DOMINATION & EQUITABLE DOMINATION NUMBERS OF GRAPHS

Theorem 4.1 For any square of a cycle graph C_n^2, the domination and equitable domination number is $\left\lceil \frac{n}{5} \right\rceil$, where $n \ge 3$.

Proof.

Let $G \cong C_n^2$ be a square of a cycle graph on n vertices and $2n$ edges where $n \ge 3$ and let D be a dominating set of the graph G. Clearly any minimum dominating set of the graph G must contain the vertices from the set $\{v_1, v_2, ..., v_n\}$.

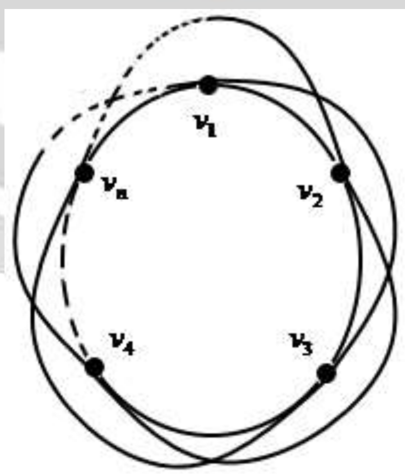


Fig. 4.1. Square of a cycle graph C_n^2

Notice that for every consecutive five vertices, the middle vertex dominates the remaining vertices. Hence $\gamma(G) = \left\lceil \frac{n}{5} \right\rceil$.

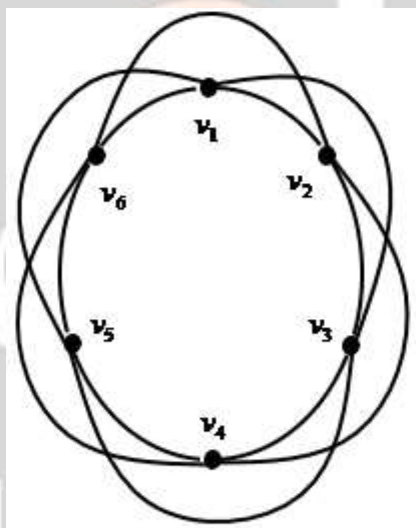
In C_n^2, degree of all vertices are equal, so if we select any one vertex from the set $\{v_1, v_2, ..., v_n\}$ it will dominate the nearby 4 vertices, but the graph G has no equitable isolated vertex. So minimum dominating and minimum equitable dominating set is the same.

Hence $\gamma(G) = \gamma_e(G) = \left\lceil \frac{n}{5} \right\rceil$.

Example 4.2

Let us find the domination and equitable domination number of square of a cycle graph C_6^2 is shown in Fig. 4.2.

Fig. 4.2. Square of a cycle graph C_6^2



In this graph, the minimum dominating and minimum equitable dominating set is $D = \{v_1, v_4\}$ and so $\gamma(G) = \gamma_e(G) = \left\lceil \frac{6}{5} \right\rceil = 2$.

Theorem 4.3

For any cycle-path-cycle graph $C_m P_{n+2} C_m$ the domination and equitable domination number is

$$\gamma(G) = \gamma_e(G) = \begin{cases} 2 + 2\left\lceil \frac{m-3}{3} \right\rceil + \left\lceil \frac{n-2}{3} \right\rceil, \text{ if } m = 3 \text{ or } m \geq 5, n \geq 0. \\ 2 + \left\lceil \frac{n+2}{3} \right\rceil, \text{ if } m = 4 \text{ and } n \geq 0. \end{cases}$$

