THEORETICAL VARIATION OF THE ELASTIC CONSTANTS OF A UNIDIRECTIONAL BAMBOO/POLYESTER LAMINA COMPOSITE ACCORDING TO THE ORIENTATION ANGLE OF THE FIBERS

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ABSTRACT

In order to contribute to the study of composite materials with bamboo fiber reinforcement, the theoretical variation of the values of the elastic constants of a unidirectional Bamboo / Polyester lamina composite, as a function of the orientation angle of the long fiber reinforcements, is studied in this article. .
The considered bamboo in the study is the "Bambusa Dissimulator" whose value of the transverse Young's modulus of the fibers is greater than that of the longitudinal Young's modulus.
From the results obtained, the values of the elastic modulus and the in-plane shear modulus of the studied lamina vary little with the orientation angle of the fibers. They increase with the amount of bamboo fibers used in the composite unlike those of the Poisson's ratio.
Moreover, the values of the latter are extreme for a fiber orientation angle around 45 °, for a given fiber content. The evolution of shear couplings and the influence of bamboo fiber content on the elastic constants of the lamina are also considered in this work.

Keywords: *Composite, Lamina, Fiber, Bamboo, Polyester, Elastic constant*

1. INTRODUCTION

Nowadays, the fibers of plants such as flax, hemp, ... are increasingly used as reinforcement of composite materials [1]. Indeed, the use of these fibers reduces environmental pollution due to the reduction in the use of synthetic fibers. In addition, the composite obtained retains its essential properties while being lighter.
In general, the value of the transverse Young's modulus of the majority of the plant fibers is lower than that of the longitudinal Young's modulus. This is not the case with bamboo fiber based on the results found by D. R. Williams et al. [2]. It is then interesting to study the elastic behavior of composites having for reinforcement of bamboo fibers.
In addition, bamboo is one of the few plants that absorb more CO_2 than it releases. Bamboo in its raw state is widely used in different fields, such as building, for example, because of its very interesting characteristics [3]. However, the use of its fibers as reinforcement of composite materials is even less well known.
The study of the elastic behavior of a lamina, base of a laminated structure, Bamboo / Polyester according to the angle of orientation of the fibers is thus studied in this article.

Indeed, the orientation of the fibers has a significant effect on the mechanical properties of the composite materials. It is reported that the Young's modulus, the Poisson's ratio and the tensile strength of most materials decrease with the degree of orientation of the fibers [4]. Cellulose fibers, for example, have significant strength in the length direction. This is significantly lower in the other directions, hence the need to align the fibers as far as possible in the direction of the length during the formulation of the composites.
Second, since long-fiber composites exhibit better mechanical behavior than short-fiber or particle composites, at least in the fiber-reinforced directions [5], the fibers in the study are long and have unidirectional arrangement.

2. CONSTITUENTS OF THE STUDIED LAMINA

The lamina studied has for reinforcement the long fibers of bamboo "Bambusa Dissimulator" and for matrix the unsaturated polyester resin.

2.1. Bamboo fibers

Bamboo fibers are extracted from bamboo, a woody plant that is part of the Gramineae family. These fibers are anisotropic. The considered bamboo in the study is the "Bambusa Dissimulator". Its necessary mechanical characteristics were measured by Brillouin spectroscopy by D. R. Williams et al. [2].

Table 1: Some mechanical characteristics of the fibers of "Bambusa Dissimulator" [2]

	Bamboo fiber
Density (g/cm^3)	1.42
Longitudinal Young’s modulus (GPa)	11.1
Transversal Young’s modulus (GPa)	14.5
Shear modulus (GPa)	8.8
Major Poisson’s Ratio	0.19

2.2. Unsaturated polyester

Unsaturated polyester is one of the most widely used thermosetting resins in composite parts manufacturing.

Table 2: Some mechanical characteristics of the unsaturated polyester resin [6]

	Unsaturated polyester resin
Density (g/cm^3)	1.2
Young’s modulus (GPa)	4
Shear modulus (GPa)	1.4
Poisson’s Ratio	0.4

3. ADOPTED APPROACHES

Generally, in the majority of laminates, a few lamina are placed at an angle because most of these have low stiffness and strength properties in the transverse direction. In order to study elasticity for the case of Bamboo/Polyester, the following hypotheses and benchmarks were adopted.

