Studies of energy contribution input by electromagnetic wave radiation by a circular black body, heated using a Solar paraboloid concentrator.

Andrianantenaina RAFIGISON\textsuperscript{1}, Adolphe RATIARISON\textsuperscript{2}, J. Eugène RANDRIANANTENAINA\textsuperscript{3}, Clin RAHARIVELO\textsuperscript{4}

\textsuperscript{1} Student, Department of physics, Faculty of Sciences, Toliara, Madagascar
\textsuperscript{2} Emeritus Professor, Department of physics, DYACO laboratory, Antananarivo, Madagascar
\textsuperscript{3} Master of conferences, Department of physics, Faculty of Sciences, Toliara, Madagascar
\textsuperscript{4} Student, Department of physics, Faculty of Sciences, Toliara, Madagascar

ABSTRACT
This work is based on the field of energy physics and particularly affects solar energy which is new and renewable to heat a circular black body placed at the focus of a dish concentrator. The principle chosen to study the energy input of the heated body is limited to the analysis of the radiation, without taking into account the thermal conduction in the air and the thermal convection in the materials. To ensure the reliability of numerical calculations and graphical representations, the MATLAB software was chosen for all operations and programming. In these cases, the results obtained are analyzed and interpreted according to the magnitudes of the constituent elements of the system (paraboloid, blackbody) and discussed according to the effect of the presence of the atmosphere on the solar constant or in its absence. This study allows researchers to determine the color of a heated body at a given temperature by studying Planck's function. For the two black body shapes studied, the wavelength calculations correspond to the maximum values of the luminances making it possible to prove that this color belongs to the Infrared (IR). For this study, we set the dimensions of the solar concentrator (height, focus), the black body and that of gray, taking into account that the latter is smaller in order to neglect radiation losses. We find that the black body heated in our study emits Infrared.

Keyword : - Paraboloid, Solar, blackbody, focus

1. Introduction
Solar energy is characterized by the constant called power flow, emitted per unit area. This power is inexhaustible and can be reunited at the focus of a dish concentrator. However, a black body placed at this point, much smaller in size than that of the concentrator opening, heats up by absorbing all the solar radiation reflected by this system. It follows that, at a given equilibrium temperature, this body emits radiation to another body. This program, which follows Planck's law, allows researchers to determine the color of the heated body by studying the luminance that is a function of this equilibrium temperature. For this study, we set the dimensions of the solar concentrator (height, focus), the black body and the gray one, making sure that the latter is as small as possible, in order to be able to neglect the losses by radiation.
2. Theoretical methods

In general, our method requires integrating Bougouer's formula [1], [2], which calculates the power radiated by a surface element \(dS_1\) to another surface element \(dS_2\) which is respectively an angle \(\theta_1\) and \(\theta_2\) with respect to the normal.

\[
d^2P_{1\rightarrow 2} = \frac{\varepsilon r I_S}{\pi} dS_2 \frac{dS_1 \cos \theta_2}{(r_{12})^2 \cos \theta_1}
\]  

\(I_S\) is the solar constant that we discuss according to the effect of the atmosphere and is the emissivity of the body considered. The integration takes place twice, the first of which is relative to the surface \(S_1\) to calculate the power received at a point of the surface \(S_2\) and the second is relative to the surface \(S_2\) to calculate the power total received on the surface \(S_2\); Three steps are followed to reach our goal.

2.1 Calculation of power received at the surface surrounding the concentrator focal point

Equation (1) can be written as:

\[
d^2P_{1\rightarrow 2} = \frac{\varepsilon r I_S}{\pi} dS \frac{dS_1}{(r_{12})^2} \left( \frac{1}{1 + \left(\frac{r}{2f}\right)^2} \right) \left( \frac{1}{2f} \right)
\]  

With \(dS\) and \(dS_1\) are respectively surface elements of the paraboloid [3] and the black body in its focus.

- \(r_{12}\): the distance between \(dS\) and \(dS_1\).
- \(f\): focal length of the parabolize.
- \(r\): distance obtained by the change of variables

Formula of change of the useful variables:

\[
\vec{r}_1(r, \varphi) = \begin{pmatrix}
rcos \varphi \\
r \sin \varphi \\
\frac{1}{4f} r^2
\end{pmatrix}
\]  

\(\vec{r}_1\): vector position of the surface element \(dS\) by report on origin \(O\) (summit of the paraboloid) of the reference mark \((O, \bar{x}, \bar{y}, \bar{z})\) considered.

