SOME OPERATIONS ON FUZZY SOFT SET THEORY

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ABSTRACT

The aim of this paper is to study some operations and results available in the literature of fuzzy soft sets.Instead of taking the notion of complement of a fuzzy soft set by Maji, now we have taken the notion of complement of a fuzzy soft set put forwarded by T J Neog and D K Sut. Also we introduced and investigate the n-th term of De Morgan Laws using fuzzy soft sets.

Keywords – *Fuzzy Set, Soft Set, Fuzzy Soft Set*

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1. INTRODUCTION

In many complicated problems arising in the fields of engineering, social sciences etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodstov [3] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that reason for these difficulties is, possibly the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodtsov [3],is free of the difficulties present in these theories. In 2011, T J Neog and D K Sut [10] put forward a new notion of complement of a soft set and in 2012, T J Neog and D K Sut [1] studied a study on some operations of fuzzy soft sets.

In recent times, researcher have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [7] introduced the concept of Fuzzy Soft Set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [2]. Recently, , T J Neog and D K Sut [9] have studied the notions of fuzzy soft union, fuzzy soft intersection, complement of a fuzzy soft set and several other properties of fuzzy soft sets along with examples and proofs of certain results.

In this paper, we have studied some operations and results available in the literature of fuzzy soft sets.Instead of taking the notion of complement of a fuzzy soft set by Maji, now we have taken the notion of complement of a fuzzy soft set put forwarded by T J Neog and D K Sut[8]. Also investigate the n-th terms of De Morgan Laws using fuzzy soft sets.

2. PRELIMINARIES

In this section, we first recall some basic definitions related to soft sets .

2.1. Soft Set [3]

A pair (F, E) is called soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words,the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε- elements of the soft set (F, E), or as the set of ε-approximate elements of the soft set.

2.2.Soft Null Set [6]

A soft set (F, A) over U is said to be null soft set denoted by $\tilde{\varphi}$ if $\forall\ \varepsilon \in A, F(\varepsilon)=\varphi$ (Null Set).

2.3. Soft Absolute Set [6]

A soft set (F, A) over U is said to be absolute soft set denoted by $\tilde{A}$ if $\forall\ \varepsilon \in A, F(\varepsilon)$=U.

2.4.Soft Subset [5]

For two soft sets (F, A) and (G, B) over the universe U,we say that (F, A) is a soft subset of (G, B) ,if

i) $A \subseteq B$

ii) $\forall\ \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A)\ \tilde{\subseteq}\ (G, B)$.

(F, A) is said to be soft superset of (G, B) if (G, B) is a soft subset of (F, A) and we write $(F, A)\ \tilde{\supseteq}\ (G, B)$.

2.5. Union of Soft Sets [6]

Union of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C), where $C = A \cup B$ and $\forall\ \varepsilon \in C$,

$$H(\varepsilon)=\begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$.

2.6. Intersection of Soft Sets [2]

Let (F, A) and (G, B) be two soft sets over a common universe U with $A \cap B \neq \emptyset$.Then intersection of two soft sets (F, A) and (G, B) is a soft set (H, C) where $C = A \cap B$ and $\forall\ \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F, A)\ \tilde{\cap}\ (G, B) = (H, C)$.

2.7. AND Operation of Soft Sets [6]

If (F, A) and (G, B) be two soft sets,then " (F, A) AND (G, B)" is a soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$,where

$H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall\ \alpha \in A$ and $\forall\ \beta \in B$,where $\cap$ is the operation intersection of two sets.

2.8.OR Operation of Soft Sets [6]

If (F, A) and (G, B) be two soft sets,then " (F, A) OR (G, B)" is a soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$,where

$K(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall\ \alpha \in A$ and $\forall\ \beta \in B$,where $\cup$ is the operation union of two sets.

2.9.Complement of a Soft Set [10]

The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow P(U)$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$.

3. A STUDY ON THE OPERATIONS ON FUZZY SOFT SETS

In this section, we first recall some basic definitions related to fuzzy soft sets and we shall endeavour to study the basic operations and results available in the literature of fuzzy soft sets. Here ,we put forward some results of fuzzy soft sets in our way.

3.1. Fuzzy Soft Set [7]

A pair (F, A) is called a fuzzy soft set over U where $F: A \rightarrow \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$.

