STUDY ON OPTIMIZATION OF FRACTIONAL FUNCTION AND THEIR APPLICATIONS

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ABSTRACT

Operations Research (OR) refers to the approach to systematic and scientific study of the operations of a system. Besides developing science, it involves the application of scientific methods, techniques and tools so as to provide optimum solutions to the operational problems of a system. During the Second World War, OR was used to find out the most effective allocation of limited military resources to various military operations and to the activities with each operation. In this way, OR began to creep into business and industry at the global level. In India, OR came into existence in 1949 with the advent of OR unit at the Regional Research Laboratory in Hyderabad.
Linear programming is a technique used for determine an optimum schedule of interdependent activities based on the available resources. The word "linear" stands for all the variables that occur in the objective function and the first degree constraints in the problems under consideration.

Keyword: - *Operations Research1, Scientific Methods2, techniques3, and Tools4*

1. OVERVIEW

The efficiency of a system is sometimes characterized by a ratio of technical and/or economical terms. Maximizing the efficiency then leads to a fractional program. Some examples are given below:

1.1 Maximization of Productivity

Gilmore and Gomory discuss a stock cutting problem in the paper industry for which under the given circumstances it is more appropriate to minimize the ratio of wasted and used amount of raw material rather than just minimizing the amount of wasted material. This stock cutting problem is formulated as a linear fractional program.
In a case study, Hoskins and Blom use fractional programming to optimize the allocation of warehouse personnel. The objective is to minimize the ratio of labor cost to the volume entering and leaving the distribution center.

1.2 Maximization of Return on Investment

In some resource allocation problems the ratio profit/capital or profit/revenue is to be maximized. A related objective is return per cost maximization. Resource allocation problems with this objective are discussed in more detail by Mjelde. In these models the term 'cost' may either be related to actual expenditure or may stand, for example, for the amount of pollution or the probability of a disaster in nuclear energy production. Depending on the nature of the functions describing return, profit, cost or capital, different types of fractional programs are encountered. For example, if the price per unit depends linearly on the output and cost and capital are affine functions, then maximization of the return on investment gives rise to a concave quadratic fractional program (assuming linear constraints).

1.3 Maximization of Return/Risk

A concave nonquadratic fractional program arises in a portfolio selection problem by Ziemba, Parkan and Brooks-Hill. For related concave and nonconcave fractional programs arising in financial planning are obtained by Ziemba. Markov decision processes may also lead to the maximization of the ratio of mean and standard deviation.
Minimization of Cost/Time
In several routing problems a cycle in a network is to be determined which minimizes the cost-to-time ratio or maximizes the profit-to-time ratio. Also, stochastic processes give rise to the minimization of cost per unit time. For example, certain maintenance problems can be formulated as a Markov decision process and, in turn, lead to a linear fractional program. Here the ratio of the expected cost for inspection, maintenance and replacement and the expected time between two inspections is to be minimized.

1.4 Maximization of Output/Input

Charnes and Cooper use a linear fractional program as a model to evaluate the efficiency of decision making units (Data Envelopment Analysis (DEA)). Given a collection of decision making units, the efficiency of each unit is obtained from the maximization of a ratio of weighted outputs and weighted inputs subject to the condition that similar ratios for every decision making unit are less than or equal to unity. The variable weights are then the efficiency of each member relative to that of the others.
In the management literature there has been an increasing interest in optimizing relative terms such as relative profit. No longer are these terms merely used to monitor past economic behavior. Instead the optimization of rates is getting more attention in decision making processes for future projects.

1.5 Non-Economic Applications

In information theory the capacity of a communication channel can be defined as the maximal transmission rate over all probabilities. This is a concave nonquadratic fractional program.
The eigenvalue problem in numerical analysis can be reduced to the maximization of the Rayleigh quotient, and hence gives rise to a quadratic fractional program which is generally not concave.
An example of a fractional program in physics is given by Falk. He maximizes the signal-to-noise ratio of a spectral filter which is a concave quadratic fractional program.

