TIME EVOLUTION OF THE COMPLEX MODULUS

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ABSTRACT

This research paper proposes an approach to model the mechanical properties of bituminous pavements towards environmental factors. In order to allow an adaptation to the available data, we have chosen a hybridized connectionist model with a time series approach while taking in account the sensitivity of the bituminous layer to temperature and traffic. The temperature will be represented as a time series in the SARIMA model (1,2,1), with a seasonal factor s = 12. In order to compensate the restricted data volume, the complex modulus will be reproduced from a three-layered RNA by a backpropagation error learning algorithm. Modifications and relations will be introduced on the data and the input parameters so as to fast-forward time calculation and the accuracy of the model.

Keyword : *Temperature, Frequency, complex modulus, artificial neural networks, time series, visco-elastic*

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1. INTRODUCTION

As far as the pavement behavior analysis is concerned, the multilayer model characterized by the homogeneous and elastic bearing of each layer under static loading is by far the most well-known model. For the case of a bituminous pavement, the viscoelastic property of the surface layer is represented by the complex modulus [1] [2]. The main problem in modeling bearing during operation remains the means to collects the data records. Since some experiments or collections require higher amount of financial resources than others, it must be pointed out that some data will be less available especially for developing countries.

In this article, our complex modulus will present a dynamic model compatible Data with different while considering the most influential factors. For the complex module, considering the most influential factors, we will present a dynamic model compatible with data of different dimensions.

2. PAVEMENTS BEARING

2.1. Structures and materials

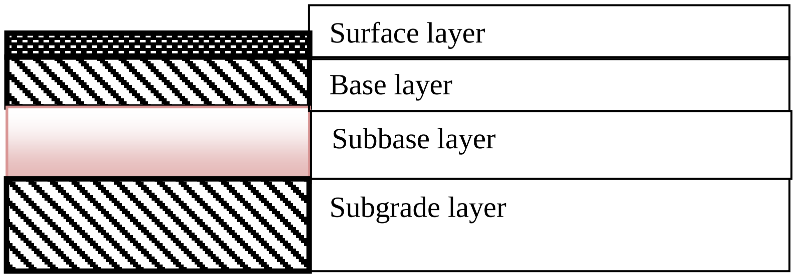

Fig -1: A pavement structure

- Subgrade layer: made of selected materials, it improves the lift of the support soil and facilitates the compaction of the upper layers.
- Subbase layer: in selected materials or in untreated gravels, its main role is to reduce the loads that are transmitted to the support soil.
- Base layer: “as dug” gravel, at this state the efforts due to the traffic are still very important as well as the environmental factors.
- Coating layer: mixture of aggregates and hydrocarbon binders, this layer is exposed to the surface and directly receives the tire loads.

The calculation of forces and deformations is generally considers an isotropic linear elastic multilayer model which requires the determination of Young's modulus values and the Poisson's ratio of each pavement layer. Due to the singular properties of bitumen, bituminous mixes are highly dependent on the loading speed as well as the temperature [3]. With respect to the elastic behavior, the complex modulus will be the module used for the coating layer. And as far as the lower layers are concerned, we can use equivalent modules directly proportional to their CBR [4].

2.2. Effect of the temperature and the traffic

Temperature presents two main mechanical effects:

- The change in the stiffness of the material: A bituminous mix that is heated becomes "soft"
- The creation of constraints and deformations within the material due to dilatation -contractions during temperature changes [5].