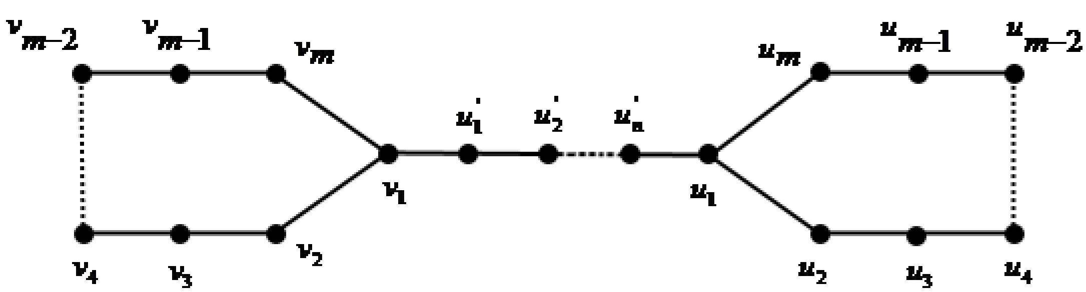
Proof.

Case 1. Let $m=3$ or $m \geq 5$, $n \geq 0$.

Let $G \cong C_m P_{n+2} C_m$ be a cycle-path-cycle graph on $2m+n$ vertices and $2m+n+1$ edges. The graph G has no equitable isolated vertex and $\deg(v_1)=\deg(u_1)=\Delta(G)$. Then any minimum dominating set of G must contain the vertices v_1 and u_1.

To minimize the size of the dominating set we will select v_1 and u_1 as two elements.

Fig4.3.The graph $C_m P_{n+2} C_m$



v_1 will dominate the vertices u'_1 and $\{v_2, v_m\}$, u_1 will dominate the vertices u'_n and $\{u_2, u_m\}$.

We know that the domination number of a cycle graph m is $\left\lceil \frac{m}{3} \right\rceil$.

That is $\gamma(C_m)=\left\lceil \frac{m}{3} \right\rceil$.

Since any two adjacent vertices in G are also equitable adjacent vertices.

In G, for any minimum equitable dominating set, v_1 is equitable dominating with vertices u'_1, v_2 and v_m & also u_1 is equitable dominating with vertices u'_n, u_2 and u_m. Thus, the dominating and equitable dominating set is the same.

Hence $\gamma(G)=\gamma_e(G)=2+\gamma(P_{m-3})+\gamma(P_{m-3})+\gamma(P_{n-2})=2+2\gamma(P_{m-3})+\gamma(P_{n-2})$

$$\gamma(G)=\gamma_e(G)=2+2\left\lceil \frac{m-3}{3} \right\rceil+\left\lceil \frac{n-2}{3} \right\rceil.$$

Case 2.

Let $m=4$ and $n \geq 0$.

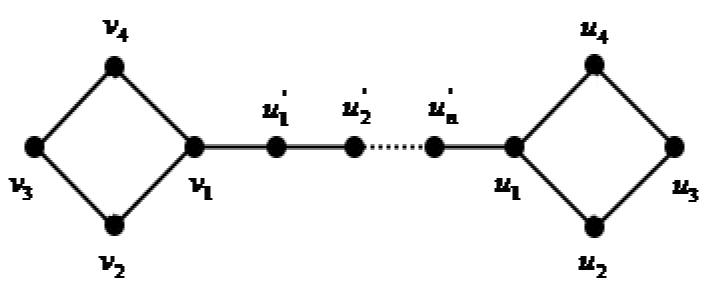


Fig 4.4. The graph $C_4P_{n+2}C_4$

Instead of selecting v_1 and u_1 if we select the vertices v_3 and u_3, we will minimize the domination number and equitable domination number.

In this case $D=\{v_3,u_3\}\cup D^{'}$, where $D^{'}=$ Dominating set of P_{n+2}.

Hence $\gamma(G)=\gamma_e(G)=2+\gamma(P_{n+2})=2+\left\lceil\frac{n+2}{3}\right\rceil$.

Example 4.4 Let us find the domination and equitable domination number of graph $C_6P_6C_6$ is shown in Fig. 4.5.

