TFSF formulation for plane wave in 2D-FDTD grid for TM propagation mode

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ABSTRACT

The description of the implementation of the simulation of a plane wave with the FDTD method is described in this paper. Studies are focused on the implementation of a 2D FDTD grid, for wave propagation in TM (Transverse Magnetic) mode. The TFSF formulation is described for the TM mode of a 2D grid in order to obtain the field correction equations. The correction of the fields requiring knowledge of the incidental fields at the TFSF limit, this work presents how these fields are calculated from an 1D FDTD grid and then injected into the 2D grid. The results of the formulations are presented in the form of a simulation illustrating the propagation of a plane wave in a 2D FDTD grid.

Keyword: *FDTD, TFSF Boundary, 2D grid, TM propagation, plane wave*

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1. INTRODUCTION

A wave incident at a receiver distant from an EM source can be approximated as a plane wave. In order to simulate a plane wave traversing an FDTD (Finite Difference Time Domain) grid, the TFSF (Total Field / Scattered Field) formulation is taken in this paper. In a 1D FDTD simulation, the TFSF formulation allows the introduction of a wave traveling in a single direction of propagation [1]. The work presented here concerns the implementation of a plane wave using the same formalisms as in 1D, on a 2D FDTD grid.

In order to implement a plane wave in a 2D FDTD simulation, besides implementing the propagation in one direction, the wave must be designed so that it forms a line perpendicular to the axis of propagation. This work presents the concepts for the realization of a plane wave traveling in the positive direction of the x axis. The grid is terminated by layers of losses in order to simulate an infinite space.

2. 2D TFSF FORMULATION

2.1. TFSF formulation for TM mode

In two dimensions, the grid is divided into a TF region and an SF region and the boundary between the two regions is defined by the perimeter formed by a rectangle. Fig. 1 illustrates a TM grid with a rectangular boundary of TFSF. The TF region is included in the TFSF boundary and the SF region is a part of the network located outside this boundary. The TF region is defined by the indices (i_d, j_d) and (i_f, j_f), of the “first” and “last” nodes of electric fields which are in the TF region. There are two possible axes of propagation in this construction, the x axis and the y axis. In the following, the wave propagation is considered to be along the x axis.

