Terminating FDTD Grid with Electrical and Magnetically loss layers

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ABSTRACT

The simulation of an infinite space in FDTD formulation can be done by the formulation of an ABC (Absorbing Boundary Condition). In this paper, the implementation of a loss-layer for the grid termination is presented. The simulations are made for a propagation in TM mode in a two-dimensional grid. The field update equations are therefore seen first with the results of a simulation without ABC. After that the behavior of a dielectric loss medium is studied. It is the absorbent property of a loss dielectric medium that will be exploited for the implementation of the FDTD grid termination. To ensure that these grid terminations do not reflect incident waves, the progressive loss factor implementation study complements the FDTD grid termination implementation.

Keyword: *2D FDTD, ABC, Loss layer, Grid termination, progressive loss*

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1. INTRODUCTION

In an FDTD formulation, the spatial grids simulation have a finite dimension. If no ABC (Absorbing Boundary Condition) is specified, electromagnetic waves incident at the edges of the grid are reflected back to the simulation space. These reflected waves can interfere with the simulation as after a certain simulation time it will no longer be possible to distinguish the wave to be studied from these reflected waves. Then, ABC's are needed to prevent electric and magnetic fields from being reflected in the problem space (FDTD grid) [1].

In order to make an FDTD grid behave like an infinite space, thus preventing waves from being reflected at its limits, grid termination with loss layers will be presented in this paper.

2. WAVES PROPAGATION IN A DIELECTRIC WITH LOSS

2.1. Field update equations for 2D FDTD TM formulation

For the 2D case, the variations in the z direction are here assumed to be zero. In this case, there appear two groups of equations governing EM wave propagation. These equations define two modes of propagation, the TM mode (Transverse Magnetic) and the TE mode (Transverse Electric). The field update equations for the TM mode, considered in this work, are given in Eq.1. [2][3]

$$E_z^{n+1}(i,j) = \frac{1-\frac{\sigma_z\Delta t}{2\varepsilon_z}}{1+\frac{\sigma_z\Delta t}{2\varepsilon_z}} E_z^n(i,j)$$

$$+\frac{\Delta t}{\left(1+\frac{\sigma_z\Delta t}{2\varepsilon_z}\right)\Delta x}\left\{H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right)-H_y^{n+\frac{1}{2}}\left(i-\frac{1}{2},j\right)\right\}$$

$$+\frac{-\Delta t}{\left(1+\frac{\sigma_z\Delta t}{2\varepsilon_z}\right)\varepsilon_z\Delta y}\left\{H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right)-H_x^{n+\frac{1}{2}}\left(i,j-\frac{1}{2}\right)\right\} \quad (1.a)$$

$$H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) = \frac{1-\frac{\sigma_{mx}\Delta t}{2\mu_x}}{1+\frac{\sigma_{mx}\Delta t}{2\mu_x}} H_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2}\right)$$

$$+\frac{-\Delta t}{\left(1+\frac{\sigma_{mx}\Delta t}{2\mu_x}\right)\mu_x\Delta y}\left(E_z^n(i,j+1)-E_z^n(i,j)\right) \quad (1.b)$$

$$H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) = \frac{1-\frac{\sigma_{my}\Delta t}{2\mu_y}}{1+\frac{\sigma_{my}\Delta t}{2\mu_y}} H_y^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right)$$

$$+\frac{\Delta t}{\left(1+\frac{\sigma_{my}\Delta t}{2\mu_y}\right)\mu_y\Delta x}\left(E_z^n(i+1,j)-E_z^n(i,j)\right) \quad (1.c)$$

2.2. Waves propagation in vacuum

Fig. 1 gives snapshots of the propagation of a sine wave of frequency 500 MHz, introduced at the node (20,30) of a grid of dimension 80 × 60. The spatial step is defined by $\Delta x = \Delta y = 0.06$ m, and the temporal step by $\Delta t = 14.14$ ns. In the figure, the wave propagation up to time step 60 is quite visible. From a certain time it is difficult to distinguish the wave from the source of the waves reflected at the limits of the grid (n = 85).

