Solving Job Sequencing Problems with Fuzzy Processing Times

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ABSTRACT

In this paper, we proposed a solution procedure to solve fuzzy job sequencing problem, where processing time being taken as Trapezoidal Fuzzy Numbers (TrFN). Here, Yager's Ranking Index method has been applied to transform the fuzzy processing time into crisp ones. The optimal solution (order) of the job and idle time for each machine is obtained by solving corresponding crisp sequencing problems using existing method. A numerical example has been considered and solved for illustration purpose.

Keyword: - *Job sequencing problems, Fuzzy processing times, Fuzzy number, Yager Ranking Index*

1. INTRODUCTION

In our daily life decision making problems, viz., job sequencing, assignment, game theory, replacement of items etc. it is sometimes required to take the decision where the values of parameters are ambiguous i.e., parameters involved in the problem are imprecise [3,5,6] . Job sequencing problem is a mathematical way out for finding a series, in which a few jobs or tasks are to be done in an order for which total processing time is minimum. Generally, in job sequencing problems, the processing times are precise valued. But in reality, it is observed that the processing times during performance of the job are imprecise. To handle impreciseness fuzzy set theory [8] plays an important role as fuzzy set is a best mathematical way for representing impreciseness or vagueness. In this paper, we have treated imprecise parameters considering fuzzy numbers. Therefore, the concept of fuzzy job sequencing problem provides an efficient framework which solves real-life problems with fuzzy processing times. In the past, a few attempts have been made in the existing literature for solving job sequencing problem with fuzzy processing times [4, 6]. In this paper, a job sequencing problem [1] has taken into consideration. The processing times are considered to be Trapezoidal fuzzy number. Then the corresponding problem has been converted into crisp equivalent job sequencing problem using defuzzification of Trapezoidal fuzzy numbers. Here, widely known Yager's ranking index method [7] has been used for defuzzification of fuzzy number. The optimal order and idle time for each machine is obtained by solving corresponding crisp sequencing problems using the existing method. Finally, to illustrate the proposed method, a numerical example has been solved and results have been presented.

2. SOME DEFINITIONS

In this section, some definitions and Yager's Ranking Index method are presented.

Definition 2.1 (*Fuzzy Set*) Let X be a non empty set. Then a fuzzy set $\tilde{A}$ in X is a set of ordered pair given by $\tilde{A}=\left\{(x,\mu_{\tilde{A}}(x)):x\in X\right\}$, where $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a function such that $0\le\mu_{\tilde{A}}(x)\le 1\,\forall x\in X$, and $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$.

Definition 2.2 (*α-Level Set*) Let $\alpha \in [0,1]$, then $\alpha-\text{cut}$ of a fuzzy set generated by fuzzy set $\tilde{A}$ is denoted by $\tilde{A}_\alpha$ and defined by $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$. The lower and upper bounds of $\tilde{A}_\alpha$, are represented by $A_L(\alpha) = \inf \tilde{A}_\alpha$ and $A_U(\alpha) = \sup \tilde{A}_\alpha$.

Definition 2.3 (*Normal Fuzzy Set*) A fuzzy set $\tilde{A}$ is called a normal fuzzy set if there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 2.4 (*Convex Fuzzy Set*) A fuzzy set $\tilde{A}$ is called convex iff for every pair of $x_1,\ x_2 \in X$, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\theta x_1 + (1-\theta)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\theta \in [0,1]$.

Definition 2.5 (*Fuzzy Number*) A fuzzy number $\tilde{A}$ is a fuzzy set which is both convex and normal.

Definition 2.6 (*Trapezoidal Fuzzy Number*) A trapezoidal fuzzy number $\tilde{A}$ is represented by (a_1, a_2, a_3, a_4) and defined by its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \end{cases}$$

2.1 Yager's Ranking Index

Yager's ranking index for a fuzzy number $\tilde{A}$ is denoted by $Y(\tilde{A})$ and is computed by the formula $Y(\tilde{A}) = 0.5\int_0^1 \left(A_L(\alpha) + A_U(\alpha)\right)d\alpha$ where $[A_L(\alpha), A_U(\alpha)]$ is a $\alpha-\text{cut}$ of the fuzzy number $\tilde{A}$. The Yager's Ranking Index [18] is a robust ranking index, which satisfies both linearity and additive property and its gives the representative value of the fuzzy number $\tilde{A}$.

Definition 2.7 (*The Yager ranking index of Trapezoidal Fuzzy Number*) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a Trapezoidal fuzzy number. Then the Yager's Ranking Index of Trapezoidal Fuzzy Number is $Y(\tilde{A}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$.

2.2 Ranking of Fuzzy Numbers

Let $Y(\tilde{A})$ and $Y(\tilde{B})$ are the Yager ranking indices of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively. Now, we have defined the ranking or order relations of two fuzzy numbers as follows: (i) $\tilde{A} > \tilde{B} \Leftrightarrow Y(\tilde{A}) > Y(\tilde{B})$, (ii) $\tilde{A} < \tilde{B} \Leftrightarrow Y(\tilde{A}) < Y(\tilde{B})$ and (iii) $\tilde{A} = \tilde{B} \Leftrightarrow Y(\tilde{A}) = Y(\tilde{B})$.

Definition 2.8 (*Processing order*) It is a sequence in which various machines are needed for completion of the job.

