Tire Modelling and Validation of Tire Diameter and Width

Alok Patnaik[1], Aditya Sahu[2]

[1]B. Tech, Mechanical Engineering Veer Surendra Sai University of Technology, Odisha, India
[2]B. Tech, Mechanical Engineering Veer Surendra Sai University of Technology, Odisha, India

ABSTRACT

A tire is a ring-shaped component that surrounds a wheel's rim to transfer a vehicle's load from the axle through the wheel to the ground and to provide traction on the surface over which the wheel travels. Moreover, they are responsible for absorbing shocks. Hence provide better ride quality and handling properties while driving. This paper provides a validation method for selecting tires for the front and rear of ATVs (all-terrain vehicles). The validation primarily focuses on achieving tripoding and slip characteristics for improving the dynamic performance of the vehicle. Data Analysis and interpretation of tire data are done in MATLAB SIMULINK and validation is done using an accelerometer and ultrasonic sensors.

Keywords: *- Tires, Slip Characteristics, Tripoding, ATV, Ride Rate, Load Transfer, MATLAB*

Introduction

When it comes to ATVs, oftentimes the first modification people make is larger tires. But why is this change made?

Aesthetically, “Bigger is Better” when it comes to ATV tires, but how big is too big? At what point do larger tires or wheels begin to negatively affect your vehicle's handling, potentially putting you into a mechanically unstable or even dangerous situation? For this, selection and proper validation of the tire diameter are necessary. Bigger tires can assuredly help you gain ground clearance and traction in sand, deep mud, snow or uneven terrain. Increasing the size of our tires will undoubtedly give our vehicle a much more aggressive look while increasing ground clearance and traction. However, there are some negative effects, which will most likely present themselves if not selected and validated with proper methods.

Some of the potential issues are as follows:

- Tire Rub on Body Panels / Suspension Components
- Increased Drive Belt Wear
- Premature Axle Failure
- Premature Wheel Bearing Failure
- Increased Roll-Over Rate

Therefore, we chose “Validation of Tire Diameter and Width”.

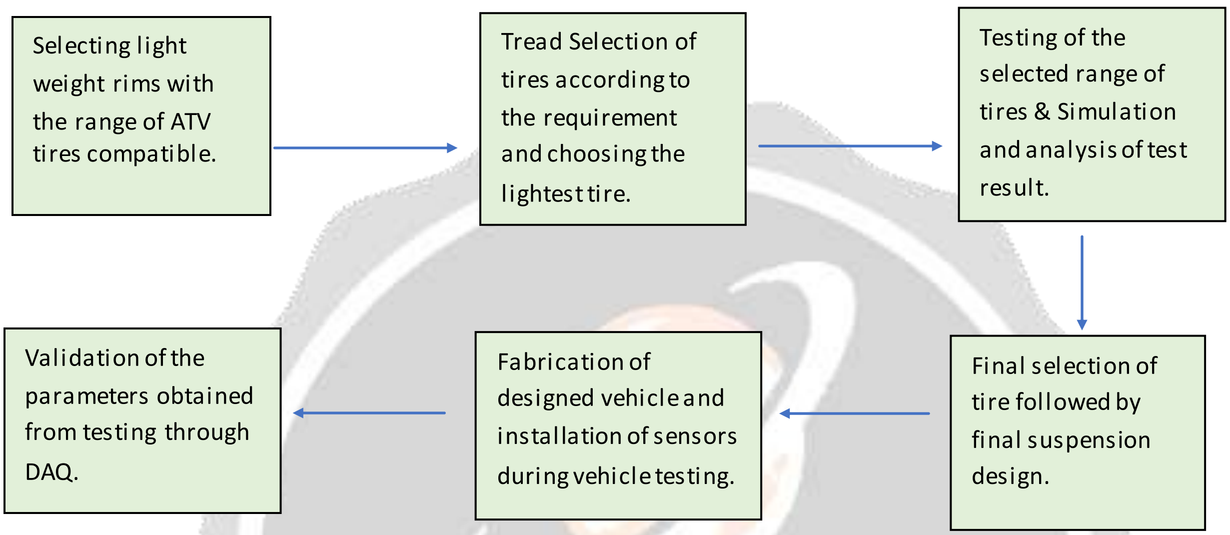
Methodology

Assumption and boundary conditions

- Front and rear track widths are chosen to be 52” and 49” respectively as larger front track width
- decreases the weight transfer, rear wheels closer help in attaining traction and weight transfer is

- inversely proportional to track width.
- Assuming 40:60 weight distribution in front and rear the wheelbase is chosen to be 58”.
- The rim of diameter 10” is selected considering the size of the wheel assembly.
- CG Height is taken to be 530 mm.
- Ride heights of 16.18” and 13.05” are chosen for front and rear respectively.
- Vertical stiffness of 400 N/mm is kept for better handling.