3.2. Fuzzy Soft Class [1]

Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

3.3. Fuzzy Soft Null Set [7]

A soft set (F, A) over U is said to be null fuzzy soft set denoted by φ if $\forall\ \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of U where $\overline{0}(x) = 0\ \forall\ x \in U$.

3.4.Fuzzy Soft Absolute Set [7]

A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall\ \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set $\overline{1}$ of U where $\overline{1}(x) = 1\ \forall\ x \in U$.

3.5. Fuzzy Soft Subset [7]

For two fuzzy soft sets (F,A) and (G,B) in a fuzzy soft class (U,E),we say that (F,A) is a fuzzy soft subset of (G,B) ,if

i) $A \subseteq B$

ii) $\forall\ \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F,A)\ \tilde{\subseteq}\ (G,B)$.

3.6.Union of Fuzzy Soft Sets [7]

Union of two fuzzy soft sets (F,A) and (G,B) in a soft class (U,E) in a fuzzy soft set (H,C), where $C = A \cup B$ and $\forall\ \varepsilon \in C$,

$$H(\varepsilon)=\begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F,A)\ \tilde{\cup}\ (G,B) = (H,C)$.

3.7.Intersection of Fuzzy Soft Sets [2]

Let (F,A) and (G,B) be two fuzzy soft sets in a soft class (U,E) with $A \cap B \neq \emptyset$.Then intersection of two fuzzy soft sets (F,A) and (G,B) in a soft class (U,E) is a fuzzy soft set (H,C) where $C = A \cap B$ and $\forall\ \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F,A)\ \tilde{\cap}\ (G,B) = (H,C)$.

3.8.AND Operation of Fuzzy Soft Sets [7]

If (F,A) and (G,B) be two fuzzy soft sets,then " (F,A) AND (G,B)" is a fuzzy soft set denoted by $(F,A) \wedge (G,B)$ and is defined by $(F,A) \wedge (G,B) = (H, A \times B)$,where $H(\alpha,\beta) = F(\alpha) \cap G(\beta), \forall\ \alpha \in A$ and $\forall\ \beta \in B$,where $\cap$ is the operation intersection of two fuzzy sets.

3.9.OR Operation of Fuzzy Soft Sets [7]

If (F,A) and (G,B) be two fuzzy soft sets,then " (F,A) OR (G,B)" is a fuzzy soft set denoted by $(F,A) \vee (G,B)$ and is defined by $(F,A) \vee (G,B) = (K, A \times B)$,where $K(\alpha,\beta) = F(\alpha) \cup G(\beta), \forall\ \alpha \in A$ and $\forall\ \beta \in B$,where $\cup$ is the operation union of two fuzzy sets.

3.10.Complement of a Fuzzy Soft Set [8]

The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where

$F^c: A \to \tilde{P}(U)$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$.

3.11. Proposition

1.$(\varphi,A)^c = (U,A)$

Proof:

Let $(\varphi,A) = (F,A)$

Then $\forall\ \varepsilon \in A$,

$$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}$$
$$= \{(x,0) : x \in U\}$$

$(\varphi,A)^c = (F,A)^c = (F^c,A)$, where $\forall\ \varepsilon \in A$,

$$F^c(\varepsilon) = (F(\varepsilon))^c$$
$$= \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}^c$$
$$= \{(x, 1 - \mu_{F(\varepsilon)}(x)) : x \in U\}$$
$$= \{(x, 1-0) : x \in U\}$$
$$= \{(x,1) : x \in U\}$$
$$= U$$

Thus $(\varphi,A)^c = (U,A)$

2.$(U,A)^c = (\varphi,A)$

Proof:

Let $(U,A) = (F,A)$

Then $\forall\ \varepsilon \in A$,

$$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}$$
$$= \{(x,1) : x \in U\}$$

$(\varphi,A)^c = (F,A)^c = (F^c,A)$, where $\forall\ \varepsilon \in A$,

$F^c(\varepsilon) = (F(\varepsilon))^c$
$= \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}^c$
$= \{(x, 1 - \mu_{F(\varepsilon)}(x)): x \in U\}$
$= \{(x, 1 - 1): x \in U\}$
$= \{(x, 0): x \in U\}$
$= \varphi$

Thus $(U, A)^c = (\varphi, A)$

3.$(F, A) \tilde{\cup} (\varphi, A) = (F, A)$

Proof:

We have

$(F, A) = \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$(\varphi, A) = \{(\varepsilon, (x, 0)): x \in U\} \forall \varepsilon \in A$
$(F, A) \tilde{\cup} (\varphi, A) = \{(\varepsilon, (x, max(\mu_{F(\varepsilon)}(x), 0))): x \in U\} \forall \varepsilon \in A$
$= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$= (F, A)$

Thus $(F, A) \tilde{\cup} (\varphi, A) = (F, A)$

4.$(F, A) \tilde{\cup} (U, A) = (U, A)$

Proof:

We have

$(F, A) = \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$(U, A) = \{(\varepsilon, (x, 1)): x \in U\} \forall \varepsilon \in A$
$(F, A) \tilde{\cup} (U, A) = \{(\varepsilon, (x, max(\mu_{F(\varepsilon)}(x), 1))): x \in U\} \forall \varepsilon \in A$
$= \{(\varepsilon, (x, 1)): x \in U\} \forall \varepsilon \in A$
$= (U, A)$

Thus $(F, A) \tilde{\cup} (U, A) = (U, A)$

5.$(F, A) \tilde{\cap} (\varphi, A) = (\varphi, A)$

Proof:

We have

$(F, A) = \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$(U, A) = \{(\varepsilon, (x, 0)): x \in U\} \forall \varepsilon \in A$
$(F, A) \tilde{\cap} (U, A) = \{(\varepsilon, (x, min(\mu_{F(\varepsilon)}(x), 0))): x \in U\} \forall \varepsilon \in A$
$= \{(\varepsilon, (x, 0)): x \in U\} \forall \varepsilon \in$
$= (\varphi, A)$

Thus $(F, A) \tilde{\cap} (\varphi, A) = (\varphi, A)$

6.$(F, A) \tilde{\cap} (U, A) = (F, A)$

Proof:

We have

$(F, A) = \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$(U, A) = \{(\varepsilon, (x, 1)): x \in U\} \forall \varepsilon \in A$
$(F, A) \tilde{\cup} (U, A) = \{(\varepsilon, (x, min(\mu_{F(\varepsilon)}(x), 1))): x \in U\} \forall \varepsilon \in A$
$= \{(\varepsilon, (x, \mu_{F(\varepsilon)}(x))): x \in U\} \forall \varepsilon \in A$
$= (F, A)$

Thus $(F, A) \tilde{\cap} (U, A) = (F, A)$

7.$(F, A) \tilde{\cup} (\varphi, B) = (F, A)$ if and only if $B \subseteq A$

Proof:

We have for (F, A)

$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)): x \in U\} \forall \varepsilon \in A$

Also,

let $(\varphi, B) = (G, B)$, Then

$G(\varepsilon) = \{(x, 0): x \in U\} \ \forall \ \varepsilon \in B$

Let $(F, A) \tilde{\cup} (\varphi, B) = (F, A) \tilde{\cup} (G, B) = (H, C)$,where $C = A \cup B$ and $\forall \ \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, \mu_{G(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, 0): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, max(\mu_{F(\varepsilon)}(x), 0)): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, 0): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

Let $B \subseteq A$

Then

$$H(\varepsilon) = \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

$= F(\varepsilon) \ \forall \ \varepsilon \in A$

Conversely, let $(F, A) \tilde{\cup} (\varphi, B) = (F, A)$

Then $A = A \cup B \Longrightarrow B \subseteq A$

8.$(F, A) \tilde{\cup} (U, B) = (U, B)$ if and only if $A \subseteq B$

Proof:

We have for (F, A)

$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)): x \in U\} \ \forall \ \varepsilon \in A$

Also, let $(U, B) = (G, B)$, Then

$G(\varepsilon) = \{(x, 1): x \in U\} \ \forall \ \varepsilon \in B$

Let $(F, A) \tilde{\cup} (U, B) = (F, A) \tilde{\cup} (G, B) = (H, C)$,where $C = A \cup B$ and $\forall \ \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, \mu_{G(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, 1): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, max(\mu_{F(\varepsilon)}(x), 1)): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}, if \ \varepsilon \in A - B \\ \{(x, 1): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, 1): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$$

Let $A \subseteq B$

Then $H(\varepsilon) = \begin{cases} \{(x, 1): x \in U\}, if \ \varepsilon \in B - A \\ \{(x, 1): x \in U\}, if \ \varepsilon \in A \cap B \end{cases}$