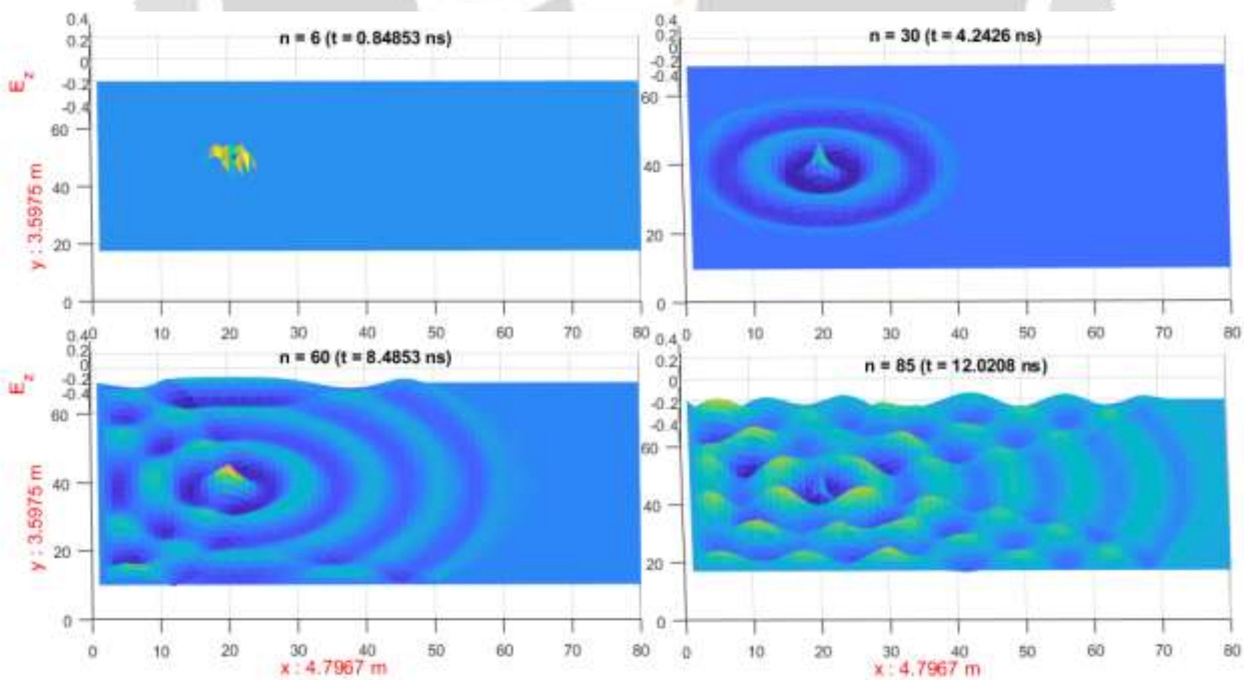

Fig.1: Snapshots of the propagation of a sine wave traveling through a vacuum in TM mode.

2.3. Wave propagation in a loss dielectric

Fig. 2 illustrates the propagation of a wave in vacuum then striking a medium formed by a rectangle of dielectric starting at the node (35,1) and ending at the node (60,60). The permittivity of the dielectric is defined by $\varepsilon_z([35,60],[1,60]) = 4$, and the permeability by $\sigma_z([35,60],[1,60]) = 1.41 \times 10^{-12}$.

Part of the wave is reflected at the Vacuum / Dielectric interface, and the wave passing through the dielectric is attenuated. The reduction in speed within the dielectric is also noted. The reflection and attenuation of the wave are due to the non-zero conductivity of the medium. As $\varepsilon_r = 4$, the wave speed in the middle is divided by 2.

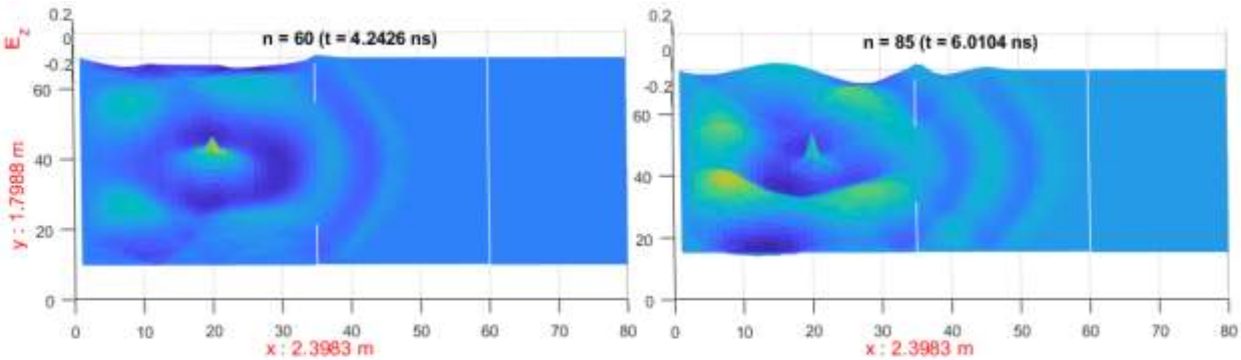

Fig.2: Snapshots of the propagation of a sine wave traveling through a dielectric in TM mode.