Flowchart of Methodology



Calculation:

Centre of Gravity Calculation

We validated the Centre of Gravity to see whether the estimated position is achieved for the chosen tire. The centre of gravity height is calculated by weighing the car when level and then raising the car 12 inches above the ground at the rear and weighing the front and vice-versa.
Before beginning the test following precautions are taken into consideration

- All fluids are filled.
- Each shock absorber is replaced with a solid link to eliminate suspension travel.
- Tires are inflated to the maximum pressure as specified by the manufacturer to eliminate any sidewall flex.

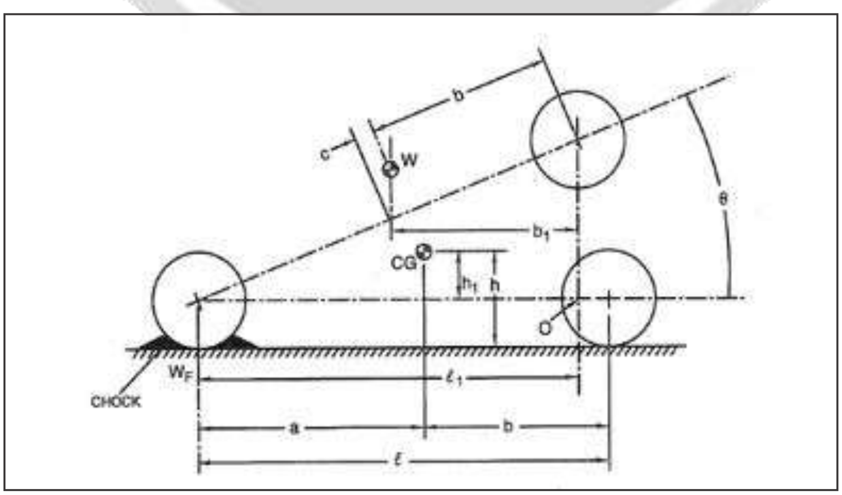


Fig 1- The vertical location of the centre of gravity

W = total weight of the vehicle
W_F = the weight of front wheels with rear elevated
b = horizontal direction from the rear axle to CG
l = wheelbase
R_{LF} = loaded radius of front wheels (axle height above ground)
R_{LR} = loaded radius of rear wheels (axle height above ground)
R_{Lcg} = loaded radius of center of gravity
tanϴ, cosϴ = tangent and cosine of the angle to which the vehicle is raised
Total weight of vehicle = 216kg = 476.28lb (pounds)
W_F = 95 kg = 209.475lb
tanϴ = (33/147); ϴ = 0.2208 rad
b = 589.28mm = 1.933ft
a = 883.92mm = 2.9ft
l = 1473.2mm = 4.83ft
l_1 = l*cos ϴ = 4.83*cos (0.2208) = 4.7127ft
These above values are obtained from the test

Taking moments about O [ref Fig 1]

$W_F l_1 = W b_1$
$b_1 = W_F l_1/W$ = (209.475*4.7127)/476.28=2.07ft
Also, b_1/b+c = cos ϴ, c = ((b1/cos ϴ)-b) = ((2.07271/0.97490)-1.933) =0.193074ft
And c = [(W_Fl / W)-b]
= ((209.475*4.83/476.28)-1.933) =0.1913055ft
Values of c from both the expression are nearly equal hence the observations are correct.
h_1= (W_fl- Wb) / W*tan ϴ
= [(209.475*4.83) - (476.28*1.933)]/476.28*0.22833=0.837846ft
R_{LF} = 254mm = 0.8334608ft
R_{LR} = 292.1mm = 0.9583479ft
R_{Lcg} = R_{LF} (b/l) + R_{LR}(a/l) = 0.83346(1.933/4.83) + 0.95834(2.9/4.83) = 0.9089621304ft
H_{cg} (CG Height) = R_{Lcg}+h_1 = 0.837846 + 0.9098962 = 1.746808ft = 532.7mm