With:

\[
\begin{cases}
r \in [0, R] \\
\varphi \in [0, 2\pi]
\end{cases}
\]

Where \(R\) is the ray of the superior opening of the parabolize.

Furthermore

\[
\frac{\partial \vec{r}_1}{\partial r} = \begin{pmatrix}
\cos \varphi \\
\sin \varphi \\
\frac{1}{2f} r
\end{pmatrix}
\]

\[
\frac{\partial \vec{r}_1}{\partial \varphi} = \begin{pmatrix}
-r \sin \varphi \\
rcos \varphi \\
0
\end{pmatrix}
\]
2.2 Calculation of the maximum temperature at the focal point of the concentrator

The maximal temperature to the foyer [3], a concentrator parabolize solar can be gotten by:

\[ C = \frac{S_c}{S_r} \]  

(9)

With:
- SC: surface collector
- Sr: receiving surface

Or:

\[ C = \frac{I_{\text{max}}}{I_s} \]  

(10)

With: \( I_{\text{max}} \): the maximal intensity in the focal volume.

And by applying the law of Stephan Boltzmann, we can associate the temperature \( T \) of a black or gray body to the maximum illumination received. That is to say:

\[ \sigma T^4 = \varepsilon_r I_{\text{max}} = \varepsilon_r C I_s \]  

(11)

With:
- \( \varepsilon_r \): Emissivity of the shape (reflector).

And while supposing that the incidental radiances are normal to the surface collector, it allows us to pull the temperature maximal attainable in the focus of a solar cooker.
And with (9), the formula (12) becomes:

\[ T = 4 \sqrt{\frac{\varepsilon_r C_l S}{\sigma}} \]

(12)

Without considering the attenuation due to the presence of the atmosphere, the calculation of the solar record is obtained by:

\[ I(r) = 3.02 \times 10^{25} \left( \frac{1}{r^2} \right) \text{ Wm}^{-2} \]

(14)

Considering the attenuation due to the presence of the atmosphere, the calculation of the solar record [4] is obtained by:

\[ I'(\theta) = 1353 e^{-[0.16 + 0.22 m(\theta)]} \cos \theta \]

(15)

With:

\[ m(\theta) = \begin{cases} 
\frac{1}{\cos \theta} & \text{si } 0 \leq \theta \leq 80^\circ \\
\left[ \cos \theta + 0.15 (93.885 - \theta)^{-1.233} \right]^{-1} & \text{si } 80^\circ \leq \theta \leq 90^\circ 
\end{cases} \]

(16)

2.3 Calculation of the power radiated by the black body toward a gray body:

The power radiated by the black body towards a gray body is thus obtained by integrating (1) and using the Residue theorem:

\[ I = \int_0^{2\pi} \frac{P(\cos t, \sin t)}{Q(\cos t, \sin t)} \, dt = 2i \pi \sum_k \text{Rés}[f, z_{ok}] \]

(17)

With \( z_{ok} \) a kineme pole as \( |z_{ok}| < 1 \) and:

\[ \text{Rés}[f, z_0] = \frac{1}{(p-1)!} h^{(p-1)}(z_0) \]

(18)

Where:

\[ h(z) = (z-z_0)^p f(z) \]

(19)

With:

\[ f(z) = \frac{1}{g(z)} \]

(20)
The pole $z_0$ of $f(z)$ is a root of the equation $g(z_0)=0$ and the $p$ order of the pole $z_0$ of $f(z)$ is the first power of the factor $(z-z_0)$ of no hopeless coefficient in the development in series of Taylor of $g(z)$ around $z_0$.

The calculation of this power requires to use the formulas of following integrals:

$$\int \frac{1}{(x^2 + b)^2} dx = \frac{1}{b} \frac{x}{\sqrt{x^2 + b}}$$

$$\int \frac{x}{\sqrt{x^2 + b}} dx = \int \frac{2x}{2\sqrt{x^2 + b}} dx = \sqrt{x^2 + b}$$

2.4 Calculation of luminance as a function of wavelength:

The luminance [5], [6], [7] as a function of the wavelength is obtained by Planck's law:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