$= G(\varepsilon) \ \forall \ \varepsilon \in B$

Conversely, let $(F, A) \tilde{\cup} (U, B) = (U, B)$

Then $B = A \cup B \Longrightarrow A \subseteq B$

9.$(F, A) \tilde{\cap} (\varphi, B) = (\varphi, A \cap B)$

Proof:

We have for (F, A)

$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}\ \forall\ \varepsilon \in A$

Also, let $(\varphi, B) = (G, B)$, Then

$G(\varepsilon) = \{(x, 0): x \in U\}\ \forall\ \varepsilon \in B$

Let $(F, A) \tilde{\cap} (\varphi, B) = (F, A) \tilde{\cap} (G, B) = (H, C)$,where

$C = A \cap B$ and $\forall\ \varepsilon \in C$,

$H(\varepsilon) = \{(x, min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))): x \in U\}$

$= \{(x, min(\mu_{F(\varepsilon)}(x), 0)): x \in U\}$

$= \{(x, 0): x \in U\}$

Thus $F, A) \tilde{\cap} (\varphi, B) = (\varphi, A \cap B)$

10.$(F, A) \tilde{\cap} (U, B) = (F, A \cap B)$

Proof:

We have for (F, A)

$F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}\ \forall\ \varepsilon \in A$

Also, let $(U, B) = (G, B)$, Then

$G(\varepsilon) = \{(x, 1): x \in U\}\ \forall\ \varepsilon \in B$

Let $(F, A) \tilde{\cap} (U, B) = (F, A) \tilde{\cap} (G, B) = (H, C)$,where

$C = A \cap B$ and $\forall\ \varepsilon \in C$,

$H(\varepsilon) = \{(x, min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))): x \in U\}$

$= \{(x, min(\mu_{F(\varepsilon)}(x), 1)): x \in U\}$

$= \{(x, \mu_{F(\varepsilon)}(x)): x \in U\}$

Thus $(F, A) \tilde{\cap} (U, B) = (F, A \cap B)$

3.12. Proposition

1.$((F, A) \tilde{\cup} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$

2. $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cap} (G, B)^c$

Ahmad and Kharal [2] proved by counter examples that these propositions are not valid.However the following inclusions are due to Ahmad and Kharal [2].They proved these results with the definition of complement initiated by Maji et al.[7].Below we are giving the proof in our way.

3.13.Proposition

For fuzzy soft sets (F, A) and (G, B) over the same universe U,we have the following

1.$((F, A) \tilde{\cup} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cup} (G, B)^c$

2. $((F, A) \tilde{\cap} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cap} (G, B)^c$

Proof :

1.Let$(F, A) \tilde{\cup} (G, B) = (H, C)$,where $C = A \cup B$ and $\forall\ \varepsilon \in C$,

$$H(\varepsilon)=\begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x))\}, if\ \varepsilon \in A - B \\ \{(x, \mu_{G(\varepsilon)}(x))\}, if\ \varepsilon \in B - A \\ \{(x, max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon \in A \cap B \end{cases}$$

Thus

$((F, A) \tilde{\cup} (G, B))^c = (H, C)^c = (H^C, C)$,where $C = A \cup B$ and $\forall\ \varepsilon \in C$,

$H^c(\varepsilon) = (H(\varepsilon))^c$

$$= \begin{cases} (F(\varepsilon))^c, & \text{if } \varepsilon \in A - B \\ (G(\varepsilon))^c, & \text{if } \varepsilon \in B - A \\ (F(\varepsilon) \cup G(\varepsilon))^c, & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, 1-\mu_{F(\varepsilon)}(x))\}, if\ \varepsilon \in A-B \\ \{(x, 1-\mu_{G(\varepsilon)}(x))\}, if\ \varepsilon \in B-A \\ \{(x, 1-max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon \in A\cap B \end{cases}$$

$$= \begin{cases} \{(x, 1-\mu_{F(\varepsilon)}(x))\}, if\ \varepsilon \in A-B \\ \{(x, 1-\mu_{G(\varepsilon)}(x))\}, if\ \varepsilon \in B-A \\ \{(x, min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon \in A\cap B \end{cases}$$