Hypotheses

- The fibers are continuous and parallel
- Fibers and matrix follow Hooke's law
- There are no voids in the lamina
- The diameters and the space between the fibers are uniform
- The adhesion between the fibers and the matrix is perfect

Benchmarks used

Let be two systems of rectangular axes (1, 2) and (x, y).
The axes 1 and 2 are called the material axes or the local axes. The direction of the axis 1 is that of the fibers. As for the axis 2, it is directly perpendicular to the axis 1.
The axes x and y are called the off-axes or the global axes.
The angle between the two axis systems is denoted by an angle θ.

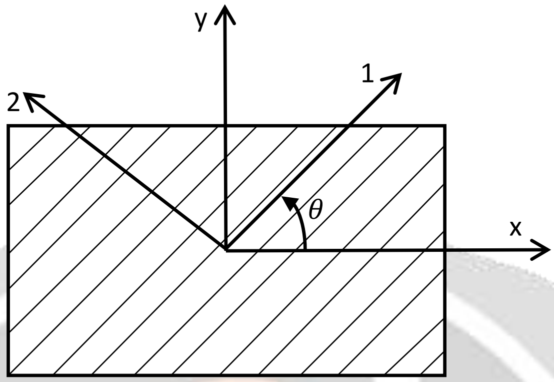

Fig 1 : Axis systems of an angle lamina

The elastic behavior of a material is illustrated through the values of its elastic constants.

Elastic constants in directions 1 and 2

The elastic constants in the directions 1 and 2 are: the longitudinal Young's modulus E_1, the transverse Young's modulus E_2, the in-plane shear modulus G_{12}, and the Poisson's ratio ν_{12} and ν_{21}.

To obtain the values of these elastic constants, the Mixture Law [7] is used:

$$\boldsymbol{E_1 = V_m E_m + V_f E_{f1}} \quad (1)$$

$$\boldsymbol{1/E_2 = V_m/E_m + V_f/E_{f2}} \quad (2)$$

$$\boldsymbol{1/G_{12} = V_m/G_m + V_f/G_{f12}} \quad (3)$$

$$\boldsymbol{\nu_{12} = V_m \nu_m + V_f \nu_{f12}} \quad (4)$$

$$\boldsymbol{\nu_{21} = \nu_{12}\, E_2/E_1} \quad (5)$$

- E_m, G_m, V_m are respectively the Young's modulus, the in-plane shear modulus and the volume fraction of the matrix.
- E_{f1}, E_{f2}, G_{f12}, V_f are the longitudinal Young's modulus, the transverse Young's modulus, the in-plane shear modulus and the volume fraction of the fibers.
- ν_m et ν_{f12} are the Poisson's ratios of the matrix and fibers

Elastic constants in directions x and y

A unidirectional lamina falls under an orthotropic material category. In Axis Systems (1,2) and (x,y), the Hooke's Law is written as follows:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (6)$$

with $[S] = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (7)$$

with $[\bar{S}] = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix}$

Where: σ_i is the normal stress applied to the material in the direction i

τ_{ij} is the shear stress applied to the material in the direction j and present in the plane having normal axis i

ε_i is the relative deformation of the material in the direction i

γ_{ij} is the angular deformation undergone by the material in the plane ij

[S] is the compliance matrix and S_{ij} are its elements

$[\bar{S}]$ is the transformed reduced compliance matrix and $\bar{S}_{ij}$ its elements

The elements of [S] and $[\bar{S}]$ have for expression:

$$\boldsymbol{S_{11}} = \frac{\boldsymbol{1}}{\boldsymbol{E_1}} \quad (8)$$

$$\boldsymbol{S_{22}} = \frac{\boldsymbol{1}}{\boldsymbol{E_2}} \quad (9)$$

$$\boldsymbol{S_{66}} = \frac{\boldsymbol{1}}{\boldsymbol{G_{12}}} \quad (10)$$

$$\boldsymbol{S_{12}} = \boldsymbol{S_{21}} = -\boldsymbol{S_{11}}.\boldsymbol{\nu_{12}} \quad (11)$$

$$\boldsymbol{\bar{S}_{11}} = \boldsymbol{S_{11}}c^4 + (\boldsymbol{2S_{12}} + \boldsymbol{S_{66}})s^2c^2 + \boldsymbol{S_{22}}s^4 \quad (12)$$

$$\boldsymbol{\bar{S}_{12}} = \boldsymbol{S_{12}}(s^4 + c^4) + (\boldsymbol{S_{11}} + \boldsymbol{S_{22}} - \boldsymbol{S_{66}})s^2c^2 \quad (13)$$