1.6 Indirect Applications

There are a number of management science problems that indirectly give rise to a fractional program.
A concave quadratic fractional program arises in location theory as the dual of a certain minimax multi-facility location problem. In large-scale mathematical programming, decomposition methods reduce the given linear program to a sequence of smaller problems. In some of these methods the sub problems are linear fractional programs. The ratio originates in the minimum-ratio-rule of the simplex method. Fractional programs are also met indirectly in stochastic programming, as first shown by Charnes and Cooper and by Bereanu.

2. LITERATURE SURVEY

In 2003 Dantzig[25] found the best assignment of 70 people to 70 jobs and still shows his success. To compute large needed to view all permutation to select the best allocation is impossible. He observed that by using the simplex algorithm takes just a few moments to find the answer and also found that the answer is at the corner of polygon formed by the constraints of the problem.

In 1956, Isbell and Marlow [1], first identified an example of LFP problem and solved it by a sequence of linear programming problems.

In 1962 Charnes and Cooper[2] considered variable transformation method to solve LFPP and the updated objective function method were developed for solving the LFP problem by Bitran and Novaes (1973)[3]. Gilmore and Gomory(1963)[4], Martos (1964)[5], Swarup (1965)[10], Wagner and Yuan (1968)[13], Pandey and Punnen (2007) [8]and Sharma et al.
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In 1980, Sharma, J. K., Gupta, A. K., & Gupta, M. P.[9] solved the LFP problem by various types of solution procedures based on the simplex method developed by Dantzig (1962).In (1981),[15] Singh, H.C. in his paper made a useful study about the optimality condition in fractional programming. Also in 1981 [24] Kornbluth and Steuer considered Multi - criterion linear fractional programming (MCLFP) problem and presented a simplex –based solution procedure to find all weakly efficient vertices of the augmented feasible region.

In 1985, Benson[23] in his article showed that the procedure suggested by Kornbluth and Steuer for computing the numbers to find break points may not work all the time and he proposed a failsafe method for computing these numbers. Also in[21,22], amount of work which has been done since 1980, on solving the problem of fractional mathematical programming, in particular, the case of bi criterion which has received a considerable attention.

In 1993, the concept of Multi-Objective Programming (MOP), has become popular among researchers during the past few years due to the fact that many single objective optimization methods are not able to help practitioners reach desirable solutions [18-20].

In 2002[17] Chdhas-Rachael solves a system of linear of inequalities in which the objective function is expressed as one of the constraint along with the given set of linear constraints of the problem.

In 2007& 2008, Tantawy [11,12]proposed two different approaches namely; a feasible direction approach and a duality approach to solve the LFP problem. In 2011 Enkhbat, Bazrsadand Enkhbatyan gave a method for Fractional Programming [14]. Fang, Gao, Sheu and Xing (2009) gave global optimization for a class of Fractional Programming Problems [28].

Recently Mojtaba, Borza et al. (2012)[6] solved the LFP problem with interval coefficients in objective function which is based on Charnes and Cooper technique (1962). Odior (2012) [7] solved the LFP problem by algebraic approach which depends on the duality concept and the partial fractions.

More recently Basiya K. Abdulrahim (2013) [32] solve the Quadratic fractional programming problem via feasible direction development and modified simplex method. P. Pandian & M. Jayalakshmi give the solution of fully fuzzy linear fractional programming problem [31]. Nejmaddin A. Nawkhass [30] solve the a special case of QFPP by using Wolfe's method alongside with the modified simplex approach.

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3. CONCLUSION

Many interesting problems in and outside management decision making give rise to the optimization of one or several ratios. Much effort has been devoted to the analysis of such nonconcave programs. However, the theoretical basis is still not broad enough, especially for sum-of-ratios problems and, to a lesser extent, for multi-objective fractional programs. The computational experience with fractional programs is also quite limited. Major progress has been made for concave single-ratio and max- min fractional programs. But much more work is necessary in case of other fractional programs of interest in applications, especially for the sum-of ratios fractional program.

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