Again,

$((F,A)\ \tilde{\cup}\ (G,B))^c = (F^c, A)\ \tilde{\cup}\ (G^c, B) = (I, J)$, say where $J = A \cup B$ and $\forall\ \varepsilon \in J$,

$$I(\varepsilon) = \begin{cases} F^c(\varepsilon), & \text{if } \varepsilon \in A-B \\ G^c(\varepsilon), & \text{if } \varepsilon \in B-A \\ F^c(\varepsilon)\ \cup\ G^c(\varepsilon), & \text{if } \varepsilon \in A\cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F^c(\varepsilon)}(x))\}, if\ \varepsilon \in A-B \\ \{(x, \mu_{G^c(\varepsilon)}(x))\}, if\ \varepsilon \in B-A \\ \{(x, max(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x)))\}, if\ \varepsilon \in A\cap B \end{cases}$$

$$= \begin{cases} \{(x, 1-\mu_{F^c(\varepsilon)}(x))\}, if\ \varepsilon \in A-B \\ \{(x, 1-\mu_{G^c(\varepsilon)}(x))\}, if\ \varepsilon \in B-A \\ \{(x, max(1-\mu_{F^c(\varepsilon)}(x), 1-\mu_{G^c(\varepsilon)}(x)))\}, if\ \varepsilon \in A\cap B \end{cases}$$

We see that $C = J$ and $\forall\ \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

Thus $((F,A)\ \tilde{\cup}\ (G,B))^c\ \tilde{\subseteq}\ (F,A)^c\ \tilde{\cup}\ (G,B)^c$

2. Let $(F,A)\ \tilde{\cap}\ (G,B) = (H,C)$,where $C = A\cap B$ and $\forall\ \varepsilon \in C$,

$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$

Thus

$((F,A)\ \tilde{\cap}\ (G,B))^c = (H,C)^c = (H^C, C)$,where $C = A\cup B$ and $\forall\ \varepsilon \in C$,

$H^c(\varepsilon) = (F(\varepsilon) \cap G(\varepsilon))^c$

$= \{x, 1-min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}$

$= \{x, max(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\}$

Again,

$(F,A)^c\ \tilde{\cap}\ (G,B)^c = (F^c, A)\ \tilde{\cap}\ (G^c, B) = (I, J)$, say where $J = A\cup B$ and $\forall\ \varepsilon \in J$,

$I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$

$= \{(x, min(\mu_{F^c(\varepsilon)}(x), \mu_{G^c(\varepsilon)}(x))$

$= \{x, min(1-\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\}$

We see that $C = J$ and $\forall\ \varepsilon \in C, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $((F,A)\ \tilde{\cap}\ (G,B))^c\ \tilde{\supseteq}\ (F,A)^c\ \tilde{\cap}\ (G,B)^c$

$= \{x, min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}$

3.14.Proposition(De Morgan Inclusions)

For fuzzy soft sets (F,A) and (G,B) over the same universe U,we have the following-

1. $(F,A)^c\ \tilde{\cap}\ (G,B)^c\ \tilde{\subseteq}\ ((F,A)\ \tilde{\cup}\ (G,B))^c$
2. $((F,A)\ \tilde{\cap}\ (G,B))^c\ \tilde{\subseteq}\ (F,A)^c\ \tilde{\cup}\ (G,B)^c$

Proof:

1.Let $(F,A)\ \tilde{\cup}\ (G,B) = (H,C)$,where $C = A\cup B$ and $\forall\ \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A-B \\ G(\varepsilon), & \text{if } \varepsilon \in B-A \\ F(\varepsilon)\ \cup\ G(\varepsilon), & \text{if } \varepsilon \in A\cap B \end{cases}$$

$$= \begin{cases} \{(x, \mu_{F(\varepsilon)}(x))\}, if\ \varepsilon \in A-B \\ \{(x, \mu_{G(\varepsilon)}(x))\}, if\ \varepsilon \in B-A \\ \{(x, max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon \in A\cap B \end{cases}$$

Thus

$((F,A)\ \tilde{\cup}\ (G,B))^c = (H,C)^c = (H^C, C)$,where $C = A\cup B$ and $\forall\ \varepsilon \in C$,