$$\boldsymbol{\bar{S}_{22}} = \boldsymbol{S_{11}}s^4 + (\boldsymbol{2S_{12}} + \boldsymbol{S_{66}})s^2c^2 + \boldsymbol{S_{22}}c^4 \quad (14)$$

$$\boldsymbol{\bar{S}_{16}} = (\boldsymbol{2S_{11}} - \boldsymbol{2S_{12}} - \boldsymbol{S_{66}})sc^3 - (\boldsymbol{2S_{22}} - \boldsymbol{2S_{12}} - \boldsymbol{S_{66}})s^3c \quad (15)$$

$$\boldsymbol{\bar{S}_{26}} = (\boldsymbol{2S_{11}} - \boldsymbol{2S_{12}} - \boldsymbol{S_{66}})s^3c - (\boldsymbol{2S_{22}} - \boldsymbol{2S_{12}} - \boldsymbol{S_{66}})sc^3 \quad (16)$$

$$\boldsymbol{\bar{S}_{66}} = \boldsymbol{2}(\boldsymbol{2S_{11}} + \boldsymbol{2S_{22}} - \boldsymbol{4S_{12}} - \boldsymbol{S_{66}})s^2c^2 + \boldsymbol{S_{66}}(s^4 + c^4) \quad (17)$$

Where : $c = cos\ (\theta)$

$s = sin\ (\theta)$

The elastic constants in the x and y directions are then defined as follows:

$$\boldsymbol{E_x} = \frac{\boldsymbol{1}}{\boldsymbol{\bar{S}_{11}}} \quad (18)$$

$$\boldsymbol{E_y} = \frac{\boldsymbol{1}}{\boldsymbol{\bar{S}_{22}}} \quad (19)$$

$$\boldsymbol{G_{xy}} = \frac{\boldsymbol{1}}{\boldsymbol{\bar{S}_{66}}} \quad (20)$$

$$\boldsymbol{\nu_{xy}} = -\frac{\overline{S}_{12}}{\overline{S}_{11}} \tag{21}$$

$$\boldsymbol{\nu_{yx}} = -\frac{\overline{S}_{12}}{\overline{S}_{22}} \tag{22}$$

$$\boldsymbol{m_x} = -\overline{S}_{16}.\boldsymbol{E_1} \tag{23}$$

$$\boldsymbol{m_y} = -\overline{S}_{26}.\boldsymbol{E_1} \tag{24}$$

4. RESULTS AND COMMENTS

In order to better understand the variations of the elastic constants, the curves corresponding to fiber volume contents of 15%, 30%, 45%, 60%, 70%, 75%, 80%, 85% and 90% were considered in this article.
The following figures show the variations of the elastic constants of the Bamboo / Polyester lamina studied as a function of the orientation angle of the fibers.

4.1. Elastic modulus in direction x : E_X

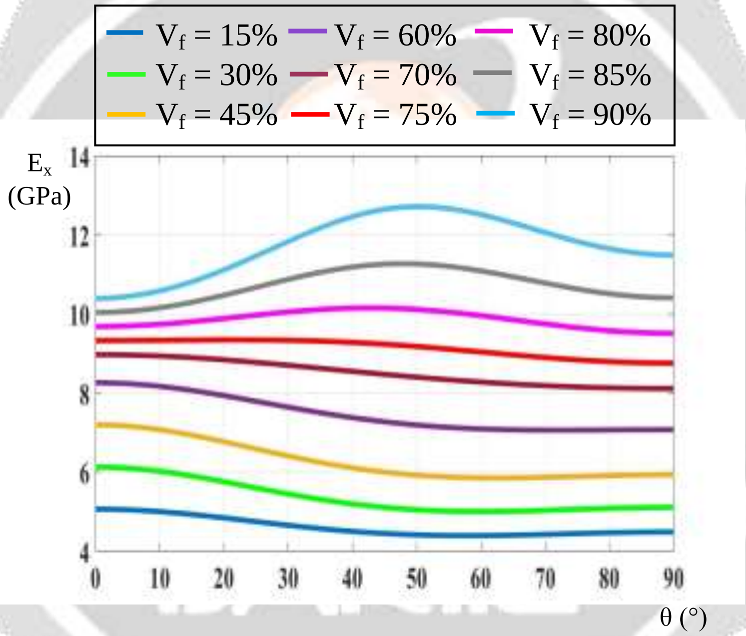

Fig 2 : Variation of the elastic modulus in direction x E_x according to θ

Fig 2 shows a small variation in the value of the Elastic modulus in direction x of the Bamboo / Polyester lamina studied as a function of the orientation angle of the fibers for a given fiber volume content.
The value of E_x also increases with the amount of fiber in the composite.
The representative curves of $E_x(\theta)$ obtained for contents of bamboo fibers greater than 75% are different from those obtained with contents of less than 75%.

