Stress Concentration Analysis and validation through Cutout Orientation in Plates

Pooja Kumar Budhner, J.S.Shitole, S.S.Kathale

ME Student, Department of Mechanical Engineering, DKCOE, Bhigwan, Maharashtra
Assistant Professor , Department of Mechanical Engineering, DKCOE, Bhigwan, Maharashtra
Assistant Professor , Department of Mechanical Engineering, DKCOE, Bhigwan, Maharashtra

Abstract

The structural analysis should guide the design of the any structural components like (spacecraft's, airplane parts or pressure vessel) and provide a high degree of confidence. The analysis should be an integral part of the design process, thus minimizing design effort and time by eliminating redesign caused by failure during structural verification testing.

Keywords*- *Stress Concertation, Optimization, CAE, Ansys, FEM

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1. Introduction

An important benefit of performing stress analyses is the ability to determine design sensitivities and to conduct trade studies. Thus, effective optimization of the structure can be achieved, enhancing reliability while reducing cost and weight. Openings/cut-outs are made into structures in order to satisfy some service requirements, results in strength degradation. In practice different shape of holes are used for different applications for an example manhole of any pressure vessel is either circular or elliptical while the window or door of an airplane is rectangular hole having chamfer of some radius at corners. This hole/opening works as stress raisers and may lead to the failure of the structure/machine component. Hence it is an important aspect of stress analysis to predict stress concentration for regular or irregular holes. The irregularity in the hole shape may be because of chemical degradation. Under the effect of external loading and chemical process some irregular shapes may evolved. It is necessary to know stress distribution around such irregular shaped hole which may be useful to know hole shape evolution. Stress concentration is localization of high stresses mainly due to discontinuities in continuum, abrupt changes in cross section and due to contact stresses. To study the effect of stress concentration and magnitude of localized stresses, a dimensionless factor called Stress Concentration Factor (SCF), denoted as K_t.

$$K_t = \frac{\sigma\ \mathrm{max}}{\sigma\ \mathrm{nom}}$$

Where, σ max is maximum stress at the discontinuity and σ nom is nominal or background stress. The stress concentration factor can be determined analytically by applying elasticity theory.

Stress analysis of the critical elements under various loading conditions is carried out by the researchers for safe design of the element. Stress is measured by experimental methods or analytical/numerical methods.

2. Literature Survey:

J. Rezaeepazhand N, M. Jafari: This paper describes that the high stress concentration at the edge of a cutout is of practical importance in designing of the engineering structures. The SCFs of these type cutouts usually are determined either experimentally or numerically using finite element methods

M Mohan Kumar, Rajesh S, Yogesh H and Yeshaswini B R: This paper describes that the plates with variously shaped cut-out are often used in engineering structures and the understanding of the effect of cut-out on the load bearing capacity and stress concentration of panels is very important in designing of structures. Allowable stress design is a design format extensively used in structural design.

Murilo Augusto Vaz, Julio Cesar Ramalho Cyrino and Gilson Gomes da Silva: In this paper the three-dimensional stress concentration factor (SCF) at the edge of elliptical and circular holes in infinite plates under remote tension has been extensively investigated considering the variations of plate thickness, hole dimensions and material properties, such as the Poisson's coefficient.

Lotfi Toubal, Moussa Karama and Bernard Lorrain: This paper showed a non-contact measurement method, namely electronic speckle pattern interferometer (ESPI), was used to investigate the tensile strain field of a composites plate in the presence of stress concentrations caused by a geometrical defect consisting of circular hole.

Muskhelishvili's complex variable method. The solution presented here can be a handy tool for the designers.

3. FINITE ELEMENT METHOD:-

3.1 Discretize the continuum

The solution region is divided into elements. Different type of elements may be used for this as well as different element shapes. Examples of such elements include; line elements (1D) – bars and beams, plane elements (2D) – plates and shells, and solid elements (3D).

3.2 Election of interpolation functions

These are chosen to represent variation of a field variable over the element and are expressed in terms of nodal variables. They are often polynomials with degrees depending on number of nodes of the element.

3.3 Computation of element properties

Matrix equations are formulated expressing the properties of individual elements from either of the formulations given above. The matrix equation of the stiffness method is given by

$$\{F^e\} = [K^e]\ \{d^e\}$$

3.4 Assembly of element properties to obtain the system equations

Properties of the overall system are a sum of properties of element properties. Element matrix equations all have the same form as the system matrix equation except the latter has more terms because it includes terms for all nodes in the system. The assembly procedure is guided by the fact that at a node where elements are interconnected, the value of the field variable is the same for each element sharing that node.

3.5 Imposing boundary conditions

System equations are modified to account for boundary conditions. Nodal values of the dependent variables or nodal loads are imposed.