3. Results:

3.1 The power received at the surface surrounding the concentrator's focus:

The equation (1), allows us to get:

$$P_{1\rightarrow 2} = 4\pi f^2 \epsilon_r I_3 R_1^2 \left\{ \text{Arctan} \left( \frac{H}{\sqrt{f}} \right) - \frac{\sqrt{H}}{\frac{H}{f} + 1} \right\}$$

By taking the following numeric values:

$$\{ f = 0,25\, \text{cm} \}
\{ \epsilon_r = 0,96 \}
\Rightarrow \text{without attenuation:}

The maximal value of the solar constant is

$$I_{S_{\text{max}}} = 1353\, \text{Wm}^{-2}$$

So we have:

$$P_{1\rightarrow 2} = 1019,62 \, R_1^2 \left\{ \text{Arctan} \left( \sqrt{4H} \right) - \frac{\sqrt{4H}}{\left(4H + 1\right)} \right\}$$

And at various values of $R_1$ given in the following Table 1:
Table 1: Values of ray of the circular black body.

<table>
<thead>
<tr>
<th>$R_1$ (en m)</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
</tr>
</thead>
</table>

While making vary the height $H_S$ of 0 to 0.5m, we have the following curves:

![Graph showing power received at varying heights](image)

**Fig 1:** Representative curve of the power received in a circular surface without attenuation, according to the height.

- With attenuation of the atmosphere:
  
  The maximal value of the solar constant is:
  
  $I'_{\text{max}}(\theta) = 925.26 \, \text{W.m}^{-2}$  

  Then we have:
  
  $P_{1_{\rightarrow 2}} = 697.27 \, R_1^2 \left\{ \text{Arctan} \left( \frac{\sqrt{4H}}{4H + 1} \right) \right\}$

  and with previous values, we have the following curves:
3.2 The maximal temperature to the concentrator’s focus:

While fixing the value of $f$ and for the following cases:

- Without attenuation of the atmosphere:

In this case we get:

$$ T = \frac{418.23}{ \sqrt{R_1} } H^\frac{3}{2} $$  \hspace{1cm} (29)

For the previous values, we have the following curves:
Fig 3: Representative curve of the maximal temperature to the circular focus according to the height.

- With the attenuation of the atmosphere:

In this case, we get:

\[ T = \frac{380,33}{\sqrt{R_1}} H^3 \]  

(30)

These curves are:

Fig 4: Curve representative of the maximum temperature at the rectangular focus according to the height of the dish.

3.3 The power radiated by the black body toward a gray body:

Is \( d^2 \Phi_{1 \rightarrow 2} \) the elementary energy flux emitted by the surface element \( d\Sigma_1 \) de la surface \( (\Sigma_1) \) at the temperature \( T_1 \) which will be received by the surface element \( d\Sigma_2 \) from the surface \( (\Sigma_2) \). The total (hemispheric) flux emitted by the surface \( (\Sigma_1) \) is:

Using the Formula Bouguer (1), the infinitesimal flux emitted by the surface element \( dS_1 \) et intercepted by the surface element \( d\Sigma_2 \) is:

\[ d^2 \Phi_{1 \rightarrow 2} = \frac{\sigma T_1^4}{\pi} dS_1 \frac{d\Sigma_2 \cos \theta_2}{(r_{12})^2} \cos \theta_1 \]  

(31)

The two surface elements \( dS_1 \) and \( dS_2 \) are separated by the distance \( |\vec{r}_2 - \vec{r}_1| \) that we note by \( r_{12} \).
The two surface elements $dS_1$ and $dS_2$ are separated by $h$.

The black surface ($S_1$) emits electromagnetic radiation to the gray surface ($S_2$). As the two surfaces are circular then note respectively their radii by $R_1$ and $R_2$. They are elongated and of the same axis.

Since the two surfaces are parallel, then: $\theta_1 = \theta_2 = \theta$, and that:

$$\cos \theta = \frac{h}{r_{12}}$$

From where equation (31) becomes:

$$d^2 \Phi_{1 \rightarrow 2} = h^2 \frac{\sigma T_1^4}{\pi} dS_1 \frac{dS_2}{(r_{12})^4} \quad (32)$$

First, by integrating on the surface ($S_1$) one can write:

$$d\Phi_{1 \rightarrow 2} = h^2 \frac{\sigma T_1^4}{\pi} \int_{S_1} \frac{dS_2}{(r_{12})^4} \quad (33)$$