3. MATCHING LAYERS WITHOUT LOSSES WITH LAYERS WITH LOSSES

3.1. Principles of the method

In the event of a loss, the characteristic impedance of a loss medium is given at Eq.2.

$$\eta = \sqrt{\frac{\mu\left(1-j\frac{\sigma_m}{\omega\mu_0}\right)}{\varepsilon\left(1-j\frac{\sigma}{\omega\varepsilon_0}\right)}} = \eta_0\sqrt{\frac{\mu_r\left(1-j\frac{\sigma_m}{\omega\mu_0}\right)}{\varepsilon_r\left(1-j\frac{\sigma}{\omega\varepsilon_0}\right)}} \quad (2)$$

When $\sigma_m/\mu_0 = \sigma/\varepsilon_0$, the terms in parentheses are equal and therefore cancel each other out. With these terms canceled, the characteristic impedance is indistinguishable from the case without loss (Eq.3). This is Berenger's principle of PML (Perfectly Matched Layer).[4]

$$\eta|_{\frac{\sigma_m}{\mu_0}=\frac{\sigma}{\varepsilon_0}} = \eta|_{\sigma_m=\sigma=0} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} \quad (3)$$

As shown in Eq.4, the reflection coefficient of a wave normally incident on a flat boundary is proportional to the difference in impedance of one side and the other of the interface. If the material in one side is lossless while that in the other side is loss with $\sigma_m/\mu_0 = \sigma/\varepsilon_0$, then the impedances are matched. With the matched impedances, there will be no reflection from the interface. Therefore, a lossy layer can be used to complete the grid. The fields will dissipate in the lossy region, and if the region is large enough, they could become small by the time they meet the end of the grid. Upon reflection at the end of the grid, the fields should propagate through the loss layer where they would decay even further.

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = 0 \quad (4)$$

The definition of loss layers ending the grid is done by redefining the multiplication coefficients of the fields at Eq.5 for the electric field and Eq.6 for the magnetic field.

$$c_{ezez} = \frac{1-\frac{\sigma_z \Delta t}{2\varepsilon_z}}{1+\frac{\sigma_z \Delta t}{2\varepsilon_z}} = \frac{1-pez}{1+pez}\ c_{ezhy} = \frac{\Delta t}{\left(1+\frac{\sigma_z \Delta t}{2\varepsilon_z}\right)\Delta x} = \frac{1}{(1+pez)}\frac{\Delta t}{\varepsilon_z \Delta x}\ ;\ c_{ezhx} = \frac{-\Delta t}{\left(1+\frac{\sigma_z \Delta t}{2\varepsilon_z}\right)\varepsilon_z \Delta y} = \frac{-1}{(1+pez)}\frac{\Delta t}{\varepsilon_z \Delta y} \quad (5.a)$$

$$c_{hxhx} = \frac{1-\frac{\sigma_{mx} \Delta t}{2\mu_x}}{1+\frac{\sigma_{mx} \Delta t}{2\mu_x}} = \frac{1-pmx}{1+pmx}\ ;\ c_{hxez} = \frac{-\Delta t}{\left(1+\frac{\sigma_{mx} \Delta t}{2\mu_x}\right)\mu_x \Delta y} = \frac{-1}{(1+pmx)}\frac{\Delta t}{\mu_x \Delta y} \quad (6.a)$$

$$c_{hyhy} = \frac{1-\frac{\sigma_{my} \Delta t}{2\mu_y}}{1+\frac{\sigma_{my} \Delta t}{2\mu_y}} = \frac{1-pmy}{1+pmy}\ ;\ c_{hyez}\left(i+\frac{1}{2},j\right) = \frac{\Delta t}{\left(1+\frac{\sigma_{my} \Delta t}{2\mu_y}\right)\mu_y \Delta x} = \frac{1}{(1+pmy)}\frac{\Delta t}{\mu_y \Delta x} \quad (6.b)$$

For the implementation of the loss layers, instead of varying the values of the conductivities σ, loss factors are defined. These loss factors are defined in Eq. 7, and for the pairing of loss layers they must have the same values.

$$pez = pe2d(i,j) = \frac{\sigma_z \Delta t}{2\varepsilon_z} \quad (7.a)$$

$$pmx = pm2d\left(i,j+\frac{1}{2}\right) = \frac{\sigma_{mx} \Delta t}{2\mu_x} \quad (7.b)$$

$$pmy = pm2d\left(i+\frac{1}{2},j\right) = \frac{\sigma_{my} \Delta t}{2\mu_y} \quad (7.c)$$

3.2. Implementation and results

Fig. 3 shows the implementation snapshots of 15 cell thickness loss layers to terminate the grid. The values of the loss factors are defined by $pe = pm = 0.05$.