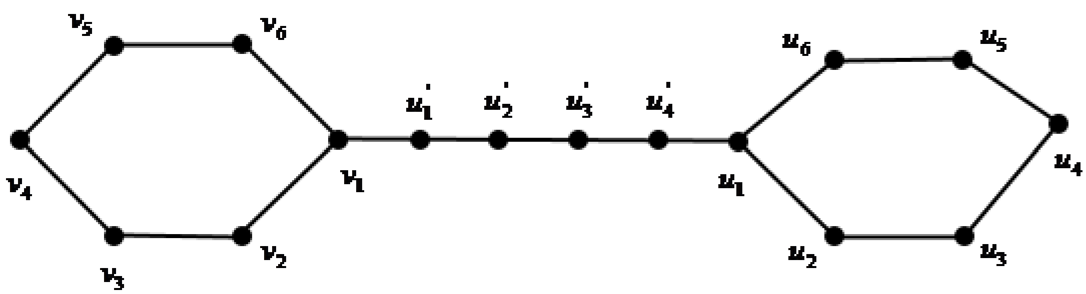


Fig. 4.5. The graph $C_6P_6C_6$

In this graph, the minimum dominating and minimum equitable dominating set is $D=\{v_1,v_4,u_1,u_4,u_2^{'}\}=5$ and so $\gamma(G)=\gamma_e(G)=5$.

Theorem 4.5

For any cycle-path-cycle graph G be the line graph of $C_mP_{n+2}C_m$. That is $L(C_mP_{n+2}C_m)$, the domination and equitable domination number is

$$\gamma(G)=\gamma_e(G)=\begin{cases}2+2\left\lceil\frac{m-2}{3}\right\rceil+\left\lceil\frac{n-2}{3}\right\rceil, \text{ if } m=3 \text{ or } m\geq 5, n\geq 0.\\ 2+\left\lceil\frac{n+2}{3}\right\rceil, \text{ if } m=4 \text{ and } n\geq 0.\end{cases}$$

Proof.

Case 1. Let $m=3$ or $m\geq 5$ and $n\geq 0$.

Let $G \cong L(C_m P_{n+2} C_m)$ be the line graph of cycle-path-cycle on $2m+n+2$ vertices and $2m+n+3$ edges where $m \geq 3, n \geq 0$. G has no equitable isolated vertex and there are six vertices with maximum equitable degree $\deg(v_1) = \deg(u_1) = \deg(v_2) = \deg(v_{m+1}) = \deg(u_2) = \deg(u_{m+1}) = \Delta(G)$. Then any minimum dominating set of G must contain the vertices v_1 and u_1.

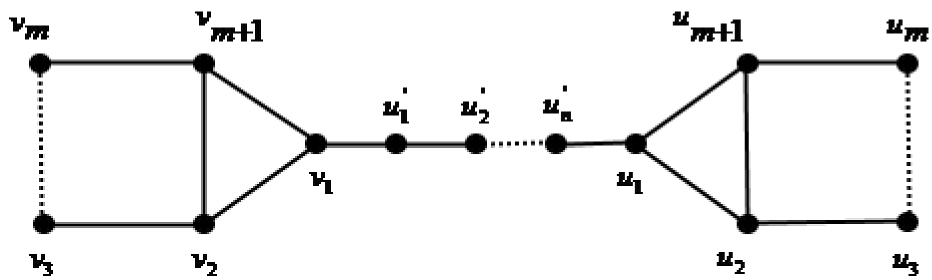


Fig. 4.6. The graph $L(C_m P_{n+2} C_m)$

To minimize the size of the dominating set we will select v_1 and u_1 as two elements.

$\therefore$ v_1 will dominate the vertices $u_1^{'}$ and $\{v_2, v_{m+1}\}$, u_1 will dominate the vertices $u_n^{'}$ and $\{u_2, u_{m+1}\}$.

We know that the domination number of a cycle graph C_{m+1} is $\left\lceil \frac{m+1}{3} \right\rceil$.