As a result to the kinetic susceptibility of the bituminous mixture, which is inherited from the properties of the binder, the speed of the traffic strongly influences its behavior. Although the relation between velocity and frequency is according to the structure depth and the rigidity of the latter, we can use the following relation as a first approximation:

$$f_{Hz} = 0{,}45.\,V_{km/h}$$

2.3. The modulus of elasticity

The linear modulus of elasticity of a material is given by HOOKE Law:

$$E = \frac{\sigma}{\varepsilon}$$

σ,E,ε : are respectively the state of constraints, the modulus of elasticity and the relative deformation

For a visco-elastic material, the bearing is governed by the complex modulus E *:

$$E^* = \frac{\sigma_0 e^{i\omega t}}{\varepsilon_0 e^{i(\omega t-\varphi)}} = |E^*| e^{i\varphi}$$

Often called rigidity module, | E* | is the standard for complex module. φ is the phase shift angle of the material. It intervenes in the quantification of the energy dissipated in the material at each loading cycle. And ω is the pulsation corresponding to the loading.
The complex representation of constraints and deformations are:

$$\sigma^*(t) = \sigma_0 e^{i\omega t}$$
$$\varepsilon^*(t) = \varepsilon_0 e^{i(\omega t - \varphi)}$$

Thus, the modulus of elasticity can be determined experimentally by measuring the corresponding deformations at a given constraints state. Olard and Di Benedetto have presented a model, based on that of Kelvin Voigt, which allows a better characterization of the visco-elastic properties of bituminous materials. [6]

3. EQUATION OF PARAMETERS

3.1. Temperature in time series

Temperature has a direct influence on materials. As measurements of degradations and deformations in a real operating environment can not be measured continuously at regular intervals, temperature variations will be introduced into the models as a random variable following a time series model {y1, y2, y3, ..., yT}, with well-defined modeling parameters [7] [9]

$$y(t) = f(w, x(t)) + e(t)$$

Where y (t) is the variable to predict at time t, f is a linear or non-linear function; w is the parameter vector of the model. We will consider that the parameters are invariant or have relatively small variations over time. x (t) = (y (t-ih), x1 (t-jh), x2 (t-kh), ...) is the vector of inputs or explanatory variables; t = 1, ..., T; T is the number of observations available; h is the sampling interval; i, j, k are the delay indices, i belongs {1, ..., I}, j belongs {1, ..., J}, k belongs {1, ..., K); and I, J, K are the maximum delays for the different inputs. They are determined by correlation study or association rules. e (t) is a random term that can be considered as the prediction error called residue, behaving like a white noise..
After defining the variables and the function f, the identification of the model consists in looking for the criterion function w. This identification takes into account the differences between the observed output and the output of the model.

3.2. Complex module by a connectionist approach

The tests on the complex modulus allowed us to get complex modulus values for different frequency and temperature values [8]. The modeling approach of the complex module will be a connectionist approach with a multilayered network. The latter makes it possible to adapt to the dataset while offering a multivariate modeling.