3.6 Solution to the system equations

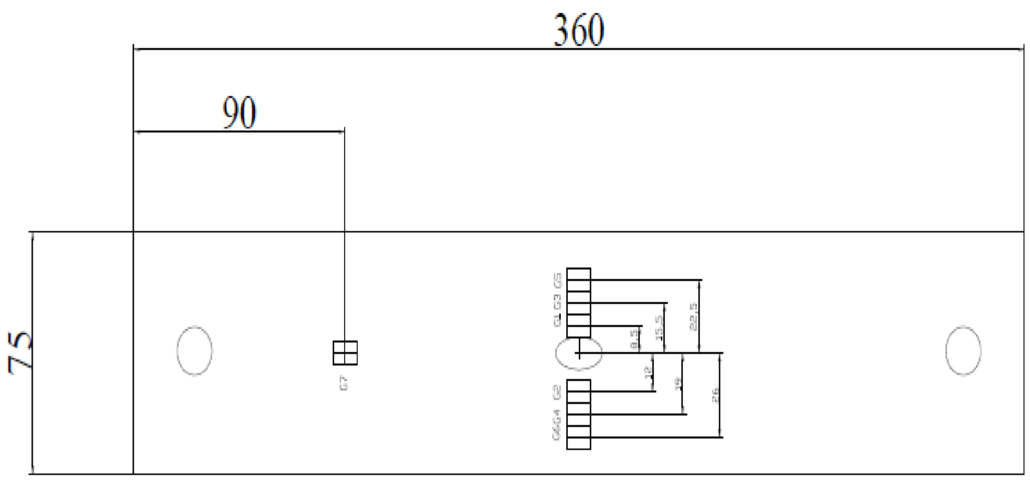
A set of simultaneous equations are solved to obtain unknown nodal values of the problem. The equations are algebraic if the system is steady and if it is unsteady, or a function of time, then a set of linear or nonlinear ordinary differential equations is solved.

3.7 Computation of additional results

Additional results depend on the system being solved; example for structural problems stresses may be required while for a heat conduction problem, heat fluxes may be required.

4. Preparation of Specimen (Ellipse Cutout)

The material of plate is Aluminum alloy selected because it is generally used in aerospace for light weight application for example like space shuttles, aeroplanes etc. of size 360 X 75 X 3mm as shown in Fig 3.12. On this plate two circular holes of dia 10 mm at the near to end of this plate are drilled for clamping purpose. A centrally elliptical hole in a plate of major axis 20 mm and minor axis 10 mm (major axis parallel to applied force) is made with help of (W-EDM) machine to control the internal residual stress near to the cutout which is negligible in this case. After this the strain gauges of 120 Ω resistances are taken for the mounting purpose near to the high stress localized area which is got from the Ansys results for measuring the corresponding strain value.



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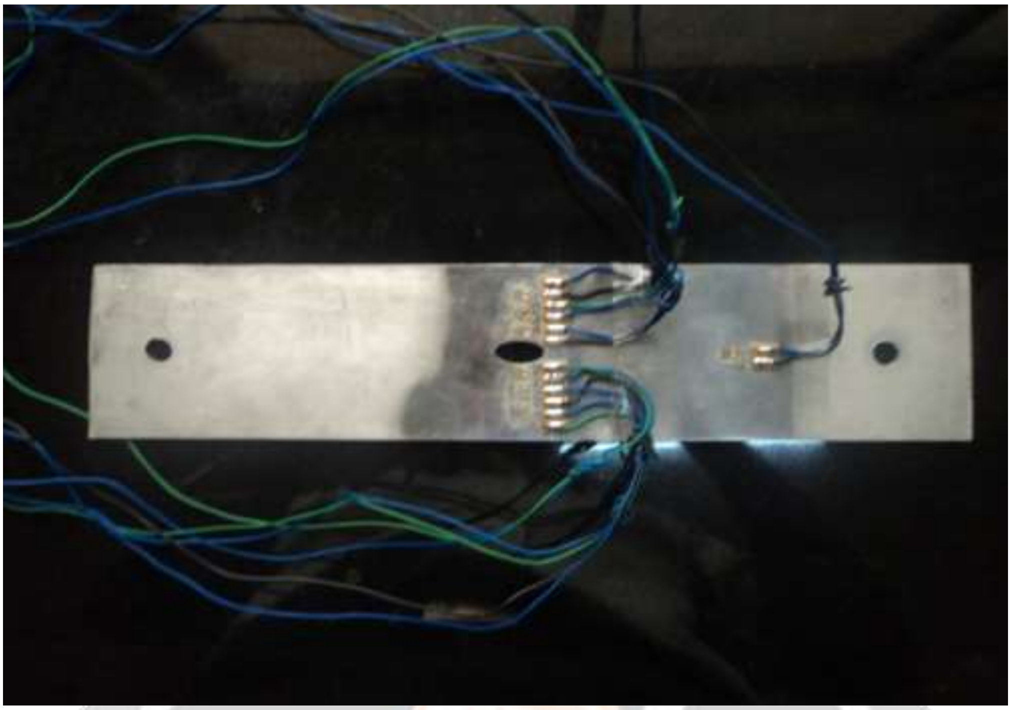
Drawing for ellipse cutout and strain gauge mounting

Same strain gauge mounting procedure is again done on this specimen as before explained in square cutout and final specimen after strain gauge mounting



Elliptical cutout specimen

Experimentation can be done for detection of SCF of different cut-out and orientation for different notches like circular, square, triangular and elliptical by changing radius ratio and angle of orientation etc. The experimentation can be done first in software that is ANSYS and its validation can be done practically by electrical strain gauges.