Load transfer due to the Roll Moment

Ride frequency =1.06Hz(front), 1.36(rear)
Ride rate: $(4 \times \Pi^2 \times F^2 \times M_{sp})/1000$ N/mm
=1.595 N/mm(front), 3.28 N/mm(rear left),
3.93N/mm(rear right)
M_{sp} = Sprung mass

$K_{\Phi f}$ = front roll rate (Nm/deg roll)
t_f = front track width(m) =52" =1.3208m
$K_{LF} = \text{LF wheel rate} \left(\frac{N}{m}\right) =$ 1.595 N/mm= 1595 N/m
$K_{RF} = \text{RF wheel rate} \left(\frac{N}{m}\right) =$ 1.595 N/mm= 1595 N/m

$$K_{\Phi f} = \pi\left(\frac{(t_f)^2 K_{LF} K_{RF}}{180\,(K_{LF}+K_{RF})}\right)$$

$$= \frac{\pi \times 1.3208^2 \times 1595 \times 1595}{180 \times (1595+1595)}$$

=**24.27 Nm/deg roll**

$K_{\Phi r}$ = *rear roll rate* (Nm/degree roll)
t_r = *rear track width*(m)= 49" =**1.2446** m

K_{LR} = LR wheel rate $\left(\frac{N}{m}\right)$ = 3.28253 N/mm= 3282.53 N/m

K_{RR} = RR wheel rate $\left(\frac{N}{m}\right)$ = 3.93904 N/mm= 3939.04 N/m

$$K_{\phi R} = \pi\left(\frac{(t_r)^2 K_{LR} K_{RR}}{180(K_{LR}+K_{RR})}\right)$$

$$= \pi \times \frac{1.29^2 \times 3282.53 \times 3939.04}{180 \times (3282.53+3939.04S)}$$

$$= \mathbf{48.32} \text{ Nm/deg roll}$$

H= CG to roll axis distance(m)= 0.33528 m

W=Vehicle weight (N)= 216 * 9.8 N

$$\frac{\phi_r}{A_y} = \textbf{Roll Gradient from ride spring} \left(\frac{Degree}{g}\right)$$

$$\frac{\phi_r}{A_y} = \frac{-W.H}{K_{\phi F} + K_{\phi R}}$$

$$= \frac{-216 \times 0.33528}{24.27+48.32}$$

$$= \mathbf{-0.5976 \text{ deg/g}}$$

From '**Optimum G'** the value for Roll gradient for a low **downforce** car lies between **0.2 to 0.7 deg/g** which satisfies the design to the permissible range .

- Load transfer due to roll moment at the front (F_{fsM})

$$F_{fsM} = \frac{k_{sf\varphi}}{T_f} = \frac{k_{sf} \times m_s \times a \times d}{T_f(k_{sf}+k_{sr}-m_s \times g \times d)}$$

F_{fsM} = load transfer due to roll moment at front axle

m_s= total vehicle mass (kg)

a = lateral acceleration (m/sec^2)

d = perpendicular distance of CG from roll axis(m)

$$F_{fsM} = \frac{24.27 \times 216 \times 0.63g \times 0.335}{1.3208 \times (24.27 + 48.38 - 216 \times g \times 0335)} = -\mathbf{18.2904N}$$

- Load transfer due to roll moment at rear (F_{rsM})=

$$F_{rsM} = \frac{k_{sr\varphi}}{T_r} = \frac{k_{sr} \times m_s \times a \times d}{T_r(k_{sf}+k_{sr}-m_s \times g \times d)}$$