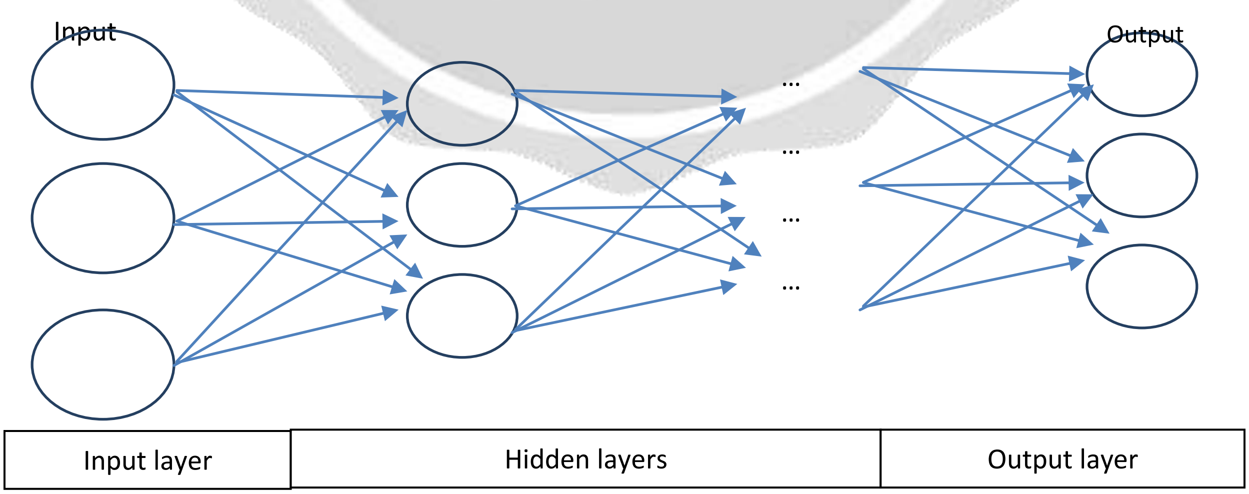

Fig -2: Structure of a multilayer network

The learning phase consists of modifying the synaptic coefficients of the network until the desired behavior is obtained. The learning will follow an algorithm of backpropagation errors.

- Forward propagation :

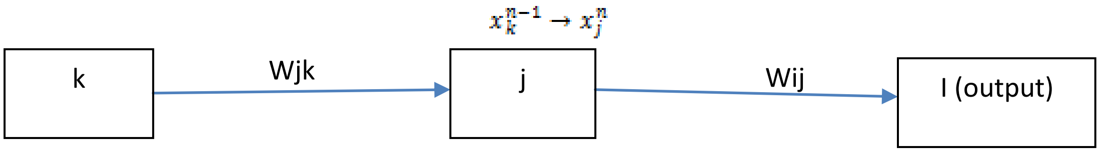

Fig -3: Forward exit propagation

$$x_k^{n-1} \to x_j^n : \sigma(p) = \sigma\left(\sum W_{jk}^n . x_k^{n-1}\right)$$

Given o be the output of the neuron, c the desired output, $e^n{}_i$ the representation of the output error:

$$e^n{}_i = \sigma'\left(\sum W_{ij}^n . x_j^{n-1}\right) . (c_i - o_i)$$

- Backward propagation :

The relation obtained by minimizing the gradient through the outputs of the transfer functions in each neuron is:

$$e^{n-1}{}_j = \sigma'\left(\sum W_{jk}^{n-1} . x_k^{n-2}\right) \sum_{succ\ j} e^n{}_i \, W^n{}_i$$

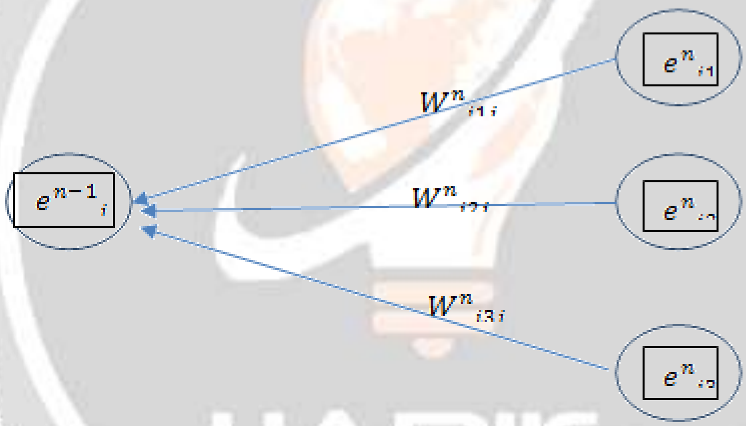

Fig -4: Backwards error propagation

The correction of the synaptic coefficients between two neurons is obtained by the relation

$$W_{ij}{}^n = W_{ij}{}^n + \varepsilon . e_i^n . x_j^{n-1}$$

To optimize the network and reduce the computation time, we introduce a momentum term α ∈] 0; 1 [which keeps in memory the corrections:

$$W_{ij}{}^n(t) = W_{ij}{}^n(t-1) + \alpha . \varepsilon . e_i^n . x_j^{n-1} + (1-\alpha)\left(W_{ij}{}^n(t-1) - W_{ij}{}^n(t-2)\right)$$