$H^c(\varepsilon)$=$(H(\varepsilon))^c$

$$=\begin{cases}(F(\varepsilon))^c, & \text{if } \varepsilon \in A-B\\ (G(\varepsilon))^c, & \text{if } \varepsilon \in B-A\\ (F(\varepsilon)\cup G(\varepsilon))^c, & \text{if } \varepsilon \in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,1-\mu_{F(\varepsilon)}(x))\}, if\ \varepsilon\in A-B\\ \{(x,1-\mu_{G(\varepsilon)}(x))\}, if\ \varepsilon\in B-A\\ \{(x,1-max(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,1-\mu_{F(\varepsilon)}(x))\}, if\ \varepsilon\in A-B\\ \{(x,1-\mu_{G(\varepsilon)}(x))\}, if\ \varepsilon\in B-A\\ \{(x,min(1-\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)))\}, if\ \varepsilon\in A\cap B\end{cases}$$

Again,

$(F,A)^c \tilde{\cap} (G,B)^c$=$(F^c,A)\ \tilde{\cap}\ (G^c,B)=(I,J)$, say where $J=A\cup B$ and $\forall\ \varepsilon\in J$,

$I(\varepsilon)$=$F^c(\varepsilon)\cap G^c(\varepsilon)$

$=\{(x,min(\mu_{F^c(\varepsilon)}(x),\mu_{G^c(\varepsilon)}(x))$

$=\{x,min(1-\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)\}$

We see that $J\subseteq C$ and $\forall\ \varepsilon\in J, I(\varepsilon)\subseteq H^c(\varepsilon)$

Thus $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A)\ \tilde{\cup}\ (G,B))^c$

2. Let $(F,A)\tilde{\cap}\ (G,B)\ =\ (H,C)$,where $C=A\cap B$ and $\forall\ \varepsilon\in C$,

$H(\varepsilon)=F(\varepsilon)\cap G(\varepsilon)$

$=\{x,min(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\}$

Thus

$((F,A)\tilde{\cap}\ (G,B))^c$ = $(H,C)^c$ = (H^C,C),where $C=A\cup B$ and $\forall\ \varepsilon\in C$,

$H^c(\varepsilon)=(F(\varepsilon)\cap G(\varepsilon))^c$

$=\{x,1-min(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\}$

$=\{x,max(1-\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)\}$

Again,

$(F,A)^c\ \tilde{\cup}\ (G,B)^c\ =\ (F^c\ ,\ A)\ \tilde{\cup}\ (G^c,B)=(I,J)$, saywhere $J=A\cup B$ and $\forall\ \varepsilon\in J$,

$$I(\varepsilon)=\begin{cases}F^c(\varepsilon), & \text{if } \varepsilon \in A-B\\ G^c(\varepsilon), & \text{if } \varepsilon \in B-A\\ F^c(\varepsilon)\cup G^c(\varepsilon), & \text{if } \varepsilon \in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,\mu_{F^c(\varepsilon)}(x))\}, if\ \varepsilon\in A-B\\ \{(x,\mu_{G^c(\varepsilon)}(x))\}, if\ \varepsilon\in B-A\\ \{(x,max(\mu_{F^c(\varepsilon)}(x),\mu_{G^c(\varepsilon)}(x)))\}, if\ \varepsilon\in A\cap B\end{cases}$$

$$=\begin{cases}\{(x,1-\mu_{F^c(\varepsilon)}(x))\}, if\ \varepsilon\in A-B\\ \{(x,1-\mu_{G^c(\varepsilon)}(x))\}, if\ \varepsilon\in B-A\\ \{(x,max(1-\mu_{F^c(\varepsilon)}(x),1-\mu_{G^c(\varepsilon)}(x)))\}, if\ \varepsilon\in A\cap B\end{cases}$$

We see that $C=J$ and $\forall\ \varepsilon\in C, H^c(\varepsilon)\subseteq I(\varepsilon)$

Thus $((F,A)\tilde{\cap}\ (G,B))^c\ \tilde{\subseteq}\ (F,A)^c\tilde{\cup}(G,B)^c$

3.15:Proposition (De Morgan Laws)

For fuzzy soft sets $(F_1,A),(F_2,A),\ldots,(F_n,A)$ over the same universe U.weve the following-

1.$((F_1,A)\tilde{\cup}(F_2,A)\tilde{\cup}\ldots\ldots.\tilde{\cup}(F_n,A))^c$=$(F_1,A)^c\ \tilde{\cap}\ (F_2,A)^c\ \tilde{\cap}\ldots\ldots.\tilde{\cap}(F_n,A)^c$