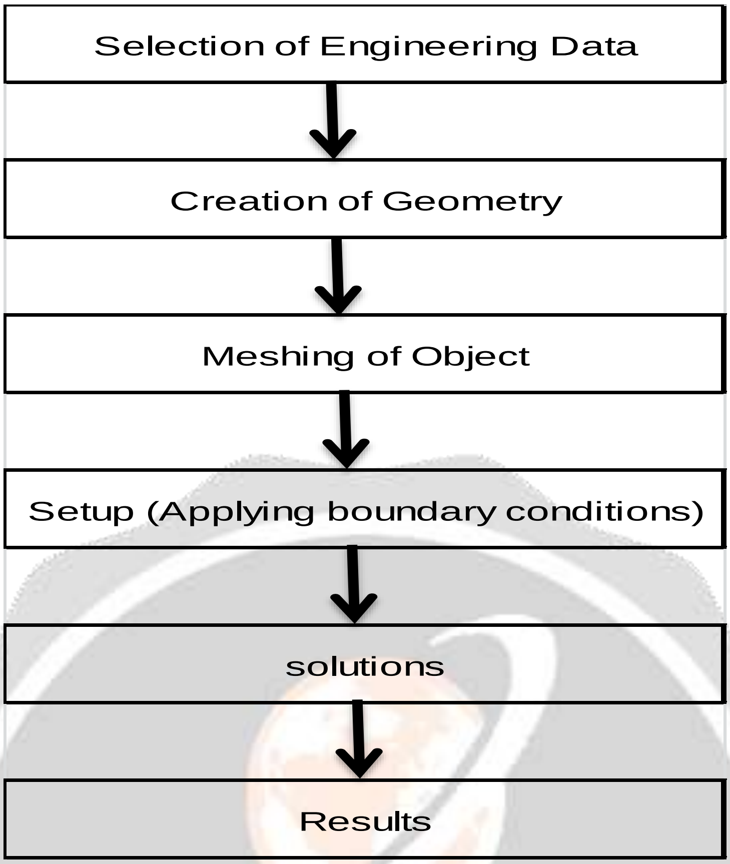
During experimentation the parameter that is radius ratio and orientation of notch can be varied as follows.

1) Bluntness ($\frac{r}{R}$):- 0.1 to 1.0

2) Orientation of cut-out: - 0° to 90° (Depend on the shape of cut-out / notch)

3) Cut-out will be in the shape: - circular, square, triangular and elliptical. [5-6]

For FEA experimentation there will be some standard procedure to follow which has been explained with the help of following flowchart.



Flow chart for FEA experimentation

During FEA we have to follow some standard steps for that, shown in above fig. 3.14. First the selection of engineering material that is engineering data collection is done then creation of geometry is done in modeling, then next step is meshing of created geometry by using different meshing methods like mapped face meshing, automatic method etc. After meshing the boundary conditions are applied like applying loads, fixed supports and applying constraint equation etc. then after applying boundary conditions the equations are solved according to conditions that is solutions and we can get the respective results like stresses, displacements, strain etc.

5. Results and Discussion

With the experimentation on different shaped cutout the following results are acquired by FEA method and experimental (Electrical strain gauge method) which are discussed successively.

5.1. SQUARE CUTOUT:

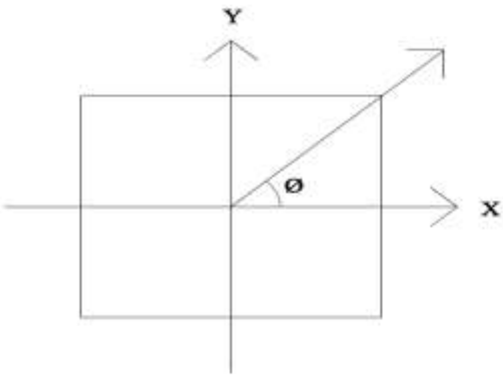
For square cutout the following parameters are consider for experimentation

Parameters:-

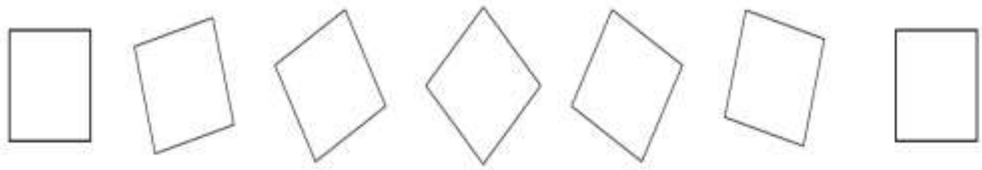
Bluntness (w) - 0.1, 0.25, 0.3, 0.5, 0.75, 0.9 and 1.0

Orientation - 0˚, 15˚, 30˚, 45˚, 60˚, 75˚, and 90˚

Following Fig 4.1 shows the square cutout and ϕ is the angle of rotation with respect to x axis the square cutout is rotated counterclockwise like 0˚, 15˚, 30˚, 45˚, 60˚, 75˚ and 90˚, for different values of bluntness like 0.1, 0.25, 0.3, 0.5, 0.75, 0.9 and 1.0. Fig 4.2 shows the different rotational conditions of the cutouts



Rotation of square cutout



ϕ=0˚ ϕ=15˚ ϕ=30˚ ϕ=45˚ ϕ=60˚ ϕ=75˚ ϕ=90˚

Rotated cutouts for square

For a square shape the acute angle between two sides is 90 ˚ so that we can rotate a square only up to 90 ˚ after it can come on a same position same as on 0 ˚ that is for ϕ=0˚ or ϕ=90˚ will be the same geometry. In this case 0 ˚ means any two opposite sides of a square are parallel to the applied load (force) direction is considered.