4. EVOLUTION OF THE COMPLEX MODULUS IN REAL ENVIRONMENT

4.1. Temperature forecast

During the passage of a rolling load, the speed (V) and the temperature are assumed to be constant. Since it is impossible for a developing country to establish a database with temperatures available at any time, we have decided to consider only a sample of data on the average monthly temperature:

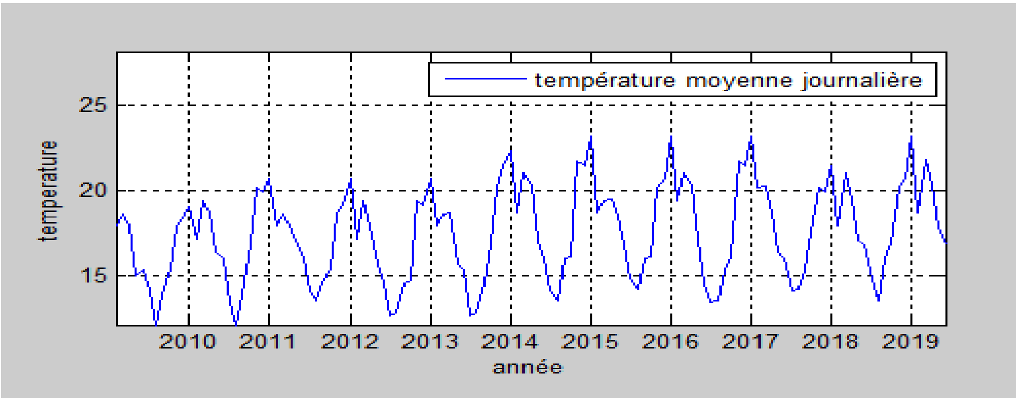

Fig -5: Average daily temperature [°C] of the city of Antananarivo for a month

Intuitively, we are able to say that the average daily temperature is a seasonal process. And according to Fig-5, the series is not stationary. The ideal form to represent this series is an ARIMA model (p, d, q) containing a seasonal factor s = 12. Stationarization of the process is carried out with the Dickey and Fuller tests [9] with an acceptance threshold of 5%. And in order to have the minimal optimal model representative of the series, we will apply Coin method [9], with the matrix of autocorrelation. For estimator calculation, the maximum likelihood will be used.

After calculation, we find the estimators of model ARIMA (1,2,1): φ_1 = 0,02; θ_1 = -0,98. And following the integration steps, we find the results of **Fig-6**

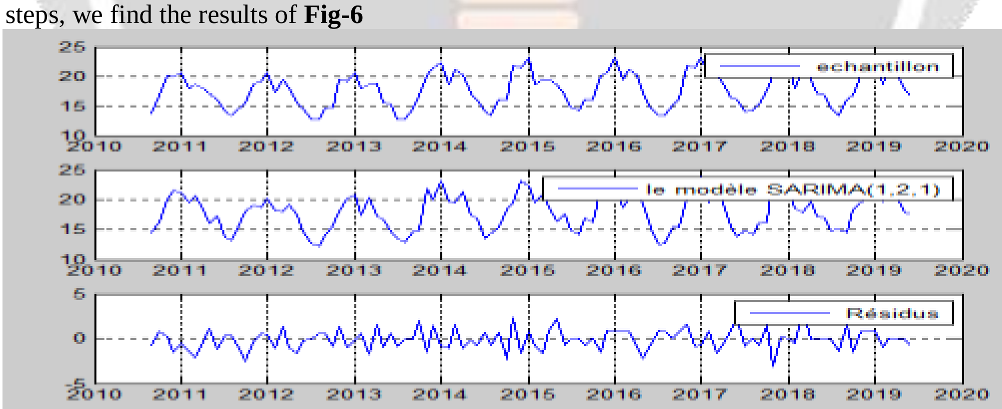

Fig -6: Reproduction of the observed data model

For the calculation, it was necessary to assume that the law of errors configured as such: we have proposed a normal distribution law $\aleph(0, \sigma^2)$,, with zero initial states.

