SOME RESULTS AND SEPARATION OF FUZZY TOPOLOGICAL SPACE

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ABSTRACT:

We study some separation properties of Fuzzy topological space. As the standard definition of Fuzzy compactness doesn't hold good in our definition of Fuzzy Hansdroff space, we introduce a new definition of power compactness and prove some interesting results associated with it.

Fuzzy Topological Subgroup

Rosenfeld (1971) defines fuzzy topological subgroup as below.

1.1 Fuzzy Topological Space Subgroup

A fuzzy subset A of a group G is called a fuzzy subgroup of G if, for all x, y in G, the following conditions are satisfied:

(1) $A(x, y) \geq \min\{A(x), A(y)\}$

(2) $A(x^{-1}) \geq A(x)$

In other words, a fuzzy topological subgroupoid A of group G will be called a fuzzy subgroup of G if $A(x^{-1}) \geq A(x), \forall x \in G$

Clearly when A is considered as an ordinary subset of G then $A(x) = 1$ and $A(y) = 1$ when $x, y\ A$.

Consequently from above definition, we have

$$A(xy^{-1}) \geq \min\{A(x), A(y^{-1})\}$$
$$\geq \min\big(A(x), A(y)\big), by\ condition\ \text{(ii)}$$
$$= \min\{1,1\}$$

i.e. $A(xy^{-1}) = 1$ which in turn implies that $xy^{-1} \in A$. Thus, $\forall x \in A, y \in A$ implies that $xy \in A$.

1.2 Fuzzy Subgroup Redefined

A fuzzy set A on a group G is called a fuzzy subgroup of G if for all $x, y\ G$, the following conditions are satisfied:

(i) $A(xy) \geq A(x).A(y)$

(ii) $A(x') \geq A(x)$

Here we observe that any fuzzy subgroup under the definition of Rosenfeld is also a fuzzy subgroup under our definition but not conversely.

If A is a fuzzy subgroup over G according to Rosenfeld, then we have $A(xy) \geq \min\{A(x), A(y)\} \geq A(x).A(y), \forall x, y \in G$. From this we infer that A is a FSG under our definition when it is a fuzzy subgroup (FSG) under the definition of Rosenfeld.

Conversely let us assume that A is a fuzzy subgroup over G under our definition i.e.

$$(xy) \geq A(x).A(y), \forall x, y \in G$$

Then $A(x).A(y) \geq \min(\ A(x), A(y)\}$ is not true for all $x, y\ G$.

Therefore, A is not fuzzy subgroup under the definition of Rosenfeld though it is fuzzy definition of Rosenfeld though it is a fuzzy subgroup under our definition.

1.3 Example

We will show by an example that the intersection of any family of fuzzy topological subgroup of a G is a fuzzy subgroup of G.

Here we will provide an example to establish the fact that intersection of two fuzzy topological subgroups over a groups G is again a fuzzy topological subgroup of G.

Let $G = (e, a, b, c)$ Define a binary operation '.' in G by the following multiplication table.

.	**e**	**A**	**b**	**c**
E	**e**	**A**	**b**	**c**
a	**a**	**E**	**c**	**b**
b	**b**	**C**	**e**	**a**
c	**c**	**b**	**a**	**e**

To determine the element of G assigned to a.b, we look at the intersection of the row labeled by a and the column headed by b.

Also, $a^2 = b^2 = c^2 = e$.

All the group axioms are satisfied. This group is usually called as Klein 4 – group named after Felix Klein (1849 – 1925)

Let λ_i, where $0 \leq i \leq 5$ be the number lying in the closed interval [0,1] such that $\lambda_0 > \lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5$.

Let us define fuzzy subsets $A, B: G \rightarrow [0,1]$ by the setting

$$A(e) = \lambda_0, A(a) = \lambda_5, A(b) = \lambda_2, A(c) = \lambda_5 and\ B(e) = \lambda_1, B(a) = \lambda_3, B(b) = B(c) = \lambda_4$$

It just remain to show that $A \cap B$ is also a fuzzy subgroup of G.

We have,

$A \cap B)\ (e) = \min\{A(e), B(e)\} = \min\{\lambda_0, \lambda_1\} = \lambda_1$

$= \min(A(a), B(a)\} = \min\{\lambda_5, \lambda_3\} = \lambda_5$

$= \min\{A(b), B(b)\} = \min\{\lambda_2, \lambda_4\} = \lambda_4$

$= \min\{A(c), B(c)\} = \min\{\lambda_5, \lambda_4\} = \lambda_5$

Then we get

(i) $A(\cap B)(ea) = (A \cap B)(a)$

$= \lambda_5 \geq \lambda_1 . \lambda_5$

$= (A \cap B)(e). (A \cap B)(a)$

$\therefore (A \cap B)(ea) \geq (A \cap B)(e). (A \cap B)(a)$

(ii) $A(\cap B)(eb) = (A \cap B)(b)$

$= \lambda_4 \geq \lambda_1 . \lambda_4$

$= (A \cap B)(e). (A \cap B)(b)$

$\therefore (A \cap B)(eb) \geq (A \cap B)(e). (A \cap B)(b)$

(iii) $A(\cap B)(ec) = (A \cap B)(c)$

$= \lambda_5 \geq \lambda_1 . \lambda_5$

$= (A \cap B)(e). (A \cap B)(c)$

$\therefore (A \cap B)(ec) \geq (A \cap B)(e). (A \cap B)(c)$

(iv) $A(\cap B)(ee) = (A \cap B)(e)$

$= \lambda_5 \geq \lambda_5 . \lambda_5$

$= (A \cap B)(e). (A \cap B)(e)$

$\therefore (A \cap B)(ee) \geq (A \cap B)(e). (A \cap B)(e)$

(v) $A(\cap B)(aa) = (A \cap B)(e)$

$= \lambda_1 \geq \lambda_5 . \lambda_5$

$= (A \cap B)(a). (A \cap B)(a)$

$\therefore (A \cap B)(aa) \geq (A \cap B)(a). (A \cap B)(a)$

Similarly we can show that the result holds for other elements.

We also have

Again in this case too

$A(x, y) \geq A(x). A(y) holds.$

$\therefore A(xy) \geq A(x). A(y) holds\ in\ all\ cases.$

Again if $x \in T, x^{-1} \in T$.

$\therefore A(x) = 1 = A(x^{-1})$

$\therefore A(x^{-1}) \geq A(x) holds.$

And of $x \notin T, x^{-1} \notin T$

$\therefore A(x) = 0\ and\ A(x^{-1}) = 0$

$\therefore A(x^{-1}) \geq A(x) again\ holds.$

It follows that A is fuzzy subgroup of G in all cases. Conversely, we suppose that A is a fuzzy subgroup of the group G. Then, we claim that T is a subgroup of G.

By hypothesis A is the characteristic function of $T \subseteq G$. Now if $x, y \in T, then\ A(x) = 1 = A$.

Since A is a fuzzy subgroup of G. Therefore

$A(xy) \geq A(x). A(y)$

= 1.1
= i
$\therefore A(xy) = 1 \Rightarrow xy \in T$
Thus $x \in T, y \in T \Rightarrow xy \in T$
We also have
$A(x^{-1}) > A(x) = 1$
$A(x^{-1}) = 1 \Rightarrow x^{-1} \in T$
i.e. $x \in T \Rightarrow x^{-1} \in T$
Hence, T is a subgroup of G.

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