4.2. Elastic modulus in direction y : E_Y

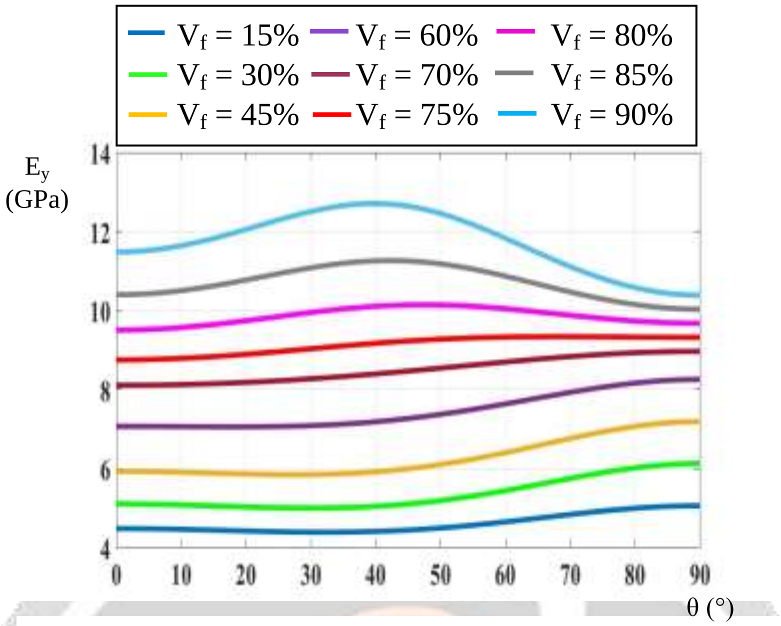

Fig 3 : Variation of the elastic modulus in direction y E_y according to θ

The elastic modulus in direction y of the Bamboo / Polyester lamina studied shows a small variation as a function of the orientation angle of the fibers similar to the elastic modulus in direction x due to symmetry.
And the larger the amount of fiber in the composite, the higher the value of E_y.
The remarks made on the curves of $E_x(\theta)$ are valid for those of $E_y(\theta)$.

4.3. In-plane Shear modulus : G_{xy}

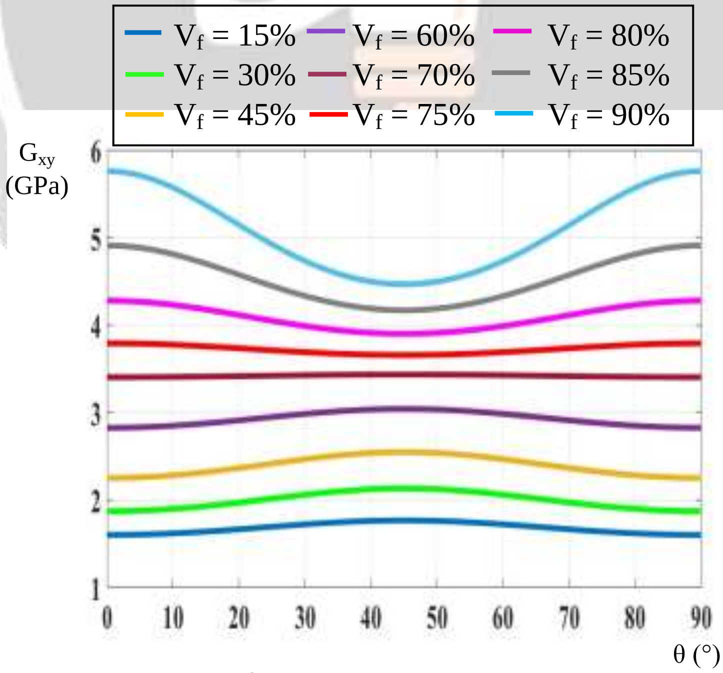

Fig 4 : Variation of the in-plane shear modulus G_{xy} according to θ

From Fig 4, the value of the shear modulus of the lamina varies little as a function of the orientation angle of the fibers, especially for a fiber volume content of less than or equal to 75%.