F_{rsM} = *load transfer due to roll moment at rear axle*

$$F_{rsM} = \frac{48.38 \times 216 \times 0.63g \times 0.335}{1.2446 \times (24.27+24.27-216 \times g \times 0.335)}$$

$$= -\mathbf{36.04865N}$$

- Load transfer due to sprung mass inertia force

$$F_{fsF} = \frac{m_{sf} \times a \times h_f}{T_f} \quad \&$$

$$F_{rsF} = \frac{m_{sr} \times a \times h_r}{T_r}$$

h_f & h_r = front & rear roll center heights

m_{sf} & m_{sr} = sprung mass distributed to the roll centres at front & rear axles

$$m_{sf} = \frac{m_s \times b_s}{L} \quad \& \quad m_{sr} = \frac{m_s \times a_s}{L}$$

L = Wheel base

a_s& b_s= perpendicular distances of front & rear axles from CG

$m_{sf} = \frac{216*0.59}{1.4732}$ $m_{sr} = \frac{216*0.824}{1.4732}$

=71.2869 kg **= 99.56kg**

$F_{fsF} = \frac{71.2869*0.63*9.81*0.32}{1.3208}$

=**107.4 N at front**

$F_{rsF} = \frac{(99.56*0.63*9.81*0.129)}{1.2446}$

=**63.95 N at rear**

- Load transfer due to un-sprung mass inertia forces

$$F_{fuF} = \frac{m_{uf} \times a \times h_{uf}}{T_f} \quad \& \quad F_{ruF} = \frac{m_{ur} \times a \times h_{ur}}{T_r}$$

m_{uf} & m_{ur} = un-sprung masses at front & rear

h_{uf} & h_{ur} = height of roll centres of un-sprung masses of front & rear

$F_{fuF} = \frac{(22.6*0.63*9.81*0.2667)}{1.3208}$

=**28.2 N at front**

$F_{ruF} = \frac{(22.8*0.63*9.81*0.2921)}{1.244}$

= **33 N at rear**

- Total load transfer at front

$F_f = F_{fsM} + F_{fsF} + F_{fuF}$

= (-18.2904+ 107.4 + 28.2) N

= **117.3096 N**

- Total load transfer at rear

$F_r = F_{rsM} + F_{rsF} + F_{ruF}$

= (- 36.04865+ 63.95 N +33) N

= **60.88 N**

which gives the desirable spool set up at the rear

Literature Review

The calculation was performed to find the approximate rear tire radius and width by satisfying tripoding at the required load transfer. The Calculation was also conducted to determine an approximate front tire radius and width based on slip characteristics. According to the result of calculations and as per availability a range of tires for Front and Rear was selected. The suspension was designed for the selected range. Theoretically, it was seen that the selected range complies with the designed suspension system after which the perfect setup was taken into consideration. Hence, the exact tires for the Rear and Front were finalized.

Description and working of the testing set-up

1. Cornering and Drive Brake Test

- The tire was attached with a shaft that has a 3-component Force matrix Sensor to measure all 3 forces and two moments.
- It has a sensor to measure the rotational velocity of the tire.
- The whole setup along with the shaft has the freedom to rotate along the vertical axis which will result in a camber change in the tire.
- A sensor mounted on the shaft measures the deformation of the tire under various conditions and as we know the diameter of the tire, we can determine the loaded radius of the tire.
- Another tire Pressure Monitoring Sensor (TPMS) helps in determining the pressure change in the tire under

various conditions. Similarly, the slip ratio is also estimated using a three-dimensional wireless MEMS accelerometer.
- The whole setup along with the shaft can also rotate about the vertical axis relative to the velocity of the carpet so that we can attain different slip angle values.
- The tire was placed on a carpet which has properties nearly equal to cement. The carpet was placed between two rollers and the velocity of the carpet can be taken as road velocity
- This process is done for 3 different tires and at different tire pressures (i.e., 20 psi, 22 psi, 24 psi)
- For every 10 milliseconds, every value is recorded.

2. Dynamic tire Balancing

The tire and wheel were mounted on a balancing machine test wheel and the assembly was rotated at 100 RPM (16 to 25 kmph with recent high sensitivity sensors) or higher, 300 RPM (40 to 60 kmph with typical low sensitivity sensors), and forces of unbalance were measured by sensors. These forces were resolved into static and couple values for the inner and outer planes of the wheel and compared to the unbalance tolerance (the maximum allowable manufacturing limits). Balance weights were then fitted to the outer and inner flanges of the wheel. The tires if not checked have the potential to cause vibration in the suspension of the vehicle on which it is mounted.