5.1.1 ANSYS 14.5 Results For Square Cutout

a) Bluntness 0.1:-

Bluntness 0.1 means ($\frac{r}{R}$) = 0.1 , in this case r means fillet radius and R means inscribed circle radius now, in this case we have taken the square cutout of dimension 15 X 15 mm, for that the inscribed circle radius is 7.5mm.

Now we have, $$w \text{ (bluntness)} = \left(\frac{r}{R}\right) = 0.1 \qquad (4.1)$$

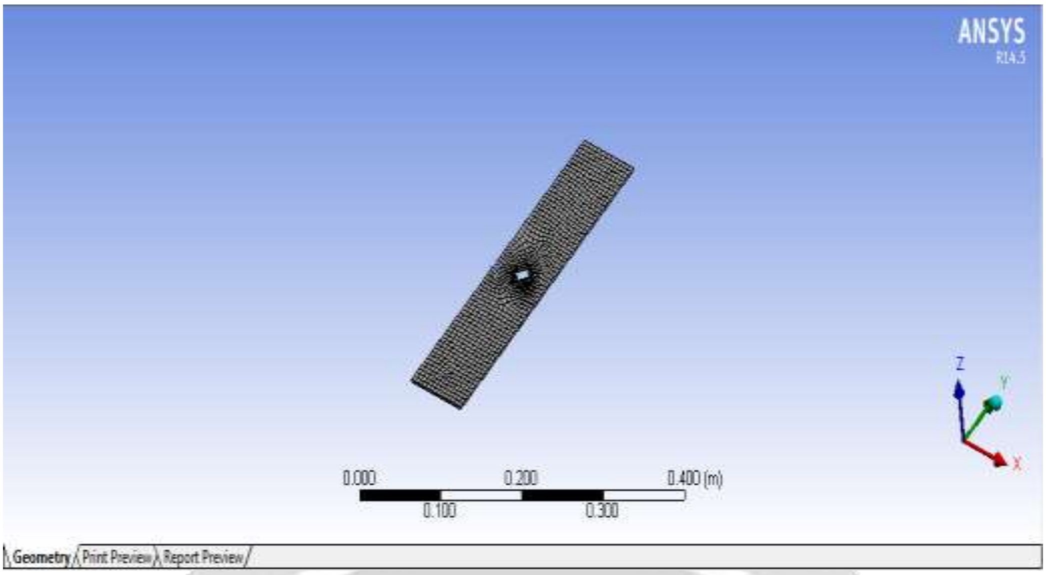
We know $w = 0.1$

$R = 7.5$

Therefore $0.1 = \frac{r}{7.5}$

$r = 0.75$

Therefore fillet radius for this condition is 0.75 is taken according to above calculations. The model of specimen is created in Ansys 14.5 workbench modeling and then after meshing is done for a specimen with a node 8111 and element size 1088.



Meshed square cutout (when w=0.1)

One end of specimen is fixed and at other end the pressure is applied 1MPa and the results are collected which are as follows