That is $\gamma(C_{m+1}) = \left\lceil \frac{m+1}{3} \right\rceil$.

Since any two adjacent vertices in G are also equitable adjacent vertices.

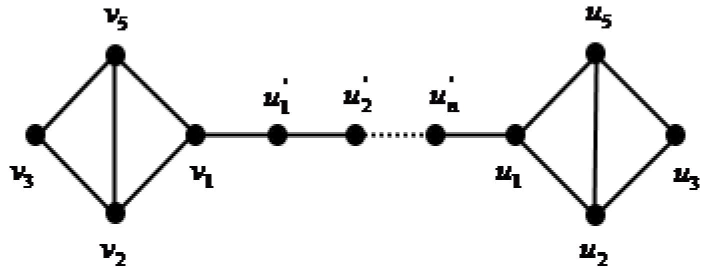
In G, for any minimum equitable dominating set, v_1 is equitable dominating with vertices $u_1^{'}, v_2$ and v_{m+1} & also u_1 is equitable dominating with vertices $u_n^{'}, u_2$ and u_{m+1}. Clearly all the vertices in G satisfies equitable adjacent. Without loss of generality we will select v_1 and u_1 to the both minimum dominating and minimum equitable dominating set. Therefore, the domination and equitable domination number is the same.

Hence $\gamma(G) = \gamma_e(G) = 2 + \gamma(P_{m-2}) + \gamma(P_{m-2}) + \gamma(P_{n-2}) = 2 + 2\gamma(P_{m-2}) + \gamma(P_{n-2})$

$$\gamma(G) = \gamma_e(G) = 2 + 2\left\lceil \frac{m-2}{3} \right\rceil + \left\lceil \frac{n-2}{3} \right\rceil.$$

Case 2. Let $m = 4$ and $n \geq 0$.

Instead of selecting v_1 and u_1 if we select the vertices v_3 and u_3, we will minimize the domination number and equitable domination number. **Fig. 4.7.** The graph $L(C_4 P_{n+2} C_4)$



In this case $D = \{v_3, u_3\} \cup D'$, where $D' =$ Dominating set of P_{n+2}.

Hence $\gamma(G) = \gamma_e(G) = 2 + \gamma(P_{n+2}) = 2 + \left\lceil \frac{n+2}{3} \right\rceil$.

Example 4.6

Let us find the domination and equitable domination number of graph $L(C_5 P_5 C_5)$ is shown in Fig. 4.8.

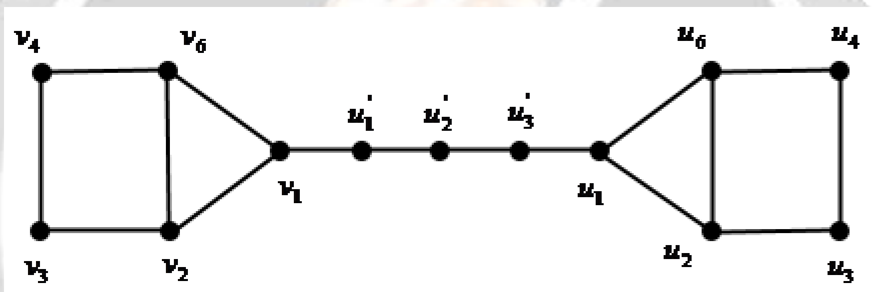


Fig. 4.8. The graph $L(C_5 P_5 C_5)$

In this graph, the minimum dominating and minimum equitable dominating set is $D = \{v_1, v_4, u_1, u_4, u_2'\} = 5$ and so $\gamma(G) = \gamma_e(G) = 5$.