2. $((F_1,A)\tilde{\cap}(F_2,A)\tilde{\cap}\ldots\ldots.\tilde{\cap}(F_n,A))^c$=$(F_1,A)^c\ \tilde{\cup}\ (F_2,A)^c\ \tilde{\cup}\ldots\ldots.\tilde{\cup}(F_n,A)^c$

Proof:

1.Let $(F_1,A)\tilde{\cup}(F_2,A)\tilde{\cup}\ldots\ldots.\tilde{\cup}(F_n,A)=(H,A)$ where $\forall\ \varepsilon\in A$,

$H(\varepsilon)=F_1(\varepsilon)\cup F_2(\varepsilon)\ldots\cup F_n(\varepsilon)$

$=\{x,max(\mu_{F_1(\varepsilon)},\mu_{F_2(\varepsilon)},\ldots\mu_{F_n(\varepsilon)}\}$

Thus

$((F_1,A)\,\tilde{\cup}\,(F_2,A)\,\tilde{\cup}\,\dots\dots\tilde{\cup}\,(F_n,A))^c = (H,A)^C = (H^C,A)$ where $\forall\ \varepsilon \in A$,

$$H^C(\varepsilon) = (H(\varepsilon))^C$$
$$= (F_1(\varepsilon) \cup F_2(\varepsilon) \cup \dots \cup F_n(\varepsilon))^C$$
$$= \{x, 1 - max(\mu_{F_1(\varepsilon)}, \mu_{F_2(\varepsilon)}, \dots \mu_{F_n(\varepsilon)})\}$$
$$= \{x, min(1-\mu_{F_1(\varepsilon)}, 1-\mu_{F_2(\varepsilon)}, \dots 1-\mu_{F_n(\varepsilon)})\}$$

Again,

$(F_1,A)^c\,\tilde{\cap}\,(F_2,A)^c\,\tilde{\cap}\dots\tilde{\cap}\,(F_n,A)^c = ({F_1}^c,A)\,\tilde{\cap}\,({F_2}^c,A)\,\tilde{\cap}\dots\tilde{\cap}\,({F_n}^c,A) = (I,A)$ say where $\forall\varepsilon \in A$,

$$I(\varepsilon) = {F_1}^c(\varepsilon) \cap {F_2}^c(\varepsilon) \cap \dots\dots\cap {F_n}^c(\varepsilon)$$
$$= \{x, min(\mu_{F_1(\varepsilon)}(x), \mu_{F_2(\varepsilon)}(x), \dots.\mu_{F_n(\varepsilon)}(x))\}$$
$$= \{x, min(1-\mu_{F_1(\varepsilon)}(x), 1-\mu_{F_2(\varepsilon)}(x), \dots.1-\mu_{F_n(\varepsilon)}(x))\}$$

Thus

$((F_1,A)\,\tilde{\cup}\,(F_2,A)\,\tilde{\cup}\,\dots\dots\tilde{\cup}\,(F_n,A))^c = (F_1,A)^c\ \tilde{\cap}\,(F_2,A)^c\ \tilde{\cap}\dots\dots\tilde{\cap}\,(F_n,A)^c$

Hence proved

2.Let $(F_1,A)\,\tilde{\cap}\,(F_2,A)\,\tilde{\cap}\,\dots\dots\tilde{\cap}\,(F_n,A) = (H,A)$ where $\forall\ \varepsilon \in A$,

$$H(\varepsilon) = F_1(\varepsilon) \cap F_2(\varepsilon) \dots \cap F_n(\varepsilon)$$
$$= \{x, min(\mu_{F_1(\varepsilon)}, \mu_{F_2(\varepsilon)}, \dots \mu_{F_n(\varepsilon)}\}$$

Thus

$((F_1,A)\,\tilde{\cap}\,(F_2,A)\,\tilde{\cap}\,\dots\dots\tilde{\cap}\,(F_n,A))^c = (H,A)^C = (H^C,A)$ where $\forall\ \varepsilon \in A$,

$$H^C(\varepsilon) = (H(\varepsilon))^C$$
$$= (F_1(\varepsilon) \cap F_2(\varepsilon) \cap \dots \cap F_n(\varepsilon))^C$$
$$= \{x, 1 - min(\mu_{F_1(\varepsilon)}, \mu_{F_2(\varepsilon)}, \dots \mu_{F_n(\varepsilon)})\}$$
$$= \{x, max(1-\mu_{F_1(\varepsilon)}, 1-\mu_{F_2(\varepsilon)}, \dots 1-\mu_{F_n(\varepsilon)})\}$$