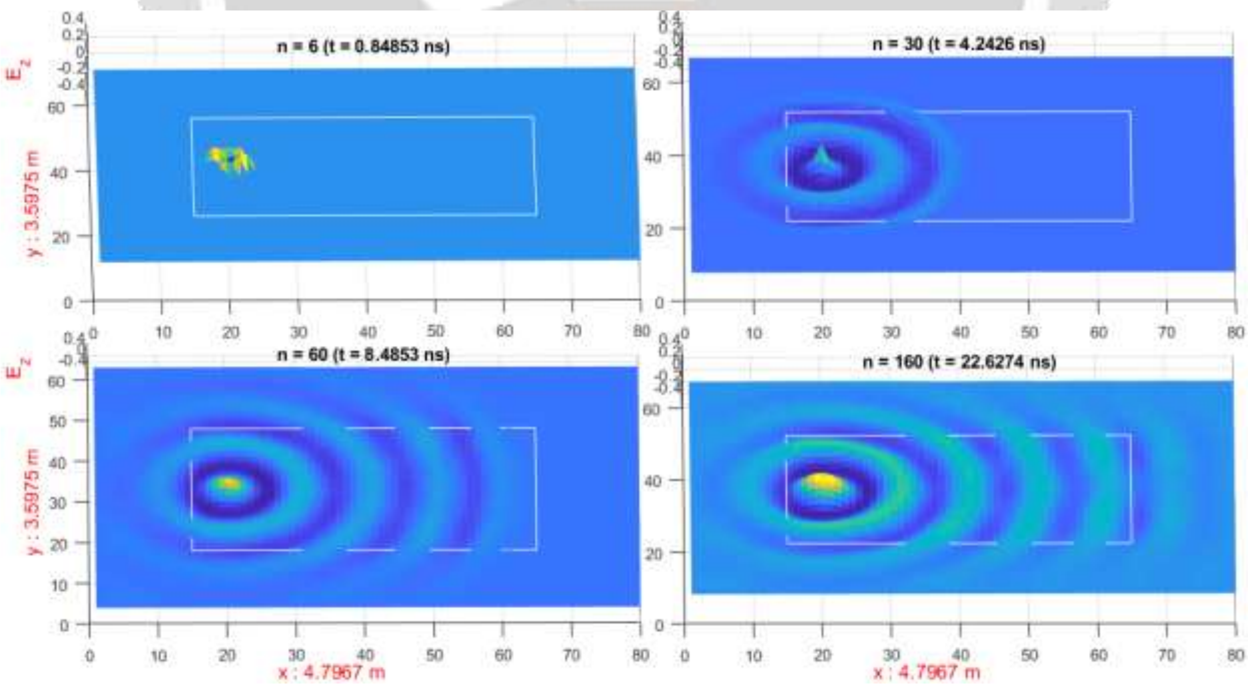

Fig.3: 2D FDTD grid terminated with loss factor 0.05 and 15 cells thickness of loss layer

The lossless and loss media impedances are matched, the fields enter the loss region without reflection. In fact, this is true in the continuous world, but approximately true in the discretized FDTD world where reflection is present. As the fields spread in the loss region, they dissipate to the point where they are almost negligible when they re-enter the lossless region.

4. REDUCED THINKING AT LOSS INTERFACES

4.1. Principles of reflection reduction

In order to implement FDTD grid terminations with absorbent layers, it appears that it is necessary to take into account the fact that if pe is high (the conductivity σ is high) then the layer is a current conductor and therefore reflects the electric field. The reflection of the field, in this case, would be as important as the value of pe was large. However, greater is the pe value, then greater is the dissipation of the wave under the loss layer.