For 0˚,

$$K_t = \frac{\sigma \max}{\sigma \text{ nom}}$$

$$= \frac{3263364.509}{1000000}$$

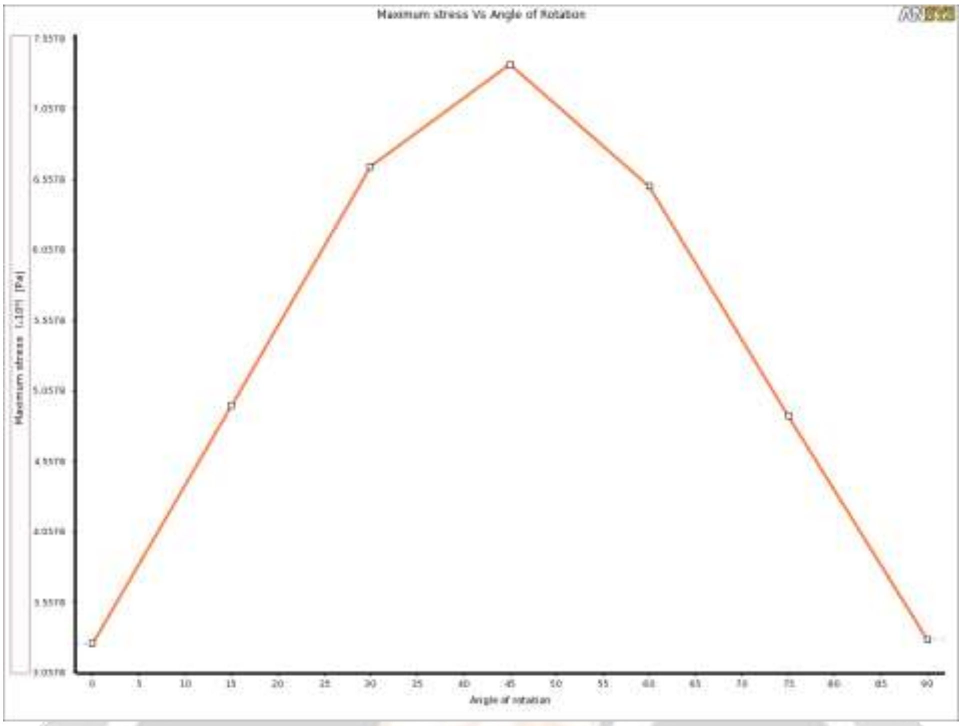
$$= 3.263364509$$

From above procedure for all angle of rotation the SCF value is calculated similarly, and shown in the following table.

SCF for square cutout w = 0.1

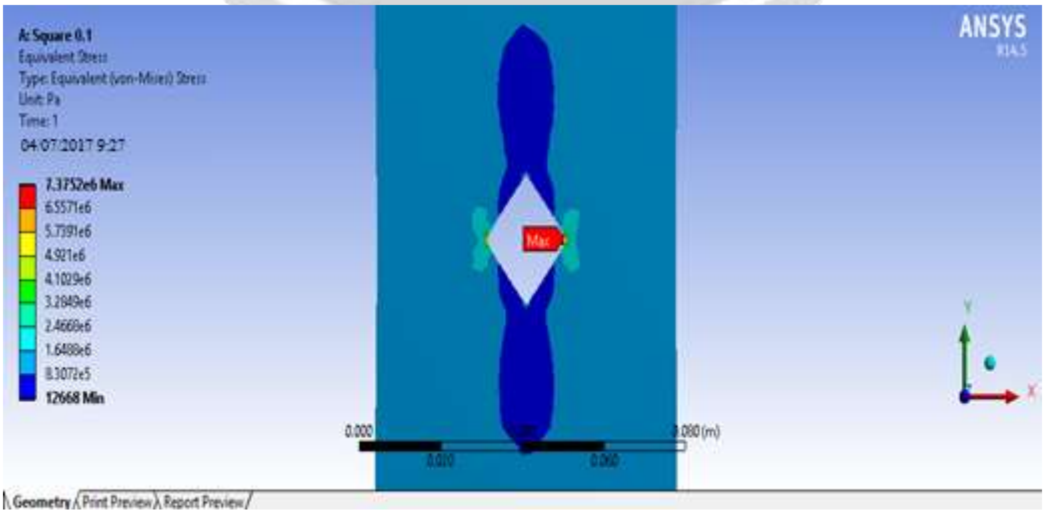
Angle of Rotation (˚)	Maximum Stress (Pa)	SCF
0	3263364.509	3.263364509
15	4954324.856	4.954324856
30	6651487.27	6.65148727
45	7375160.834	7.375160834
60	6508823.732	6.508823732
75	4879617.005	4.879617005
90	3293027.811	3.293027811

From the above results the graph is plotted against the maximum stress vs angle of rotation for bluntness value 0.1.



Maximum stress Vs Angle of rotation for w = 0.1

In above graph on x- axis angle of rotation in degree is taken and on y- axis maximum stress in MPa is taken, this graph is plotted in Ansys 14.5 tool. From the above graph it is cleared that the SCF will vary with respect to the change in angle of a rotation for a specific bluntness value, for a square cutout with bluntness 0.1 the maximum stress is observed, which is 7375160.834 Pa (that is SCF value is 7.37) when square cutout is inclined at 45˚ with respect to the fixed end as shown in following fig, and minimum stress is observed, which is 3263364.509 Pa (that is SCF value is 3.26) when square cutout is at 0˚ that is it's any two opposite sides will be parallel to the applied force. From following fig.4.5 it is cleared that the for square cutout with w = 0.1, near to two opposite corner of a square which are on vertical central axis of a square cutout the stress localization will be very less (shown in blue coloured bubbles on both side) compared to any other region in plate and other two opposite corners have maximum stress localization (shown in red mark).



Maximum stress contour for square cutout (when w = 0.1)

b) Bluntness 0.25

Bluntness 0.25 means ($\frac{r}{R}$) = 0.25, in this case r means fillet radius and R means inscribed circle radius now, in this case we have taken the square cutout of dimension 15 X 15 mm, for that the inscribed circle radius is 7.5mm

Now we have, w (bluntness) = ($\frac{r}{R}$) = 0.25 (4.2)

We know $w = 0.25$

$R = 7.5$

Therefore $0.25 = \frac{r}{7.5}$

$r = 1.875$

Therefore fillet radius for this condition is 1.875 is taken according to above calculations. The model of specimen is created in Ansys 14.5 workbench modeling and then after meshing is done for a specimen with a node 9824 and element size 1327.