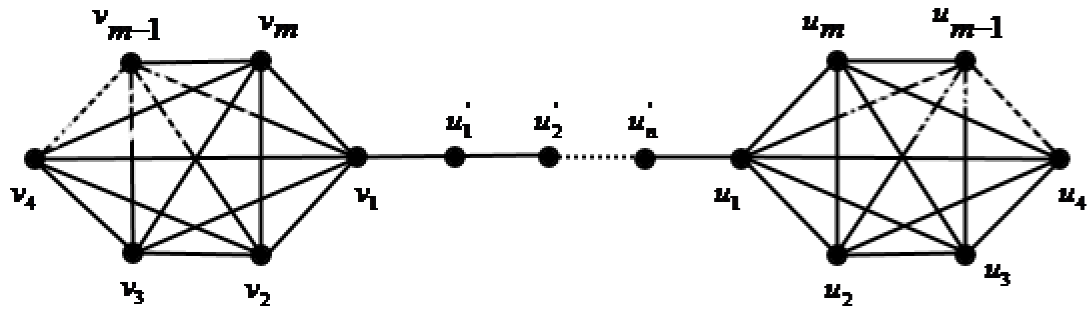
Theorem 4.7

For any complete-path-complete graph $K_m P_{n+2} K_m$, the domination number is $2 + \left\lceil \frac{n-2}{3} \right\rceil$, where $m \geq 2, n \geq 0$.

Proof.

Let $G \cong K_m P_{n+2} K_m$ be a complete-path-complete graph on $2m+n$ vertices and $2 \times \left[\frac{m(m-1)}{2} \right] + (n+1)$ edges where $m \geq 2, n \geq 0$. Note that G has no equitable isolated vertex and $\deg(v_1) = \deg(u_1) = \Delta(G)$. Then any minimum dominating set of G must contain the vertices v_1 and u_1.

Fig. 4.9. The graph $K_m P_{n+2} K_m$



To minimize the size of the dominating set we will select v_1 and u_1 as two elements.

$\therefore$ v_1 will dominate the vertices $u_1^{'}$ and $\{v_2, v_3, ..., v_m\}$, u_1 will dominate the vertices $u_n^{'}$ and $\{u_2, u_3, ..., u_m\}$.

We know that the domination number of a complete graph is one.

That is $\gamma(K_m) = 1$ for any positive integer m.

Hence $\gamma(G) = \gamma(K_m) + \gamma(P_{n-2}) + \gamma(K_m) = 1 + \left\lceil \frac{n-2}{3} \right\rceil + 1$

$$\gamma(G) = 2 + \left\lceil \frac{n-2}{3} \right\rceil.$$

Theorem 4.8

For any complete-path-complete graph $K_m P_{n+2} K_m$, the equitable domination number is $2 + \left\lceil \frac{n}{3} \right\rceil$, where $m \geq 4, n \geq 0$.

Proof.

Let $G \cong K_m P_{n+2} K_m$ be a complete-path-complete graph.

Any two adjacent vertices in K_m are equitable adjacent vertices and any two adjacent vertices in P_{n+2} except the initial and final vertices of P_{n+2} are also equitable adjacent vertices.

In G, there is no equitable isolated vertex and $\deg(v_1) = \deg(u_1) = \Delta(G)$. Then any minimum equitable dominating set of G must contain the vertices v_1 and u_1. However v_1 is not equitable dominating $u_1^{'}$ and u_1 is not equitable dominating $u_n^{'}$. Clearly the vertices in G satisfies equitable adjacent.

From Fig. 4.9, v_1 will dominate $\{v_2, v_3, ..., v_m\}$ and u_1 will dominate $\{u_2, u_3, ..., u_m\}$ vertices.

Hence $\gamma_e(G) = \gamma(K_m) + \gamma(P_n) + \gamma(K_m) = 1 + \left\lceil \frac{n}{3} \right\rceil + 1$

$$\gamma_e(G) = 2 + \left\lceil \frac{n}{3} \right\rceil.$$

Example 4.9 Let us find the domination and equitable domination numbers of graph $K_5P_6K_5$ is shown in Fig 4.10.