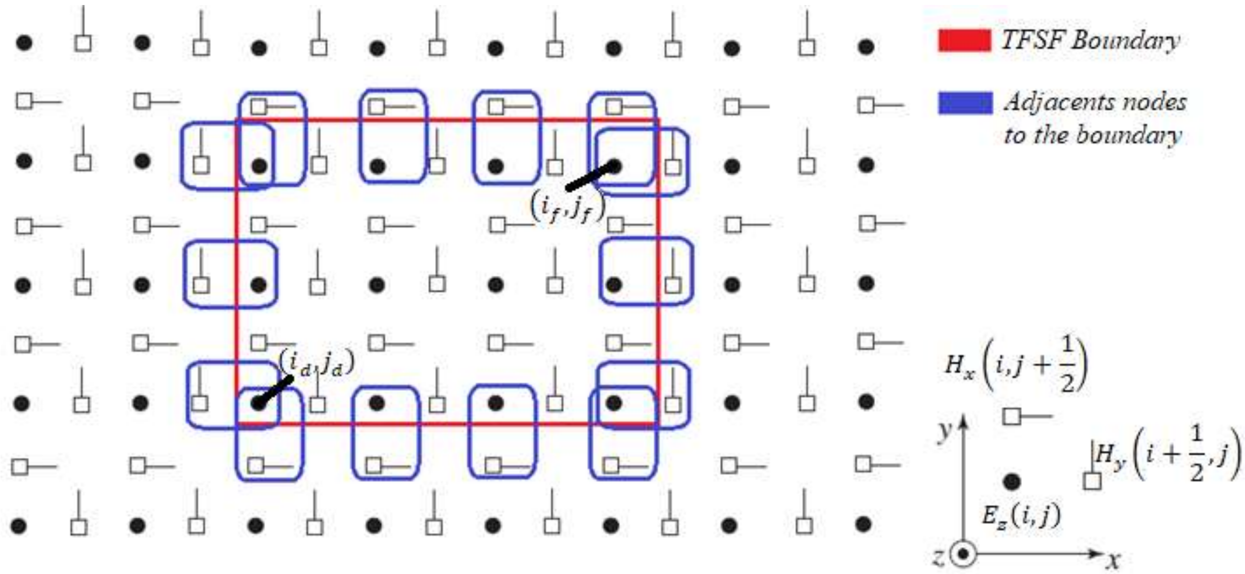

Fig.1: Delimiting a TFSF boundary in a 2D FDTD grid in TM mode

2.2. Corrections for propagation along the x axis

In Fig. 1, the electric fields adjacent to the TFSF boundary are always found in the TF region. These nodes have at least one neighbor magnetic field node located in the SF region [2]. Thus, the necessary correction would involve the addition of the incident field to neighboring magnetic fields on the other side of the TFSF boundary. The Eq.2, allow to define the corrections necessary for these electric fields. These fields relate to the electrical nodes adjacent to the left limit (Eq.2.c) and as well as those adjacent to the right limit (Eq.2.e). Nodes adjacent to the lower and upper edges of the boundary do not need correction, as there are no H_x fields incident at the boundary. The coefficients for updating the fields are defined in Eq. 1.

$$c_{ezez} = \frac{1-\frac{\sigma_z \Delta t}{2\varepsilon_z}}{1+\frac{\sigma_z \Delta t}{2\varepsilon_z}} ; c_{ezhy} = \frac{\Delta t}{\left(1+\frac{\sigma_z \Delta t}{2\varepsilon_z}\right)\Delta x} ; c_{ezhx} = \frac{-\Delta t}{\left(1+\frac{\sigma_z \Delta t}{2\varepsilon_z}\right)\varepsilon_z \Delta y} \quad (1.a)$$

$$c_{hxhx} = \frac{1-\frac{\sigma_{mx} \Delta t}{2\mu_x}}{1+\frac{\sigma_{mx} \Delta t}{2\mu_x}} ; c_{hxez} = \frac{-\Delta t}{\left(1+\frac{\sigma_{mx} \Delta t}{2\mu_x}\right)\mu_x \Delta y} \quad (1.b)$$

$$c_{hyhy} = \frac{1-\frac{\sigma_{my} \Delta t}{2\mu_y}}{1+\frac{\sigma_{my} \Delta t}{2\mu_y}} ; c_{hyez}\left(i+\frac{1}{2}, j\right) = \frac{\Delta t}{\left(1+\frac{\sigma_{my} \Delta t}{2\mu_y}\right)\mu_y \Delta x} \quad (1.c)$$

The equations for updating the electric fields located at the left limit of the FDTD grid are presented in Eq.2.a and Eq.2.b [3]. Eq.2.b shows that the update involves electric and magnetic fields found in the TF region but also magnetic field components found in the SF region. These components of magnetic fields of the SF region must be corrected by adding the incident fields to the boundary. The addition of these incident fields allows the updating of the components of the electric field involving only total fields. In order to not modify the updating equations of the FDTD formulation, the correction is made after the usual updating of the electric fields. The corrections made to the electric field components to the left of the TFSF limit are described in Eq.2.c. The electric fields at the right TFSF limit, the updates of which are given in Eq.2.d, are corrected in the same way using Eq.2.e. Eq.2.a indicate the nodes of the y axis involved by the corrections.

$$j \in \left[j_d, j_f\right] \tag{2.a}$$

$$\overbrace{E_z^{n+1}(i_d, j)}^{TF} = c_{ezez}\overbrace{E_z^{n}(i_d, j)}^{TF} + c_{ezhy}\left\{\overbrace{H_y^{n+\frac{1}{2}}\left(i_d + \frac{1}{2}, j\right)}^{TF} - \overbrace{H_y^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right)}^{SF}\right\} + c_{ezhx}\left\{\overbrace{H_x^{n+\frac{1}{2}}\left(i_d, j + \frac{1}{2}\right)}^{TF} - \overbrace{H_x^{n+\frac{1}{2}}\left(i_d, j - \frac{1}{2}\right)}^{SF}\right\} \tag{2.b}$$

$$E_z^{n+1}(i_d, j) = c_{ezez}E_z^{n}(i_d, j) - c_{ezhy}H_{yinc}^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}\right) \tag{2.c}$$

$$\overbrace{E_z^{n+1}(i_f, j)}^{TF} = c_{ezez}\overbrace{E_z^{n}(i_f, j)}^{TF} + c_{ezhy}\left\{\overbrace{H_y^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right)}^{SF} - \overbrace{H_y^{n+\frac{1}{2}}\left(i_f - \frac{1}{2}, j\right)}^{TF}\right\} + c_{ezhx}\left\{\overbrace{H_x^{n+\frac{1}{2}}\left(i_f, j + \frac{1}{2}\right)}^{SF} - \overbrace{H_x^{n+\frac{1}{2}}\left(i_f, j - \frac{1}{2}\right)}^{TF}\right\} \tag{2.d}$$

$$E_z^{n+1}(i_f, j) = c_{ezez}E_z^{n}(i_f, j) + c_{ezhy}H_{yinc}^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}\right) \tag{2.e}$$