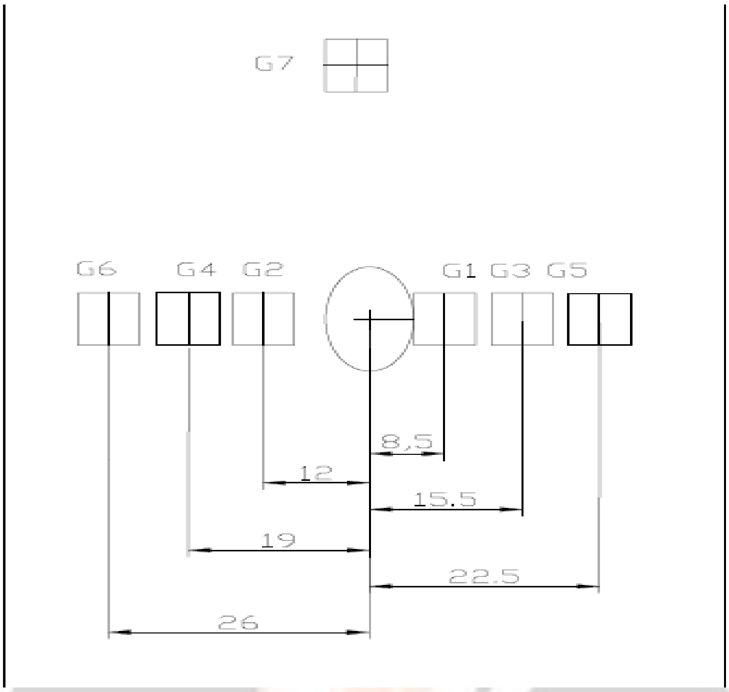
5.1.2 Experimental Validation (Electrical Strain Gauge Method)

a) Vishay micro-measurement P3 strain indicator and recorder module

Electrical strain gauge method is used for finding out the stress concentration factor for elliptical cutout; the experimental setup is shown in following Fig.4.54, P3 strain module (Strain indicator) is used for strain measurements.

Experimental setup for elliptical cutout (Electrical strain gauge method)

Description – There are total of 6 strain gauges that are bonded along transverse axis of symmetry starting from close to ellipse edge to the right end of transverse axis. Since the gauges cannot be bonded exactly at the edge of ellipse hole at right transverse edge, a polynomial is fitted to the data to extrapolate the strain to an edge as explained in procedure below, another single gauge is bonded in the middle portion of the specimen (i.e. between hole and load) to measure average strain.



Strain gauge arrangement on ellipse specimen

Procedure – Measure the distance from centre of ellipse hole to the one edge (corner) of the hole, also record the distance from center of the hole to every gauge starting from gauge1 to gauge 6 as shown in above Fig.4.55. Record the gauge factor for all 6+1 gauges. Next load the specimen in a tension tester (loading frame type) carefully connect all seven gauges as a quarter bridges to the strain indicator. Set the desired gauge factor and balance (zero) all seven gauges. Next load the specimen until load P = 212 N. Record the strain readings of all seven gauges in excel sheet as shown below along with $\frac{X}{r_0}$ values, where X = distance to gauge from the center of ellipse hole to gauge as shown in the table below, r_0 = distance from centre of ellipse hole to the one edge (corner) of the hole (5 mm).

We know nominal strain value over the plate is 20, from that we can find applied force on plate if we know the young's modulus value for the plate as follows

Young's modulus = $\frac{\text{Stress}}{\text{Strain}}$

Stress = Young's modulus X Strain

Stress = (71×10^9) N/m^2 X (20×10^{-6})

Stress = 1420×10^3 N/m^2

Now we know relation between stress, force and area from that we can calculate the applied force,

Stress = $\frac{\text{Force}}{\text{Area}}$

Force = Stress X Area

Area = Width of plate X Thickness of plate

Area = 0.075 m X 0.003 m = 0.000225 m^2

Therefore, Force = $(1420 \times 10^3$ N/m$^2)$ X (0.000225) m^2

Force = 319.5 N

Strain gauge readings for ellipse cutout (P3 Strain module readings)

Load N	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7
319.5	36	30	27	22	21	13	(Strain without stress concentration)
x distance	8.5	12	15.5	19	22.5	26	20
x/r_0	1.7	2.4	3.1	3.8	4.5	5.2	