4.4. Poisson's ratio ν_{xy}

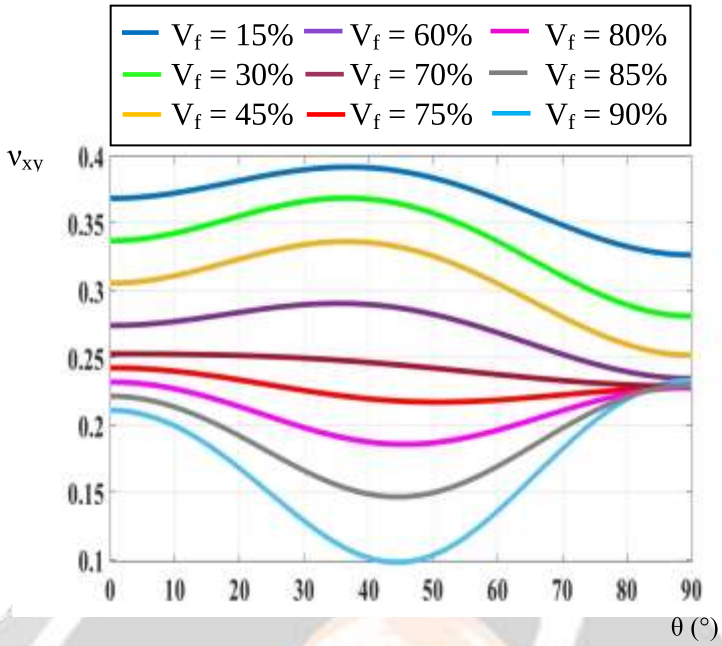

Fig 5 : Variation of the Poisson's ratio ν_{xy} according to θ

The value of Poisson's ratio ν_{xy} decreases as the amount of fiber in the composite increases.

For a given fiber content, the value of ν_{xy} is extreme for an angle of orientation of the fibers around 45 °.

4.5. Poisson's ratio ν_{yx}

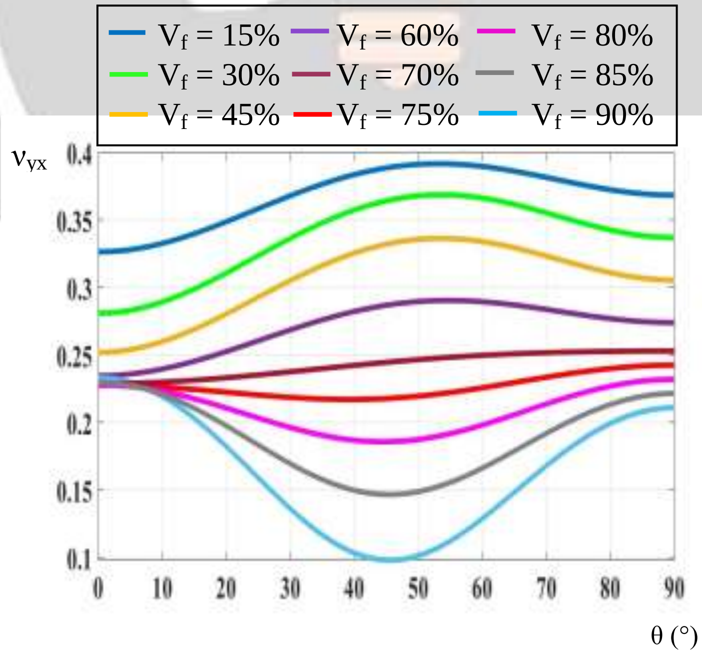

Fig 6 : Variation of the Poisson's ratio ν_{yx} according to θ

The value of the Poisson's ratio ν_{yx} of the Bamboo / Polyester lamina studied also decreases as the amount of fibers in the composite increases.
It is extreme for an angle of orientation of the fibers around 45 °.