Definition 2.9 (*Processing time*) It represents the time by a job on each machine.

Definition 2.10 (*Idle time*) It is the time for which a machine does not have a job to process.

Definition 2.11 (*Total elapsed time*) It is the time interval between starting of the first job and completion of the last job including idle time in a particular order by the given set of machines.

3. PROCESSING OF n JOBS THROUGH m MACHINES

Let there are n jobs say $J_1, J_2, ..., J_n$ be processed through m machines say $M_1, M_2, ..., M_m$ in the order $M_1M_2...M_m$. Let $\tilde{t}_{ij}$ be the fuzzy processing time taken by the *i*-th job to be completed by *j*-th machine. The well known Johnson method [2] can be extended to this problem, forming two fictitious machines G and H (say), if either or both of the following conditions are satisfied.

(i) $\min_i \left(Y(\tilde{t}_{i1})\right) \geq \max_i \left(Y(\tilde{t}_{ij})\right)\ \ \forall j = 2,3,...,m-1$

and/or

(ii) $\min_i \left(Y(\tilde{t}_{im}) \right) \geq \max_i \left(Y(\tilde{t}_{ij}) \right) \forall j = 2,3,...,m-1$

The corresponding processing times of the fictitious machines G and H (say) are given by $G_i = \sum_{j=1}^{m-1} Y(\tilde{t}_{ij})$ and $H_i = \sum_{j=2}^{m} Y(\tilde{t}_{ij})$, $i = 1,2,...,n$

4. NUMERICAL EXAMPLE AND COMPUTATIONAL RESULTS

In this section, a numerical example has been considered to illustrate the proposed solution procedure. The fuzzy sequencing problem with five jobs through three machines has been considered where the order is A_1, A_2 and A_3. Here, processing time of each machine for each job is Trapezoidal Fuzzy Number. The said sequencing problem is given below:

Jobs/Machine	Processing time $\tilde{t}_{ij}$ in hours		
	Machine A_1	Machine A_2	Machine A_3
1	(4,6,8,14)	(2,4,5.9)	(2,3,4,7)
2	(7,8,10,15)	(3,6,7,8)	(5,9,10,12)
3	(3,6,7,8)	(0,1,2,5)	(4,6,8,14)
4	(3,7,8,10)	(1,2,4,5)	(3,6,7,8)
5	(8,10,12,14)	(2,3,4,7)	(2,4,5,9)

Solution: Here, $\min(Y(\tilde{t}_{i1})) = 6$, $\max(Y(\tilde{t}_{i2})) = 6$ and $\min(Y(\tilde{t}_{i3})) = 4$ also $\min(Y(\tilde{t}_{i1})) = \max(Y(\tilde{t}_{i2}))$. Hence, Jonson method can be applied for this problem and we solved the problem for two fictitious machines G and H with corresponding processing times given by $G_i = Y(\tilde{t}_{i1}) + Y(\tilde{t}_{i2})$ and $H_i = Y(\tilde{t}_{i2}) + Y(\tilde{t}_{i3})$ where $i = 1,2,3,4,5$ .Therefore the equivalent problem involving five jobs through two fictitious machines G and H is as follows:

Jobs/Machine	Processing time in hours	
	Machine G	Machine H
1	13	9
2	16	15
3	8	10
4	10	9
5	15	9

Now, proceeding by Johnson method, the optimal sequence for five jobs through three machines is as follows:

3	2	5	1	4

Here, the minimum expected elapsed time from starting of the first job to the completion of the last job in the optimal sequence is computed and presented in Table 1.

Table 1: Minimum expected elapsed time from starting of the first job to the completion of the last job

Job	Machine A_1			Machine A_2			Machine A_3		
	Time in	Time out	Idle time	Time in	Time out	Idle time	Time in	Time out	Idle time
3	0	$0+Y(3,6,7,8)$	0	6	$6+Y(0,1,2,5)$	6	8	$0+Y(4,6,8,14)$	8
2	6	$6+Y(7,8,10,15)$	0	16	$16+Y(3,6,7,8)$	8	22	$22+Y(5,9,10,12)$	6
5	16	$16+Y(8,10,12,14)$	0	27	$27+Y(2,3,4,7)$	5	31	$31+Y(2,4,5,9)$	0
1	27	$27+Y(4,6,8,14)$	0	35	$35+Y(2,4,5,9)$	4	40	$40+Y(2,3,4,7)$	4
4	35	$35+Y(3,7,8,10)$	9	42	$42+Y(1,2,4,5)$	2+6=8	45	$45+Y(3,6,7,8)$	1

From Table 1, it is seen that expected elapsed time is $45+Y(3,6,7,8)=51$ hours and the ideal times for machine A_1, machine A_2 and machine A_3 are 9, (6+8+5+4+8)=31 and (8+6+4+1)=19 hours respectively.

5. CONCLUDING REMARKS

In this paper, we have solved job sequencing problem with fuzzy processing times. Here, we have considered trapezoidal fuzzy numbers to represents the fuzzy processing times. For this purpose, a method for solving sequencing problem with fuzzy processing times is proposed to obtain the optimal solution. A numerical example has been considered and solved to illustrate the proposed method. Finally, the sequencing problem with their order and idle time for each machine have been presented and compared. For future research one may use the entire method to solve real-life decision making problems having fuzzy parameters.

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