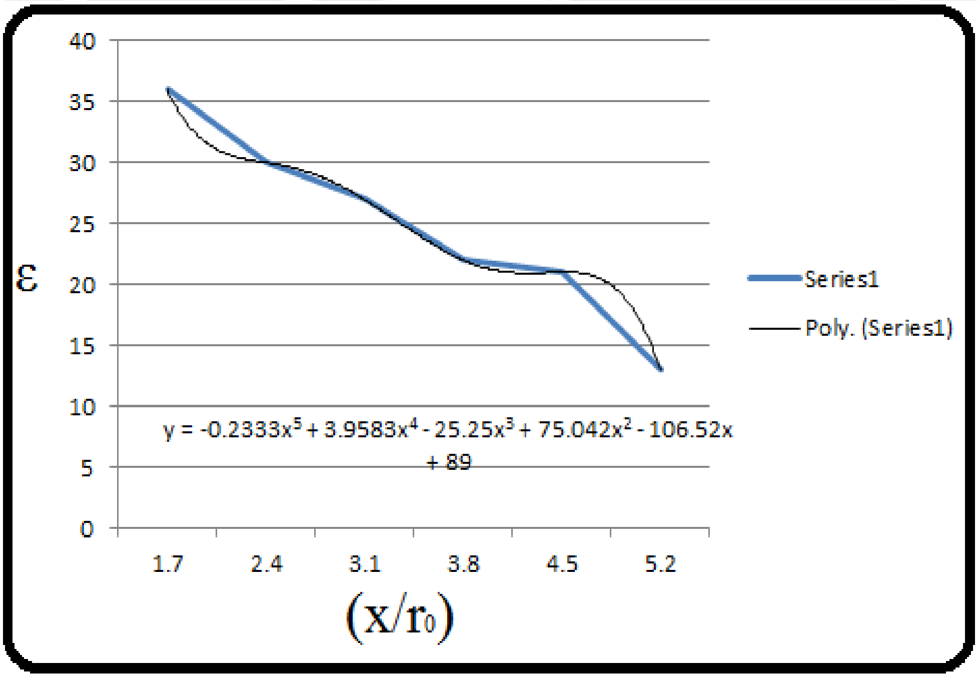
To fit data of $\left(\frac{x}{r_0}\right)$ Vs strain, determine values of a_1, a_2, a_3, a_4 and a_5. i.e,

$$\varepsilon_x = a_1 + a_2\left(\frac{x}{r_0}\right) + a_3\left(\frac{x}{r_0}\right)^2 + a_4\left(\frac{x}{r_0}\right)^3 + a_5\left(\frac{x}{r_0}\right)^4 + a_5\left(\frac{x}{r_0}\right)^5 \ldots$$

Once the data of fit $\left(\frac{x}{r_0}\right)$ Vs strain is fitted then realize that the strain at the edge of the square hole where $\left(\frac{x}{r_0}\right)$= 1 is simply equal to the sum of polynomial constants i.e.

$$\varepsilon_{r_0} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

Stress concentration factor, $K = \frac{\varepsilon_{r_0}}{\varepsilon}$



Graph for ε Vs (X/r_0) (Ellipse cutout)

From the graph, the equation fitted is as follows:

ε_x= -0.2333 $(x/r_0)^5$+3.9583$(x/r_0)^4$ + -25.25 $(x/r_0)^3$ +75.042 $(x/r_0)^2$ -106.52 (x/r_0) + 89

a_1 = -0.2333;

$a_2 = 3.9583$;

$a_3 = -25.25$;

$a_4 = 75.042$;

$a_5 = -106.52$;

$a_6 = 89$

The strain at the edge of the hole where $\left(\frac{x}{r_0}\right) = 1$ is equal to the sum of polynomial constants i.e. $\varepsilon_{r_0} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$$= -0.2333+3.9583-25.25+74.042-106.52+89$$

$$= 35.997$$

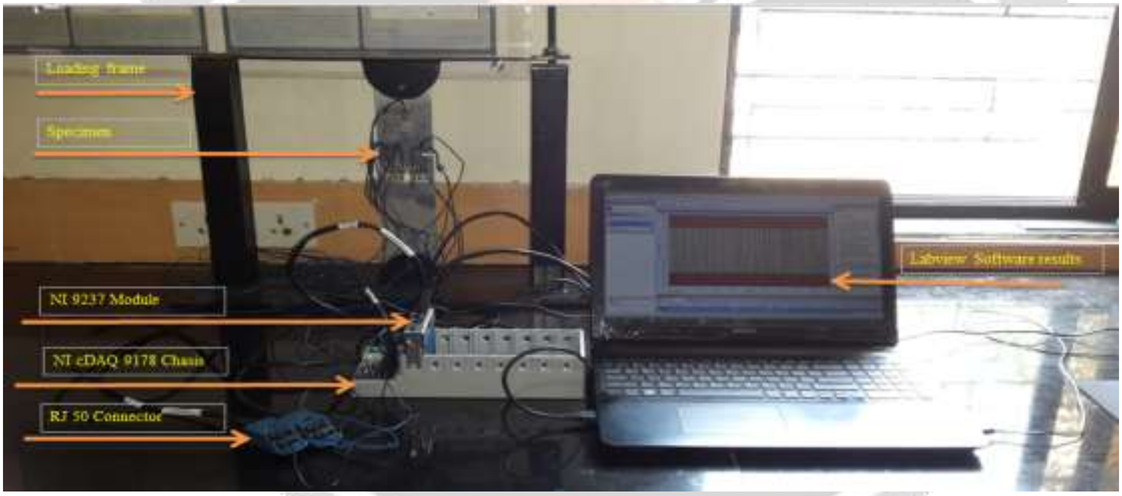
Stress concentration factor, $K = \frac{\varepsilon_{r_0}}{\varepsilon}$