At first sight, we can see the seasonality of the model with a representative variation. Furthermore, the analysis of the residues tends to validate the model which mathematical expectations fluctuates around zero from a certain horizon and their distribution approach a normal law.

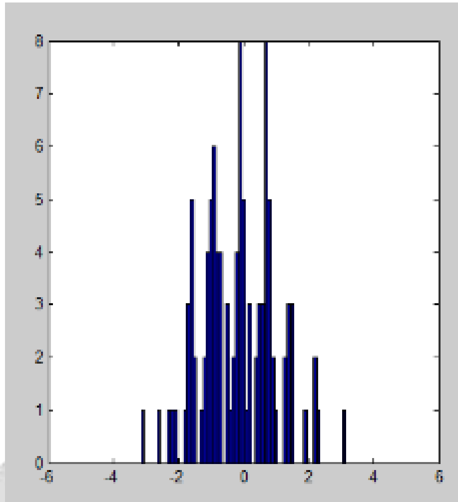

Fig -7: distribution of residues

However, this figure permits us to detect the inaccuracy of the model. These inaccuracies as well as the imperfect behavior of the residues are due to the lack of data, to the initial conditions on the residues $(\varepsilon_0) = (0)$ and $(\varepsilon_0) \sim \aleph(0, \sigma^2)$.

For a forecast on the horizon h of an ARIMA process, we use the estimators of the model such as:

$$\begin{cases} \Phi(L)(1 - L^{12})(1 - L)^d X_t = \Theta(L)\varepsilon_t \\ \Phi(L) = I - \phi_1 L - \phi_2 L^2 \ldots - \phi_p L^p \\ \Theta(L) = -\theta_1 L - \theta_2 L^2 \ldots - \theta_q L^q \\ \varepsilon_{t+h-j} = 0 \text{ pour } j < h \end{cases}$$

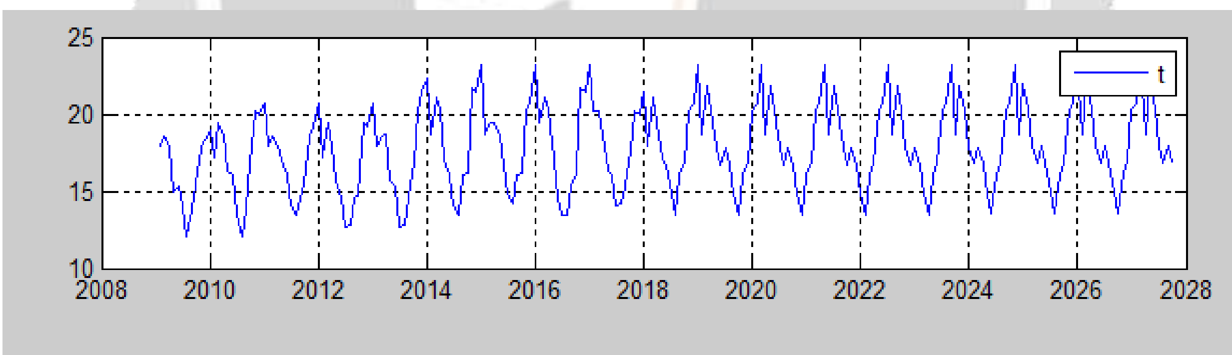

Fig -8: Prediction of the average daily temperature over a horizon of T = 100 months

4.2. Application of the complex modulus

The data used here come from laboratory tests:

