Stress Analysis of Pressure Vessels

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ABSTRACT

This paper discusses the stresses developed in thin walled pressure vessels. Pressure vessels are designed to hold gases or liquids at a pressure substantially different from atmospheric pressure. Equations of static equilibrium along with free body diagrams will be used to determine the normal stress σ_1 in the hoop direction and σ_2 in the longitudinal direction

Keyword : - *Stress, Pressure Vessel, Longitudinal, Circumferential*

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1. INTRODUCTION

Pressure vessels are compressed gas storage tanks designed to hold gases or liquids at a pressure substantially different from the ambient pressure. The design of pressure vessels encompasses many combinations of stresses. The safe design, installation, operation, and maintenance of pressure vessels are in accordance with codes such as American Society of Mechanical Engineers (ASME) boiler and pressure vessel code [1] Longitudinal and hoop stresses are produced due to internal pressure and variable bending stress due to bending moment caused by wind loads. Therefore a designer has to keep all the probable loads and moments on a pressure vessel[2]. Spherical Pressure vessel is preferred for storage of high pressure fluids. A spherical pressure vessel has approximately twice the strength of a cylindrical pressure vessel with the same wall thickness. A sphere is a very strong structure. The distribution of stresses on the sphere's surfaces, both internally and externally are equal. Spheres however, are much more costly to manufacture than cylindrical vessels.The normal stress in the walls of the container is proportional to the pressure and radius of the vessel and inversely proportional to the thickness of the walls [3] [4]. As a general rule, pressure vessels are considered to be thin-walled when the ratio of radius r to wall thickness t is greater than 10 [5]. Pressure vessels fail when the stress state in the wall exceeds some failure criterion [6] [7]. Thus, it is important to understand and quantify (analyze) stresses in pressure vessels. In this paper we will analyze the stresses in thin-walled pressure vessels (cylindrical & spherical shapes)

2. THIN WALLED CYLINDRICAL PRESSURE VESSEL

A thin-walled half cylinder subjected to internal pressure is shown in Figure 1.The normal stresses $\sigma_1(\sigma_{H)}$ and $\sigma_2(\sigma_{L)}$ acting on the side faces of this element. No shear stresses act on this face because of the symmetry of the vessel. Therefore the stresses are principal stresses .As per their direction σ_1 is called circumferential or hoop stress and σ_2 is called the longitudinal stress. These stresses can be calculated from static equilibrium equations

Several assumptions are made while using these equations

1) Plane sections remain plane
2)There are no shear stresses acting in the wall.
3)The longitudinal and hoop stresses do not vary through the wall.

4) Material is linear-elastic, isotropic and homogeneous.
5) Stress distributions throughout the wall thickness is constant

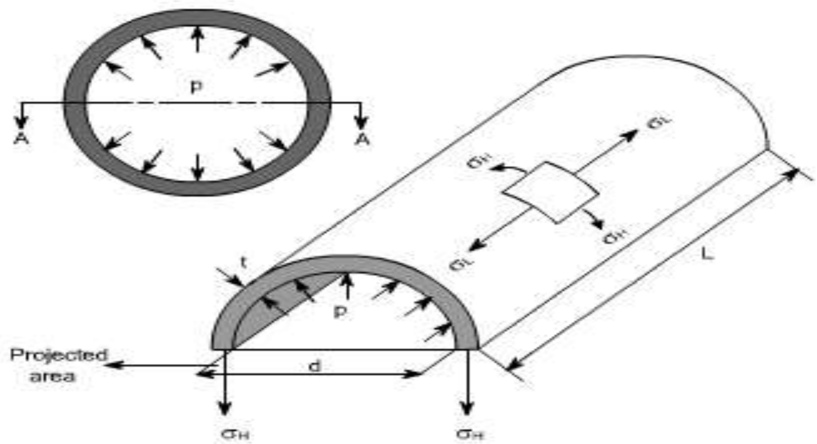

Fig-1: One half of the cylinder subjected to internal pressure

2.1 Circumferential Stress

The circumferential stress σ_1 acting in the wall of the vessel have a resultant equal to $\sigma_1(2bt)$ where t is the thickness of the wall .Also the resultant force P of the internal pressure is equal to 2pbr where r is the inner radius of the cylinder. Hence, we have the following equation of equilibrium:

$$\sigma_1 (2bt) - 2pbr = 0$$

From the above equation, the circumferential stress in a pressurized cylinder can be found:

$$\sigma_1 = pr/t \quad \text{……….} \quad 1$$

2.2 Longitudinal Stress

The longitudinal stress σ_2 is obtained from static equilibrium of a free body diagram as shown in figure-2.The stress σ_2 acts longitudinally and have a resultant equal to $\sigma_2(2\pi rt.)$The resultant force P of the internal pressure is a force equal to $p\pi r^2$.The equation of equilibrium for the free body diagram is given as

$$\sigma_2(2\pi rt) = p\pi r^2$$

$$\sigma_2 = pr/2t \quad \text{…………}2$$

3. Spherical Pressure Vessel

A similar approach can be used to derive an expression for an internally pressurized thin-wall spherical vessel. A spherical pressure vessel is just a special case of a cylindrical vessel.
To find we cut the sphere into two hemispheres. The free body diagram gives the equilibrium equation

$$\sigma_1 (2\pi rt) = p \pi r^2$$

This gives the value of stress as

$$\sigma_1 = pr/2t \ldots\ldots\ldots\ldots\ldots 3$$

Comparing equations 1, 2 and 3 the spherical geometry is twice as efficient in terms of wall stress.

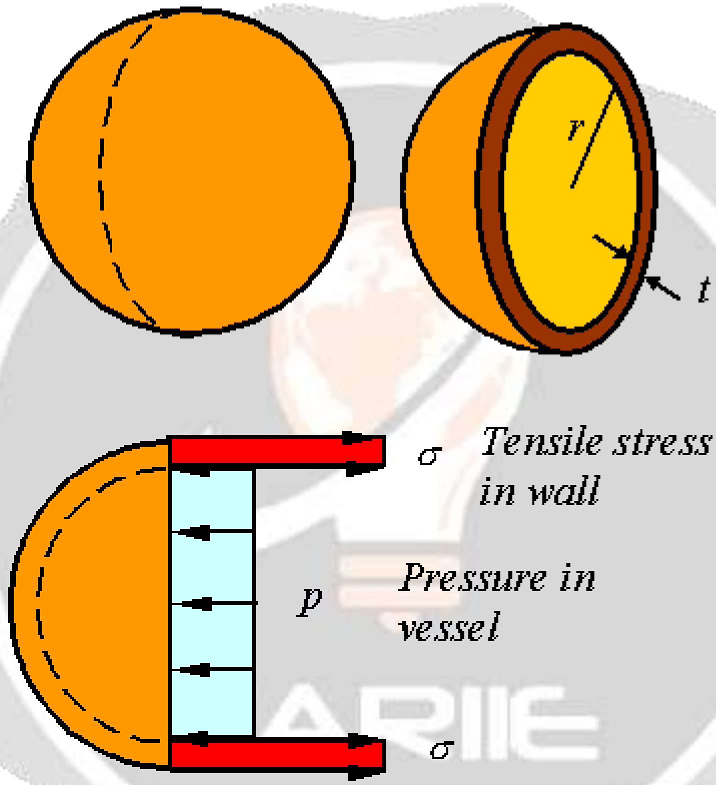

Fig -2: One half of the sphere subjected to internal pressure

4. CONCLUSIONS

This paper presented a detailed stress analysis of the stresses developed in thin-walled pressure vessels (cylindrical & spherical).The longitudinal stress, hoop stress were determined from equations of generalized Hooke's law for stress and strain under assumptions of anisotropy and homogeneity. In the end it is found that spherical pressure vessels are better than cylindrical vessels because circumferential stress is half than the stress in cylindrical vessel.

5. ACKNOWLEDGEMENT

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6. REFERENCES

[1] Rao, K. (2012) Companion Guide to the ASME Boiler and Pressure Vessel Code, Volume 1, Fourth Edition, ASME. http://dx.doi.org/10.1115/1.859865
[2] U.C. Jindal, (2010) Machine Design. 4th Edition, ISBN 978-81-317-1659-5
[3] Gere, J. and Timoshenko, S. (1997) Mechanics of Materials. 4th Edition, PWS, 549. http://dx.doi.org/10.1007/978-1-4899-3124-5
[4] Moss, D. (2013) Pressure Vessel Design Manual. Elsevier. http://dx.doi.org/10.1016/b978-0-12-387000-1.10032-4
[5] Freyer, D. and Harvey, J. (1998) High Pressure Vessels. Springer, Berlin, 11.http://dx.doi.org/10.1007/978-1-4615-5989-4
[6] Annaratone, D. (2007) Pressure Vessel Design. Springer, Berlin, 47-125. http://dx.doi.org/10.1007/978-3-540-49144-6
[7] Shigley, J. (1983) Mechanical Engineering Design. 4th Edition, McGraw-Hill, New York, 70. http://dx.doi.org/10.1115/1.3258702)