4.6. Shear coupling m_x

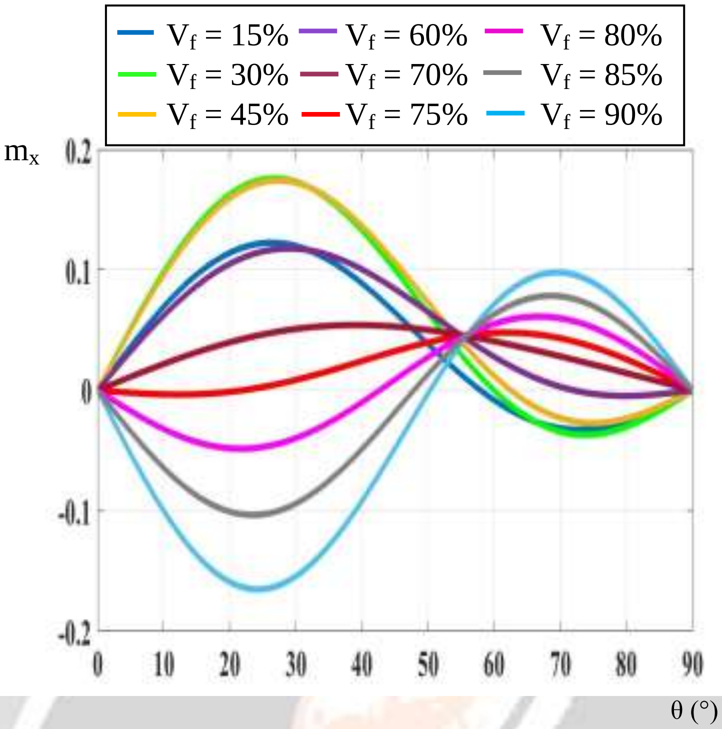

Fig 7 : Variation of the shear coupling m_x according to θ

The shear coupling m_x relates the normal stress in the x-direction to the shear strain.
Whatever the bamboo fiber content in the lamina studied, the value of m_x is approximately equal to 0.05 for an angle of orientation of the fibers at around 55 °.

4.7. Shear coupling m_y

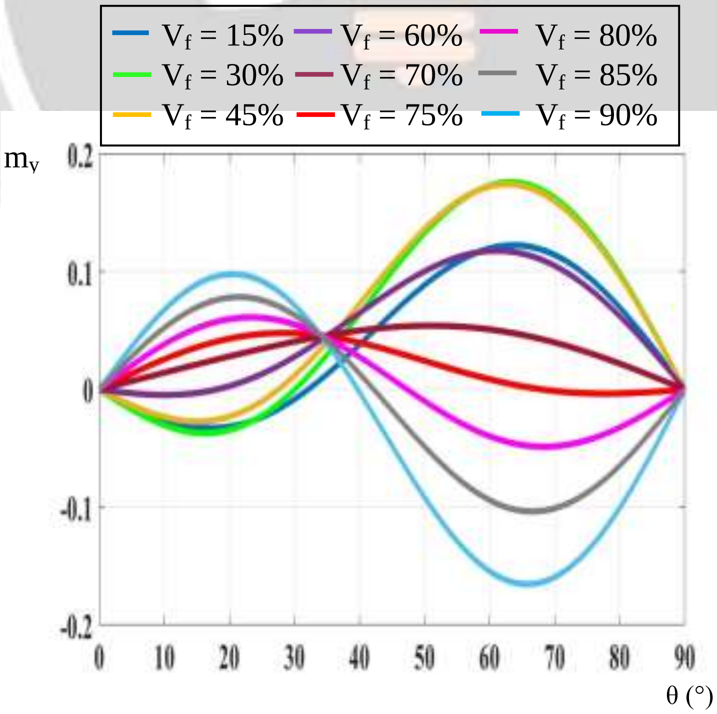

Fig 8 : Variation of the shear coupling m_y according to θ

The shear coupling relates the normal stress in the y-direction to the shear strain.
Whatever the bamboo fiber content in the lamina of the study, the value of m_y is approximately equal to 0.05 for an angle of orientation of the fibers at around 35 °.

5. CONCLUSION

The theoretical evolution of the elastic constants according to the orientation angle of the fibers of the composite lamina whose matrix is the unsaturated polyester resin and its reinforcement is the long fibers of bamboo "Bambusa Dissimulator", has been studied in this article.

According to the results obtained, for a given fiber content, the values of the elastic modulus in direction x, the elastic modulus in direction y and the in-plane shear modulus of the lamina studied vary little with the orientation angle of the fibers.

The values of E_x, E_y and G_{xy} increase with the amount of fibers introduced into the composite, unlike those of the Poisson’s ratio ν_{xy} and ν_{yx}.

The curves representative of the elastic constants obtained for contents of bamboo fibers greater than 75% have different behavior from those obtained with contents of less than 75%.

For a given fiber content, the values of ν_{xy} and ν_{yx} are extreme for a fiber orientation angle around 45 °.

The results obtained for the lamina studied also show the existence of a fiber orientation angle for which the shear coupling value is the same, regardless of the fiber content in the composite. Indeed, m_x is approximately equal to 0.05 for an angle θ around 55 °, and for an angle θ equal to 35 °, m_y is approximately equal to 0.05.

This work was done to contribute to the study of the mechanical properties of composite structures with vegetable fiber reinforcement.

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