Table 1 : Values complex modules depending on the temperature and frequency [8]

	frequency[Hz]				
temperature[°C]	**8**	**15**	**25**	**33**	**50**
-10	21837	22538	23145	23473	24603
-5	19582	20630	21327	21663	23266
0	17488	18316	19244	19790	21441
5	14728	15809	16750	17312	19459
15	12035	13212	14172	14734	16375
20	8800	10042	10809	11584	12607
25	6629	7693	8604	9077	10247
30	4438	4516	6184	6634	7506

Network characteristics:

- 3 inputs:
  - X1 = E_ref [MPa] = 8604: the reference modulus for a temperature of 25 ° C at a travel frequency of 25Hz
  - X2 = t [° C]: the ambient temperature
  - X3 = f [Hz]: the frequency
- 2 hidden layers with 6 neurons: The number of hidden layers and the number of neurons per layer have an influence on the quality of learning.
- An output: Y = E * [MPa], the complex modulus of materials

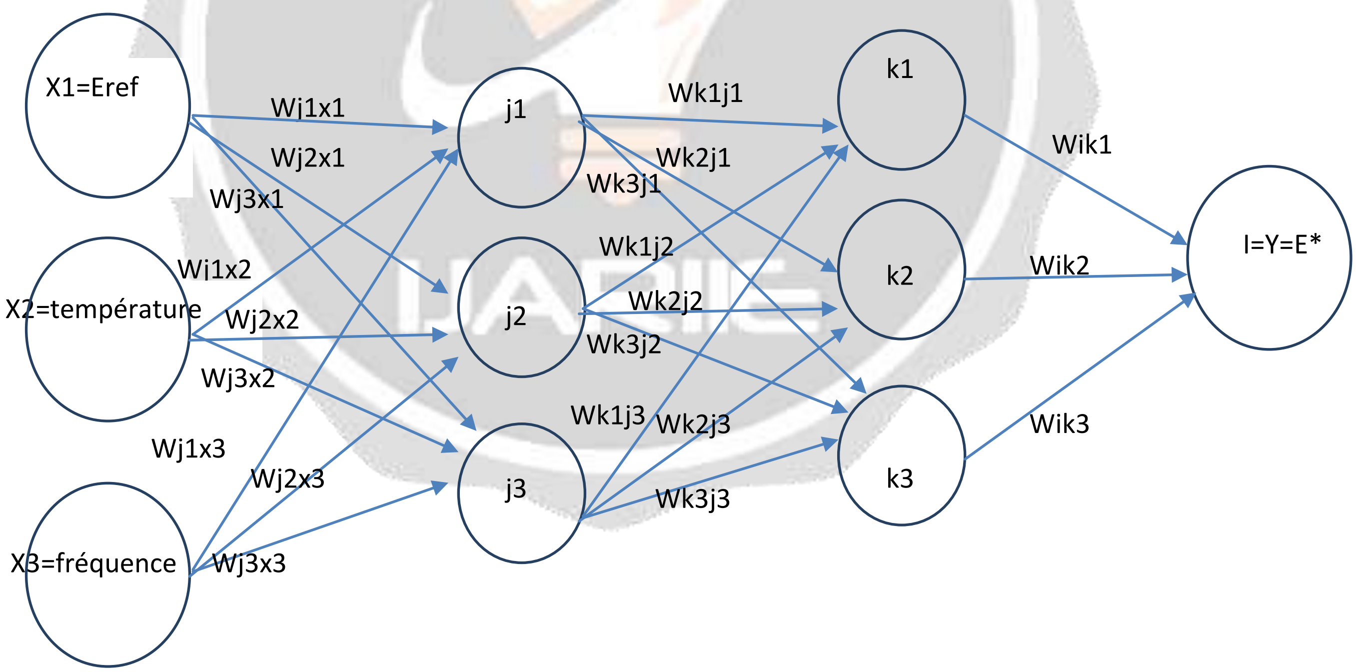

Fig -9: Complex modulus network model

The relations between the input parameters of the complex modulus are non-linear relationships. To optimize the calculation time, we will convert the temperature in ° K (0 ° K = -273.15 ° C) to have a positive value in all cases. And to lift the non-linear forms, we will use the property of the logarithmic function.