An optimal absorbent layer would therefore be a layer where absorption occurs gradually, that is to say with the value of pe increasing as the layer is crossed. This gradual increase can greatly reduce reflections at the interfaces of the absorbent layer. D.M. Sullivan introduced the auxiliary parameter xn (Eq.8) for the PML calculation, a parameter similar to pez. [5] The calculation of the loss factors pe will be done in the same way and described in Eq.9. The values of pe, in a layer of fifteen cells at the right edge of the grid are seen in Tab. 1.

$$xn(i) = \frac{\sigma(i)\Delta t}{2\varepsilon_0} \tag{8}$$

$$pe(i) = 0.333\left(\frac{i}{taille_{perte}}\right)^3; \tag{9}$$

Tab.1: Values of pe at the right edge of the grid for a loss layer consisting of 15 cells

$pe(76)$	**$pe(77)$**	**$pe(78)$**	**$pe(79)$**	**$pe(70)$**
9.86×10^{-5}	7.89×10^{-4}	0.0027	0.0063	0.0123
$pe(71)$	**$pe(72)$**	**$pe(73)$**	**$pe(74)$**	**$pe(75)$**
0.0213	0.338	0.0505	0.0719	0.0987
$pe(76)$	**$pe(77)$**	**$pe(78)$**	**$pe(79)$**	**$pe(80)$**
0.1313	0.1705	0.2168	0.2707	0.3330

4.2. Implementation and results

For a 2D simulation in TM mode, the absorbent layers of thickness $taille_{perte}$ are introduced at the limits of the grid of dimension $taille_x \times taille_y$using Eq.10. The implementation is illustrated in Fig. 4, where the increasing values of the loss factors are seen at the limits of the grid (of dimension 60×80). The source is always a sine wave of frequency 500 MHz, introduced at the node (20,30) of a grid.

$$i \in [1, taille_{perte}] \quad (10.a)$$

$$pe2d([i: taille_x - i + 1], i) = pe(taille_{perte} - i + 1) \quad (10.b)$$

$$pe2d([i: taille_x - i + 1], taille_y - i + 1) = pe(taille_{perte} - i + 1) \quad (10.c)$$

$$pe2d(i, [i: taille_y - i + 1]) = pe(taille_{perte} - i + 1) \quad (10.d)$$

$$pe2d(taille_x - i + 1, [i: taille_y - i + 1]) = pe(taille_{perte} - i + 1) \quad (10.e)$$

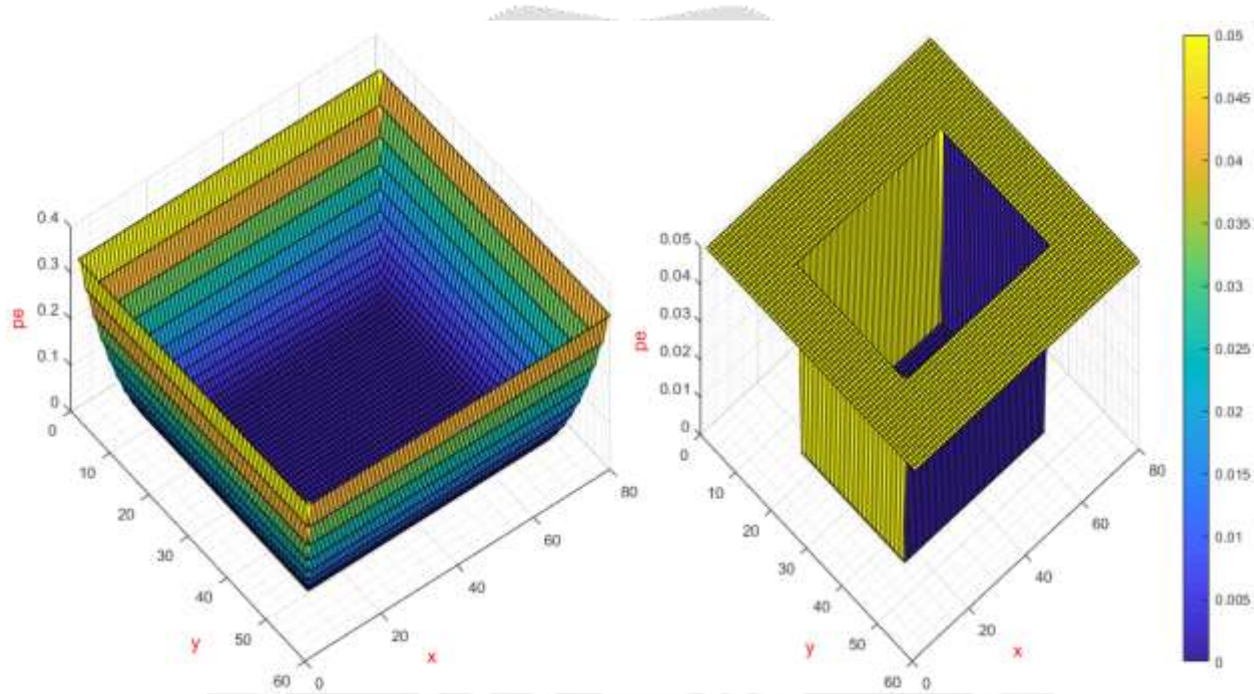

Fig.4: Illustration of the terminating grid layer implementation with progressive loss factors