$$= \frac{35.997}{20}$$

$$= 1.7999$$

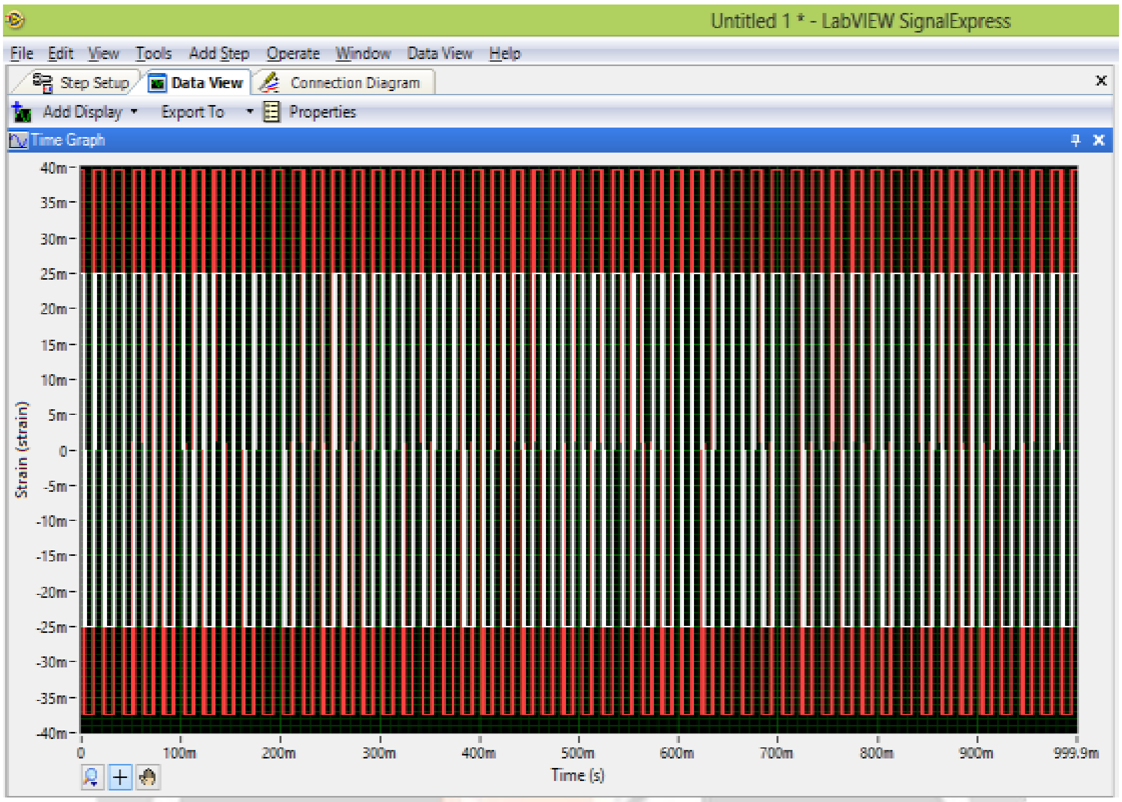
b) National Instrument cDAQ-7138 strain measurement module (LABVIEW Software)

For same specimen the SCF is calculated by using National Instrument cDAQ-7138 strain measurement module (LABVIEW Software) for strain measurements; the experimental setup is shown in following Fig.4.60.



Experimental setup for elliptical cutout (By using NI cDAQ7138)

Results of the LABVIEW 2011 software are shown in following fig.



Results of LABVIEW 2011 software for elliptical cutout

From above graph we have,

- Maximum Strain value near to cutout is 41
- Nominal strain value is 24

There for we know,

Stress concentration factor,

$$K_t = \frac{\text{Maximum strain}}{\text{Nominal strain}}$$

$$= \frac{\varepsilon 1}{\varepsilon 7}$$

$$= \frac{41}{24}$$

$$= 1.7083$$

From the above experimentation the SCF, for ellipse when its major axis is parallel to the applied force direction then it is 2.01 by FEA and 1.7999 by (Vishay Micro-measurement P3 Strain module), and 1.7083 (National Instrument cDAQ-7138 strain measurement module LABVIEW Software) electrical strain gauge method.

From above experimentation is cleared that, we can control the SCF value by changing the parameters like bluntness, cutout shape and orientation of cutout with respect to applied force,

- For square cutout the maximum SCF value is 7.37(FEA), when bluntness is 0.1 and θ = 45˚. We know generally for circular cutout the SCF value is 3, from above experimentation it is noted that for square cutout SCF value can be reduced to 3 when bluntness value 0.25, 0.3, 0.5, 0.75 and 0.9 (and θ = 0˚ or 90˚)

that means any two opposite sides of a square must be parallel to applied force at this condition SCF value is minimum for any bluntness value.

- For triangular cutout the maximum SCF value is 8.45, when bluntness is 0.1 and θ = 0˚. from experimentation it is noted that for triangular cutout SCF value cannot be reduced to 3 for any bluntness value but it can be nearly equal to 3 and the one side of a triangular must be perpendicular to the applied force at this condition SCF value is minimum for any bluntness value except (w= 0.75 and 0.9).
- For elliptical cutout the maximum SCF value is 4.9 when its major axis is perpendicular to applied load, but it should be eminent that SCF value will be minimum as compared any cutout shape which is 2.01 by (FEA) and 1.79 (Experiment strain gauge method) less than circular cutout when its major axis is parallel to applied load.

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