Equally, magnetic field nodes tangential to the TFSF boundary are always in the SF region. These nodes have a neighbor electric field node which is in the TF region. Thus, the correction needed on these nodes would involve subtracting the incident field from the electric field node located on the other side of the TFSF boundary. The Eq.3, define the corrections to be made to the updating equations for the H_x fields adjacent to the lower (Eq.3.c) and upper (Eq.3.e) edges of the limit. Eq.3.b and Eq.3.d define the irregularities when updating the H_x component of the magnetic field [3].

$$i \in \left[i_d, i_f\right] \tag{3.a}$$

$$\overbrace{H_x^{n+\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right)}^{SF} = c_{hxhx}\overbrace{H_x^{n-\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right)}^{SF} + c_{hxez}\left(\overbrace{E_z^{n}(i, j_d + 1)}^{TF} - \overbrace{E_z^{n}(i, j_d)}^{SF}\right) \tag{3.b}$$

$$H_x^{n+\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right) = c_{hxhx}H_x^{n+\frac{1}{2}}\left(i, j_d - \frac{1}{2}\right) - c_{hxez}E_{zinc}^{n}(i) \tag{3.c}$$

$$\overbrace{H_x^{n+\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right)}^{SF} = c_{hxhx}\overbrace{H_x^{n-\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right)}^{SF} + c_{hxez}\left(\overbrace{E_z^{n}(i, j_f + 1)}^{SF} - \overbrace{E_z^{n}(i, j_f)}^{TF}\right) \tag{3.d}$$

$$H_x^{n+\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right) = c_{hxhx}H_x^{n+\frac{1}{2}}\left(i, j_f + \frac{1}{2}\right) + c_{hxez}E_{zinc}^{n}(i) \tag{3.e}$$

The Eq.4, define the corrections to be made to the update equations for the H_y fields adjacent to the left (Eq.4.c) and right (Eq.4.e) edges of the TFSF limit. Eq.4.b and Eq.4.d define the inconsistencies when updating the H_y component of the magnetic field.

$$j \in [j_d, j_f] \tag{4.a}$$

$$\overbrace{H_y^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right)}^{SF} = c_{hyhy}\,\overbrace{H_y^{n-\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right)}^{SF} + c_{hyez}\left(\overbrace{E_z^n(i_d, j)}^{TF} - \overbrace{E_z^n(i_d - 1, j)}^{SF}\right) \tag{4.b}$$

$$H_y^{n+\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right) = c_{hyhy} H_y^{n-\frac{1}{2}}\left(i_d - \frac{1}{2}, j\right) - c_{hyez} E_{zinc}^n(i_d) \tag{4.c}$$

$$\overbrace{H_y^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right)}^{SF} = c_{hyhy}\,\overbrace{H_y^{n-\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right)}^{SF} + c_{hyez}\left(\overbrace{E_z^n(i_f + 1, j)}^{SF} - \overbrace{E_z^n(i_f, j)}^{TF}\right) \tag{4.d}$$

$$H_y^{n+\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right) = c_{hyhy} H_y^{n-\frac{1}{2}}\left(i_f + \frac{1}{2}, j\right) + c_{hyez} E_{zinc}^n(i_f) \tag{4.e}$$

3. IMPLEMENTATION OF A PLANE WAVE

3.1. Calculation of incident fields

To implement a TFSF boundary, it is necessary to know the incident field on each node that has a neighbor on the other side of the TFSF boundary. The incident field must be known at all these points and for each time step. Analytical expressions were used for the incident field in 1D, that is to say the expressions describing the propagation of the incident field in the continuous world. These continuous world expressions usually involve a transcendental function (such as a trigonometric function or an exponential). The computation of these functions is quite computationally expensive, at least compared to some simple algebraic computations. If the transcendental functions must be calculated at different times for each time step, that can impose a significant cost of calculation. Provided that the direction of the incident field propagation coincides with one of the axes of the grid, there is a way to make the incident field exactly match how the incident field propagates in the FDTD grid at two dimensions [4].