Fig. 4 shows the gradual increase of the value of the loss factors in the layers. The figure on the left shows the layers with progressive losses and the one on the right shows the loss layers with a fixed loss factor. The structure of the figure on the left should reduce reflections at the interfaces of the loss layers.

Fig. 5 gives the snapshots of the E_z field on the (xy) plane. In progressive factor loss layers, the wave is gradually attenuated as it penetrates the depths of the layers. With the implementation of progressive loss factors in the FDTD grid, no reflections produced by the interfaces of the loss layers are apparent.

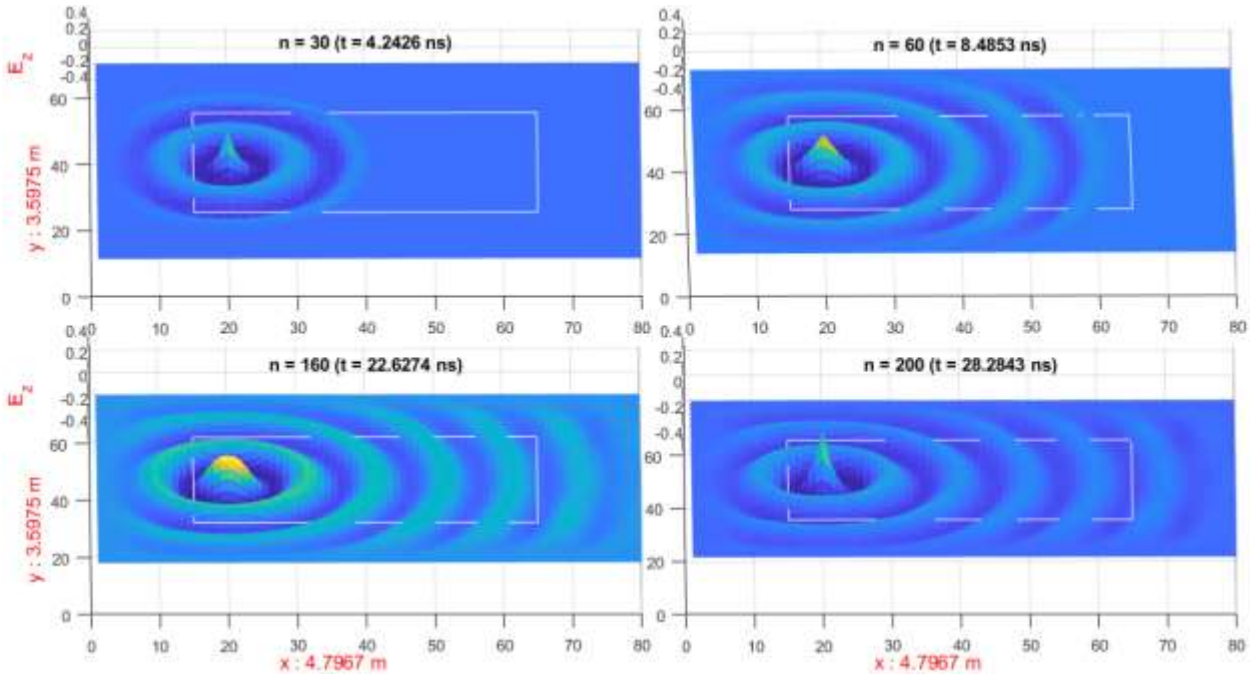

Fig.5: 2D FDTD grid finished with progressive factor loss layer with 15 cells thickness

5. CONCLUSION

ABC's application allows an FDTD grid, which is a space of finite dimension, to behave like a space of infinite dimension. The ABC formed by loss layers is suitable for simulations of infinite media. The constraint of the loss-layer grid termination implementation is to be able to reduce, or even eliminate, reflections at its interfaces. In order to manage with these constraints, the implementation of the loss layers is done with the implementation of progressive loss factors. The results obtained show the effectiveness of the method. Therefore, grid termination by progressive factor loss layers is a method that can be used in FDTD simulations as an ABC.

6. REFERENCES

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