$$Log(E*) = \begin{bmatrix} Log(E_{ref}) \\ Log(t * E_{ref}) \\ Log(f * E_{ref}) \end{bmatrix} x\ [F]$$

With a 5% threshold, we find the transfer matrix [F]:

$$[F] = \begin{bmatrix} 0.2444 & 0.2682 & 0.1249 \\ 0.2585 & 0.4898 & 0.4335 \\ 0.4552 & 0.4849 & 0.4322 \end{bmatrix} x \begin{bmatrix} 0.2232 & 0.2424 & 0.1514 \\ 0.2285 & 0.4524 & 0.4622 \\ 0.4307 & 0.4566 & 0.4672 \end{bmatrix} x [0.2320 \quad 0.1978 \quad 0.2093]$$

For a time series forecast of the complex modulus which is both compatible with the traffic data, we will consider the average daily temperatures.

In the city, we will take an average speed of 15.2km / h versus an average speed of 60km / h on the road which is typical public transport.

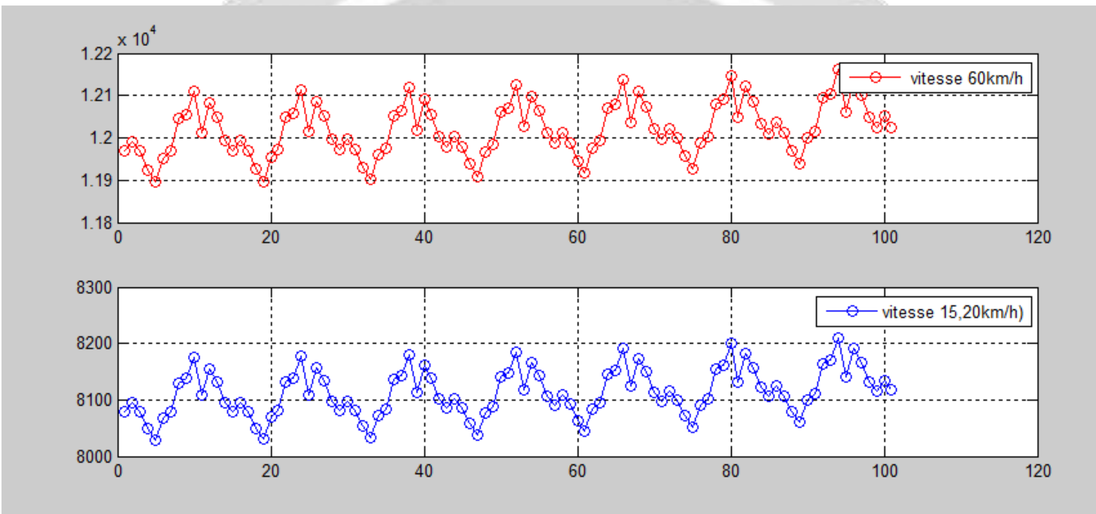

Fig -10: Forecast of the average daily values of the complex modulus from May 2019 on a 100-month horizon of the city of Antananarivo

After making the simulation, we found that the model presents a slight default to determine the complex modulus for a low temperature. But for tropical regions, this problem can be set aside.

5. CONCLUSION

The results of these investigations show us the significant evolution of the complex modulus depending on the temperature and the frequency of the cycle. The higher the temperature, the more the module decreases. There is also a reflective decrease with the speed of circulation. Residue analysis, relative errors and evaluation criteria led to a validation of the model. However, even after validation, it is clear that a larger volume of data will improve these outcomes.

These results might allow us to calculate the deformations and the states of constraints per cycle of passage for the case of pavement structures in a future work. As well as this rheological behavior per loading cycle might serve as a basis for an estimation of the states of degradation of pavements during its exploitation.

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