Again,

$(F_1,A)^c\,\tilde{\cup}\,(F_2,A)^c\,\tilde{\cup}\dots\tilde{\cup}\,(F_n,A)^c = ({F_1}^c,A)\,\tilde{\cup}\,({F_2}^c,A)\,\tilde{\cup}\dots\tilde{\cup}\,({F_n}^c,A) = (I,A)$ say where $\forall\varepsilon \in A$,

$$I(\varepsilon) = {F_1}^c(\varepsilon) \cup {F_2}^c(\varepsilon) \cup \dots\dots\cup {F_n}^c(\varepsilon)$$
$$= \{x, max(\mu_{F_1(\varepsilon)}(x), \mu_{F_2(\varepsilon)}(x), \dots.\mu_{F_n(\varepsilon)}(x))\}$$
$$= \{x, max(1-\mu_{F_1(\varepsilon)}(x), 1-\mu_{F_2(\varepsilon)}(x), \dots.1-\mu_{F_n(\varepsilon)}(x))\}$$

Thus

$((F_1,A)\,\tilde{\cap}\,(F_2,A)\,\tilde{\cap}\,\dots\dots\tilde{\cap}\,(F_n,A))^c = (F_1,A)^c\ \tilde{\cup}\,(F_2,A)^c\ \tilde{\cup}\dots\dots\tilde{\cup}\,(F_n,A)^c$

Hence proved

4.CONCLUSION

The Soft Set Theory of Molodstov[3] offers a general mathematical tool for dealing with uncertain and vague objects. At present, work on the extension of soft set theory is progressing r0-papidly. Maji et al.[7] proposed the concept of fuzzy soft set and developed some properties of fuzzy soft sets and in recent years the researchers have contributed a lot towards the fuzzification of Soft Set Theory. This paper contributes some more properties regarding fuzzy soft sets and support these propositions with proof.We further give the extension of De Morgan Law in this paper.We hope that our findings will help enhancing this study on fuzzy soft sets.

5.REFERENCES

1. Manoj Borah,Tridiv Jyoti Neog, and Dusmanta Kumar Sut,"A Study On Some Operations of Fuzzy Soft Sets",International Journal of Modern Engineering Research,vol. 2,pp. 219-225,2012.
2. B. Ahmad and A. Kharal, "On Fuzzy Soft Sets", Advances in Fuzzy Systems,Volume 2009.

3. D. A. Molodtsov, “Soft Set Theory - First Result”, Computers and Mathematics with Applications, Vol. 37, pp. 19-31, 1999.
4. D. Chen, E.C.C. Tsang, D.S. Yeung, and X.Wang , “The parameterization reduction of soft sets and its applications,” Computers & Mathematics with Applications, vol. 49, no.5-6, pp. 757–763, 2005.
5. D. Pei and D. Miao, “From soft sets to information systems,” in Proceedings of the IEEE International Conference on Granular Computing, vol. 2, pp. 617–621, 2005.
6. P. K. Maji and A.R. Roy, “Soft Set Theory”, Computers and Mathematics with Applications 45 (2003) 555 – 562.
7. P. K. Maji, R. Biswas and A.R. Roy, “Fuzzy Soft Sets”, Journal of Fuzzy Mathematics, Vol 9 , no.3,pp.-589-602,2001.
8. T. J. Neog, D. K. Sut, “On Fuzzy Soft Complement and Related Properties” , Accepted for publication in International Journal of Energy, Information and communications (IJEIC).
9. T. J. Neog, D. K. Sut, “On Union and Intersection of Fuzzy Soft Sets”, Int.J. Comp. Tech. Appl., Vol 2 (5), 1160-1176
10. T. J. Neog and D. K. Sut, “ A New Approach To The Theory of Soft Sets” , International Journal of Computer Applications, Vol 32, No 2,October 2011, pp 1-6 .
11. Xun Ge and Songlin Yang, “ Investigations on some operations of soft sets”, World Academy of Science, Engineering and Technology, 75, 2011, pp. 1113-1116.