The trick to calculating the incident field is to perform a one-dimensional auxiliary FDTD simulation that calculates the incident field. This auxiliary simulation uses the same material parameters as the two-dimensional grid, but is otherwise completely separate from the two-dimensional grid. The one-dimensional grid is simply used to find the incident fields needed to implement the TFSF limit. Each node E_z and H_y of the 1D grid can be considered as providing E_{zinc} and H_{yinc} respectively, at the appropriate moment in space-time [4][5].

Fig. 2 illustrates the 1D auxiliary grid as well as the 2D grid. The base of the vertical arrows pointing from the 1D grid to the 2D grid indicate the 1D grid nodes from which the 2D grid nodes get the incident field (only the 2D grid nodes adjacent to the TFSF boundary require the knowledge of the area of the incident).

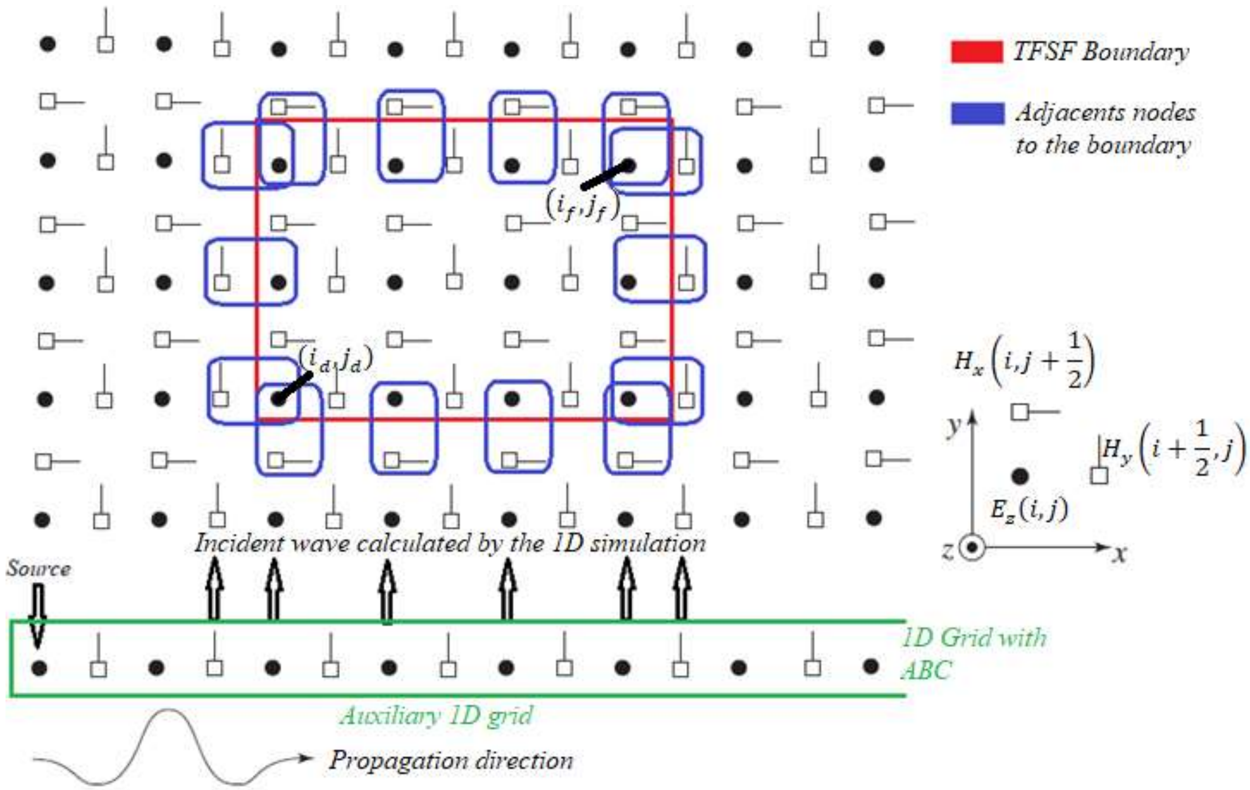

Fig.2: Introduction of incident fields to a TFSF boundary in a 2D grid in TM mode

3.2. 2D FDTD TM mode algorithm for the introduction of a plane wave

To implement a TFSF limit, in order to simulate a plane wave as a source in the simulation, it suffices to translate the field correction equations into the necessary declarations. The 2D FDTD algorithm with a TFSF implementation can be summarized as follows:

1. Update of the E_z component of the electric field

2. Correction of the value of the electric fields adjacent to the TFSF limits (Eq.2.c and Eq.2.e)

3. Update of the components H_x and H_y of the magnetic field

4. Correction of the values of the components of the magnetic fields adjacent to the TFSF limits (Eq.3.c, Eq.3.e, E.4.c and Eq.4.c)