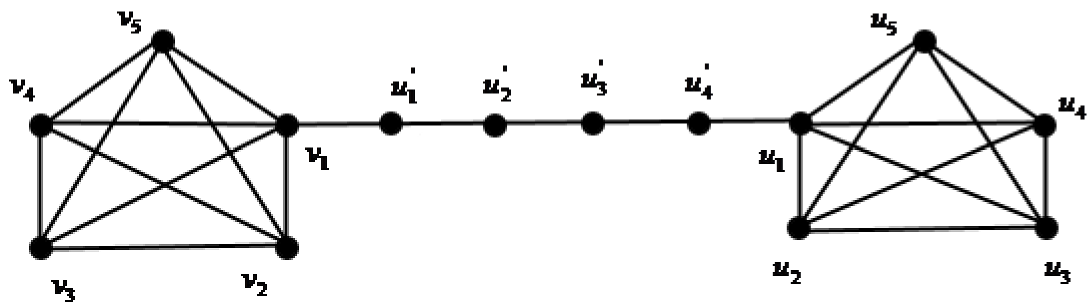


Fig. 4.10. The graph $K_5P_6K_5$

In this graph, the minimum dominating set is $D = \{v_1, u_1, u_2'\} = 2 + \left\lceil \frac{4-2}{3} \right\rceil = 3$ and so $\gamma(G) = 3$ and the minimum equitable dominating set is $D = \{v_1, u_1, u_2', u_4'\} = 2 + \left\lceil \frac{4}{3} \right\rceil = 4$ and so $\gamma_e(G) = 4$.

Remarks 4.10

In the case of complete-path-complete graph $K_mP_{n+2}K_m$, when $m = 2, n \ge 0$, the minimum dominating and minimum equitable dominating set is the same. i.e., $\gamma(G) = \gamma_e(G) = \left\lceil \frac{n+4}{3} \right\rceil$. If $m = 3, n \ge 0$, the minimum dominating and minimum equitable dominating set of complete-path-complete graph $K_mP_{n+2}K_m$ and cycle-path-cycle graph $C_mP_{n+2}C_m$ are the same.

Theorem 4.11

For any prism graph $Y_{n,2}$, the domination and equitable domination number is $\left\lfloor \frac{n+2}{4} \right\rfloor + \left\lfloor \frac{n+3}{4} \right\rfloor$, where $n \ge 2$.

Proof. Let $G \cong Y_{n,2}$ be a prism graph on $2n$ vertices and $3n$ edges where $n \ge 3$ and let D be a dominating set of the graph G. Clearly any minimum dominating set of the graph G must contain the vertices from the set $\{v_1, v_2, ..., v_n, v_1', v_2', ..., v_n'\}$.

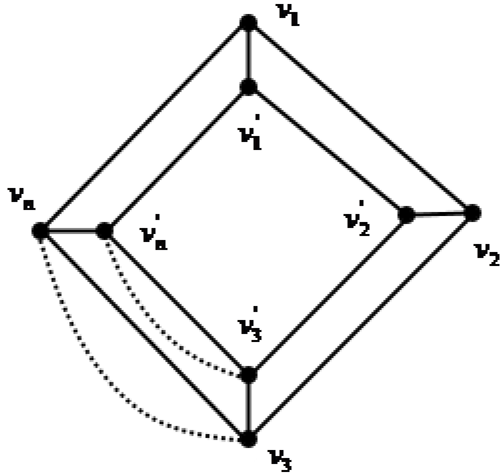


Fig. 4.11. Prism graph $Y_{n,2}$

Notice that for every vertices in both inner and outer cycle if we choose any one vertex from the above set it will dominate the nearby 3 vertices. So we will proceed like this, remaining few vertices which are dominated by our required vertex. Hence $\gamma(G) = \left\lfloor \frac{n+2}{4} \right\rfloor + \left\lfloor \frac{n+3}{4} \right\rfloor$.