Parameters the surface ($S_1$):

$$\hat{r}_1 (r_1, \varphi) = r_1 \begin{pmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{pmatrix}$$

with

$$\begin{cases} r_1 \in [0, R_1] \\ \varphi \in [0, 2\pi] \end{cases}$$

From where:
\[
\mathbf{dS_1} = \left( \frac{\partial \mathbf{r}_1}{\partial r_1} \wedge \frac{\partial \mathbf{r}_1}{\partial \varphi} \right) dr_1 d\varphi = r_1 \begin{pmatrix} \cos \varphi \\
-\sin \varphi \\
0 \end{pmatrix} \wedge \begin{pmatrix} -\sin \varphi \\
\cos \varphi \\
0 \end{pmatrix} = r_1 \begin{pmatrix} 0 \\
0 \\
1 \end{pmatrix} dr_1 d\varphi
\]

\[ r_{12}^2 = r_1^2 - 2(x_2 \cos \varphi + y_2 \sin \varphi)r_1 + r^2 + h^2 \]

\[
d\Phi_{1\to2} = h^2 \frac{\sigma T_1^4}{\pi} \int_{(S_2)} \frac{r_1 dr_1 d\varphi}{[r_1^2 - 2(x_2 \cos \varphi + y_2 \sin \varphi)r_1 + r^2 + h^2]^2}
\]

\[
d\Phi_{1\to2} = \frac{1}{2} \frac{\sigma T_1^4}{\pi} \left[ 1 - \frac{(\rho_2^2 - R_1^2 + h^2)}{\sqrt{(\rho_2^2 - R_1^2 + h^2)^2 + 4R_1^2 h^2}} \right] dS_2
\]

Now integrate with \(dS_2\)

\[
\Phi_{1\to2} = \frac{1}{2} \frac{\sigma T_1^4}{\pi} \left[ S_2 - \pi \int_0^{R_2} \frac{(R^2 - R_1^2 + h^2)}{\sqrt{(R^2 - R_1^2 + h^2)^2 + 4R_1^2 h^2}} 2R dR \right]
\]

\[ dS_2 = RdRd\theta \]

With:

\[ R \in [0, R_2] \text{ and } \theta \in [0, 2\pi] \]

\[
\Phi_{1\to2} = \frac{\pi}{2} \left( \frac{\sigma T_1^4}{\pi} \right) \left[ R_1^2 + R_2^2 + h^2 - \sqrt{(R_1^2 - R_2^2 - h^2)^2 + 4R_1^2 h^2} \right]
\]

Fig. 2: Positioning of the receiving surface element of the radiations emitted by the black body in polar coordinates.
We find that this power is:

\[
\Phi_{1 \rightarrow 2} = \frac{\pi}{2} \left( \frac{\sigma T_1^4}{\pi} \right) R_1^2 \left\{ 1 + \left( \frac{R_2}{R_1} \right)^2 + \left( \frac{h}{R_1} \right)^2 \sqrt{1 - \left( \frac{R_2}{R_1} \right)^2 - \left( \frac{h}{R_1} \right)^2} \right\}^{2} + 4 \left( \frac{h}{R_1} \right)^2 \right\} \]  
(40)

3.4 The luminance according to the length of wave:

The luminance according to the length of wave is given by the law of Planck:

\[
L_{\lambda}(T) = \frac{1.19 \times 10^{-16}}{\lambda^5} \left( \frac{1}{e^{\frac{\lambda}{\lambda_{1}}} - 1} \right) \]  
(41)

Without attenuation of the atmosphere:

For the two heights of the dish, and the values of the maximal temperatures found, we have the following curves:

![Graph showing luminance as a function of wavelength](image)

**Fig 5**: Representative curve of the luminance as a function of the wavelength for the height of 0.1m.

and:
Fig. 6: Representative curve of the luminance according to the length of wave for the height of 0.5m.

- **With attenuation of the atmosphere:**

For this case and for the two heights of the paraboloid, we have the two following curve groups:

Fig 7: Curve representative of luminance, a circular black body as a function of the wavelength for the height of 0.1m.

and:
Fig. 8: representative curve of the luminance of a circular black body according to the wavelength for the height of 0.5m.