5. Repeat the previous four steps until the fields are obtained for the desired duration

4. SIMULATION OF A PLANE WAVE SPREADING ALONG THE x-axis

For the numerical stability of the simulation, the number of Current is defined by $S_c = \frac{1}{\sqrt{2}}$. Thus, the time step of the simulation is $\Delta t = \frac{\Delta x}{\sqrt{2}c_0}$ and the space step is $\Delta x = \frac{c_0}{20 \times f_{src}}$. The wave source is defined as a wave packet generated by a modulated Gaussian pulse of frequency $f_{src} = 500\ MHz$. The dimension of the grid is 150 × 100, and it is finished with absorbent layers of size 15 cells.

The rectangular limit TFSF is defined by the first electric field node of the TF zone $((i_d, j_d) = (15{,}16))$, and the last electric field node of the TF zone $((i_f, j_f) = (135{,}85))$. Fig. 3 shows the snapshots of the wave packet propagation in the grid. The wave travels the full field area until it is absorbed into the loss layer at the end of the grid. Propagation is defined as occurring along the x-axis.

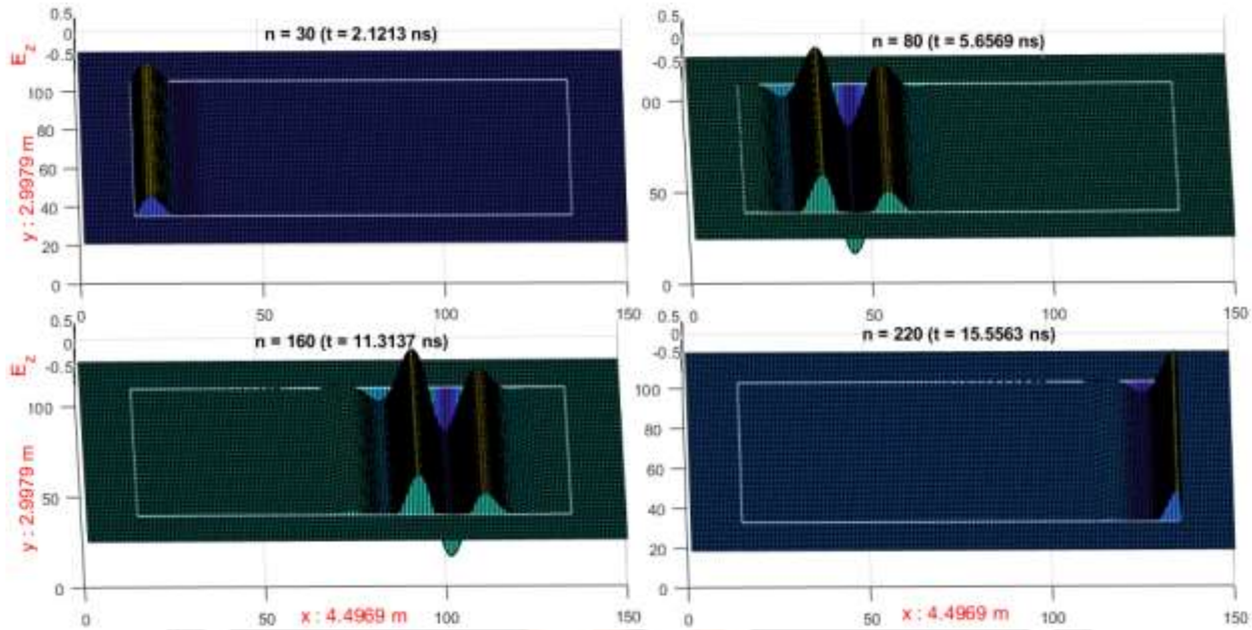

Fig.3: Snapshots of the E_z field for a plane wave consisting of a Gaussian pulse modulated in a 2D grid in TM mode.

5. CONCLUSION

Implementing a plane wave simulation in a FDTD formulation requires an understanding of the TFSF formulation. In a TFSF formulation, consideration is given to the location of the fields around and in the rectangular TFSF boundary. The formulation does not modify the main lines of an FDTD code, but adds to them the correction lines for the fields, and removes the lines necessary for the introduction of an impulse source. In this paper, the propagation of a plane wave in the positive x direction has been described for EM propagation in TM mode. By following the same method, the simulation of a propagation in the reverse direction and in the y direction can be obtained. A 2D FDTD TFSF formulation can also be described for the TE propagation mode.

6. REFERENCES

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