In $Y_{n,2}$, degree of all vertices are equal and G has no equitable isolated vertex. So, minimum dominating and minimum equitable dominating set is the same.

$$\text{Hence } \gamma(G) = \gamma_e(G) = \left\lfloor \frac{n+2}{4} \right\rfloor + \left\lfloor \frac{n+3}{4} \right\rfloor.$$

Example 4.12

Let us find the domination and equitable domination number of prism graph $Y_{5,2}$ is shown in Fig. 4.12.

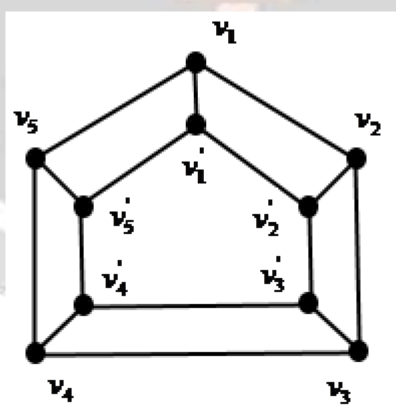


Fig. 4.12. Prism graph $Y_{5,2}$

In this graph, the minimum dominating and minimum equitable dominating set is $D = \{v_1, v_3', v_4'\} = 3$ and so $\gamma(G) = \gamma_e(G) = 3$.

5. END EQUITABLE DOMINATION NUMBERS OF GRAPHS

Theorem 5.1. For a subdivision of jellyfish graph $S(J(m,n))$, the domination number is $m+n+2$, the equitable and end equitable domination number of $S(J(m,n))$ is $m+n+4$, where $m,n \geq 2$.

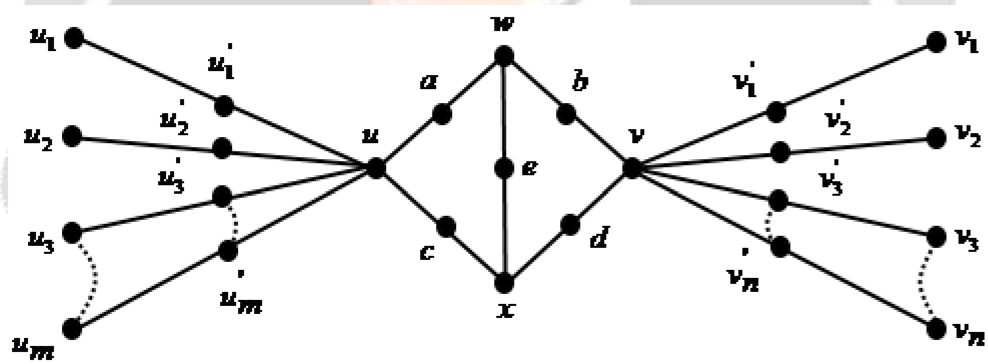
Proof. Let G be the subdivision of jellyfish graph $J(m,n)$ and it has $2m+2n+9$ vertices and $2m+2n+10$ edges.

In any dominating, equitable dominating and end equitable dominating sets, the vertices u,v must be included, since $\deg(u)=\deg(v)=\Delta(G)$. Out of the vertices a, b, c, d, w,e and x, the vertex 'w' and 'x' dominates a, b, c, d & e.

Hence the set $D=\{u_1^{'},u_2^{'},...,u_m^{'},v_1^{'},v_2^{'},...,v_n^{'},w,x\}$ is a minimum dominating set for the graph G, that means the set D will dominate all the other vertices in G.

$\therefore \gamma(G)=m+n+2$.

Now the set of equitable isolated vertices $E=\{u_1,u_2,...,u_m,v_1,v_2,...,v_n\}$ must be included in any minimum equitable and minimum end equitable dominating set and these vertices will equitable dominate the set of vertices as $\{u_1^{'},u_2^{'},...,u_m^{'},v_1^{'},v_2^{'},...,v_n^{'}\}$. If we choose the vertices w and x, it will dominate the vertices a,b,c,d and e.