- **Without attenuation:**

At various temperatures and according to the heights of the paraboloid we have the following maximum wavelength values:

- **H = 0.1m:**

<table>
<thead>
<tr>
<th>H(m)</th>
<th>T(K)</th>
<th>( L_{\lambda_{\text{max}}} ) (10^3 \text{W/m}^2/\text{Sr/m} )</th>
<th>( \lambda_{\text{max}} ) (( \mu \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>881.76</td>
<td>2.19</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>778.57</td>
<td>1.23</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>720.01</td>
<td>0.79</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>666.6</td>
<td>0.54</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Table 2: Value of the maximal wave length given out by the circular black body to various temperature for H=0.1m (case without attenuation)

We find that:

\[ \lambda_{\text{max}} \geq 3.3 \mu \text{m}. \]

- **H = 0.5m:**
Table 3: Value of the maximal wave length given out by the circular black body to various temperature for H=0,5m (case without attenuation)

<table>
<thead>
<tr>
<th>H(m)</th>
<th>0,5</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(K)</td>
<td>1612,4</td>
</tr>
<tr>
<td>(L_\lambda(T)_{\text{max}}(10^{10}\text{w/m}^2/\text{Sr/m}))</td>
<td>4,44</td>
</tr>
<tr>
<td>(\lambda_{\text{max}}(\mu\text{m}))</td>
<td>1,8</td>
</tr>
</tbody>
</table>

We find that: \(\lambda_{\text{max}} \geq 1,8\mu\text{m}\).

- **With attenuation:**

At various temperatures and according to the heights of the paraboloid we have the following maximum wavelength values:

  ➢ H = 0,1m:

Table 4: Value of the maximal wave length given out by the circular black body to various temperature for H=0,1m (case with attenuation)

<table>
<thead>
<tr>
<th>H(m)</th>
<th>0,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(K)</td>
<td>801,89</td>
</tr>
<tr>
<td>(L_\lambda(T)_{\text{max}}(10^{9}\text{w/m}^2/\text{Sr/m}))</td>
<td>1,34</td>
</tr>
<tr>
<td>(\lambda_{\text{max}}(\mu\text{m}))</td>
<td>3,6</td>
</tr>
</tbody>
</table>

We find that: \(\lambda_{\text{max}} \geq 3,6\mu\text{m}\).

  ➢ H = 0,5m:

Table 5: Value of the maximal wave length given out by the circular black body to various temperature for H=0,5m (case with attenuation)

<table>
<thead>
<tr>
<th>H(m)</th>
<th>0,5</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(K)</td>
<td>1466,3</td>
</tr>
<tr>
<td>(L_\lambda(T)_{\text{max}}(10^{10}\text{w/m}^2/\text{Sr/m}))</td>
<td>2,76</td>
</tr>
<tr>
<td>(\lambda_{\text{max}}(\mu\text{m}))</td>
<td>2</td>
</tr>
</tbody>
</table>

We find that: \(\lambda_{\text{max}} \geq 2\mu\text{m}\).

In general, the maximum lengths that characterize the light emitted by our heated black body, at a temperature T, using our solar paraboloid concentrator, are between 0.67\mu\text{m} and 4.8\mu\text{m}. That is, included in the frequency band such that: Knowing that the frequency electromagnetic waves included in the band: are designated by "infrared". As the name suggests, this frequency band is immediately below the visible red light. Our result allows us to say that our black body heated using our dish concentrator, emits infrared.
4. Conclusion:

Our study allows us to conclude that the reunited solar energy at the focus of the paraboloid concentrator increases according to its height on which its surface depends. And that the maximum temperature that the black body placed at this meeting point can reach is proportional to the depth of the paraboloid raised to the power three out of eight. And according to Planck's law, this body can radiate similarly with respect to its absorption of black electromagnetic waves placed at this point, much smaller than the opening of the concentrator, heats up by absorbing all the reflected solar radiation by this system. And at a given equilibrium temperature, this body emits radiation to another body. This program, which follows Planck's law, allows researchers to determine the color of the heated body by studying the luminance according to this equilibrium temperature. For this study, we set the dimensions of the solar concentrator (height, focus), the black body and that of gray, taking into account that the latter is smaller in order to neglect radiation losses. We find that the black body heated in our study emits Infrared.

5. ACKNOWLEDGEMENT

Our deep gratitude to Mr. RATIARISON Adolphe Andriamanga, Emeritus Professor at the Faculty of Sciences of the University of Antananarivo, Madagascar and his team, for allowing us to conduct our investigations in their DYACO laboratory.

6. Reference