. **Fig. 5.1.** Subdivision of jellyfish graph $S(J(m,n))$

Hence the set $E\cup\{u,v,w,x\}$ is a minimum equitable and minimum end equitable dominating set, since u and v are equitable isolated vertices. So $\deg(u)=\deg(v)=\Delta(G)$.

$$\therefore \gamma_e(G)=\gamma_{ee}(G)=\left|E\cup\{u,v,w,x\}\right|=(m+n)+4=m+n+4.$$

Hence $\gamma_e(G)=\gamma_{ee}(G)=m+n+4$.

Example 5.2. The subdivision of jellyfish graph $S(J(4,3))$ is shown in Fig. 5.2.

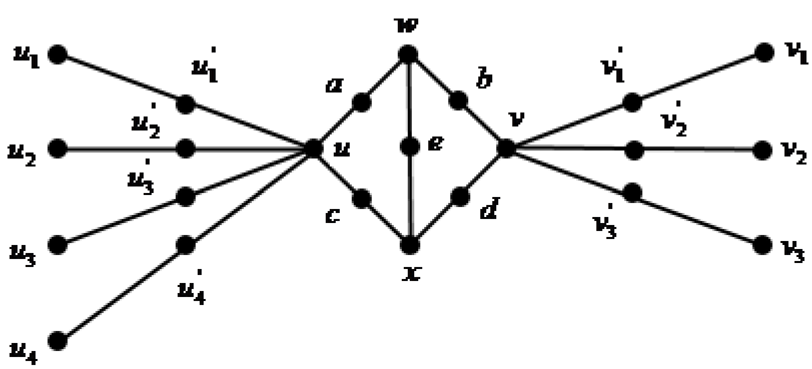


Fig. 5.2.

Subdivision of jellyfish graph $S(J(4,3))$

In this graph, the minimum dominating set is $D=\{w,x,v_1',v_2',v_3',u_1',u_2',u_3',u_4'\}=9$ and so $\gamma(G)=9$, the minimum equitable dominating and end equitable dominating set is $D=\{u,v,w,x,v_1,v_2,v_3,u_1,u_2,u_3,u_4\}=11$ and so $\gamma_e(G)=\gamma_{ee}(G)=11$.

Remarks 5.3. In the case of subdivision of jellyfish graph $J(m,n)$, when $m,n=1$, the minimum dominating is $D=\{w,x,u_1',v_1'\}$, minimum equitable dominating and minimum end equitable dominating sets i.e., $D=\{u,v,e,u_1,v_1\}$ is the domination, equitable domination and end equitable domination number is $\gamma(G)=4,\gamma_e(G)=\gamma_{ee}(G)=5$.

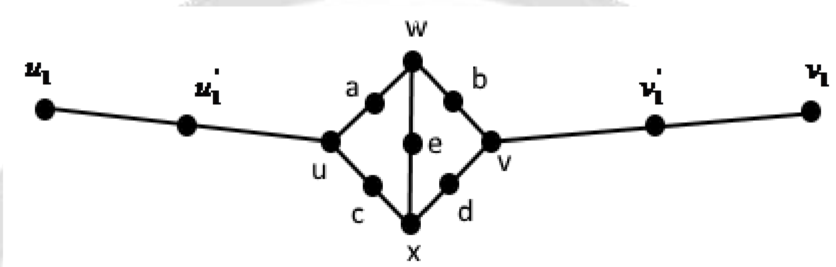


Fig. 5.3. Subdivision of jellyfish graph $S(J(1,1))$

6. CONCLUDING REMARKS

The theory of domination plays vital role and many researchers are producing a huge collection of papers in this topic. The domination concept is further classified into equitable dominating and end equitable dominating sets. In this paper we proved that the domination, equitable domination and end equitable domination number of some new graphs.

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