Dynamic balancing is better than static balance because both dynamic and static imbalances can be measured and corrected.

3. Validation through Data Acquisition System

- Accelerometers were mounted at the uprights for both front and rear and the results were recorded. While testing the vehicle, as the sensor is fixed to the wheel assembly, any change in caster, camber, toe, slip angles, roll, and pitch movement was obtained as sensor readings.
- Ultrasonic sensors were mounted at each of the air suspensions and their respective travels were recorded during testing and cornering data were taken into consideration.

Fig 2: Test Rig Setup in Testing Centre

Data Collected:

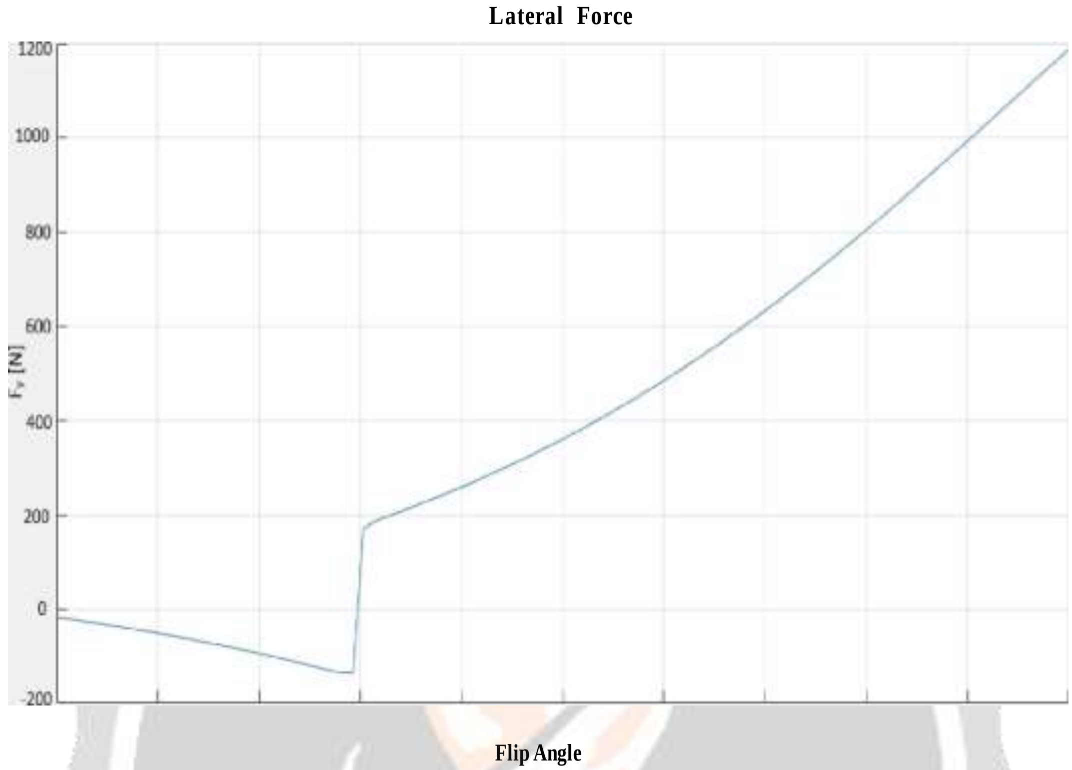


Fig.3 - Lateral force vs Slip angle

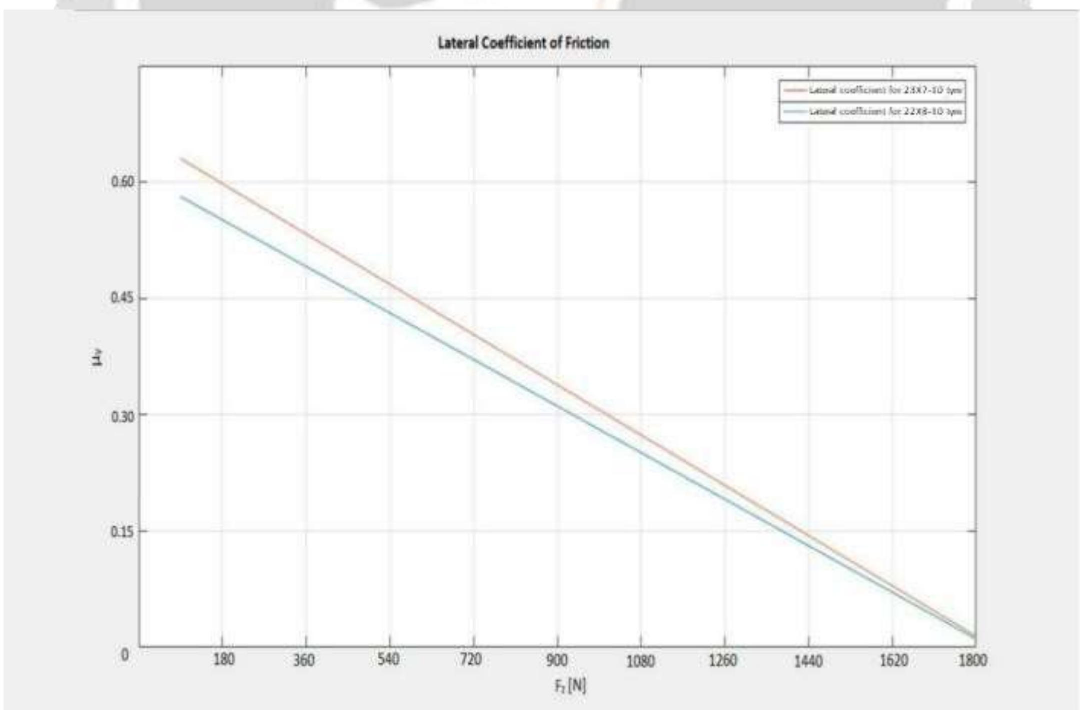


Fig 4

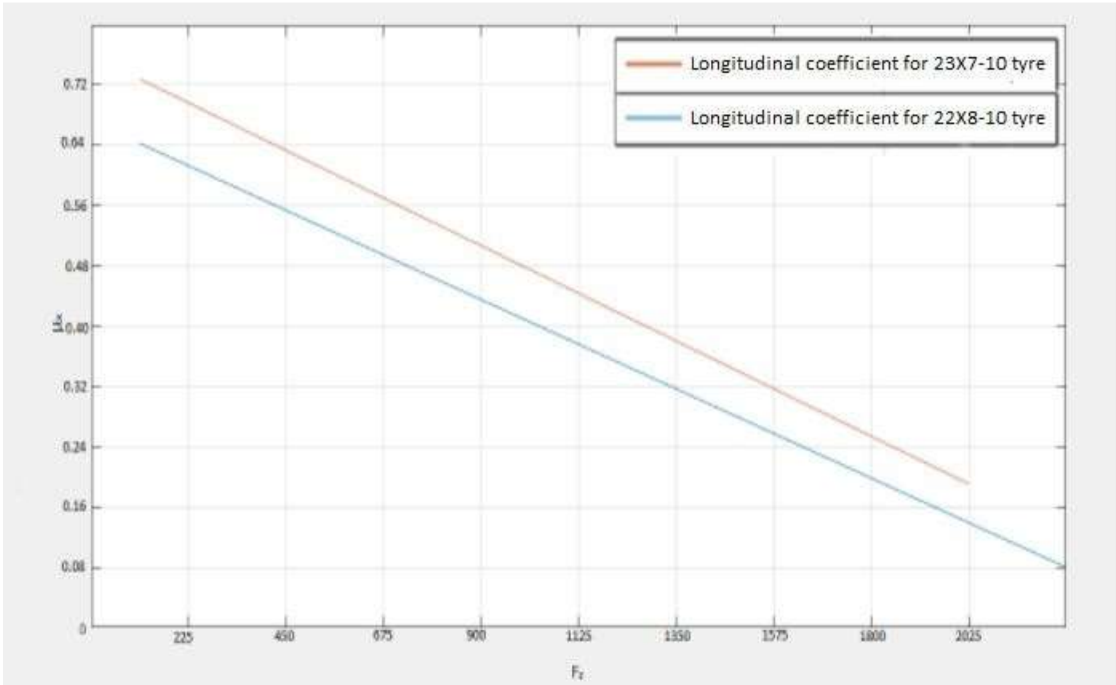


Fig.5

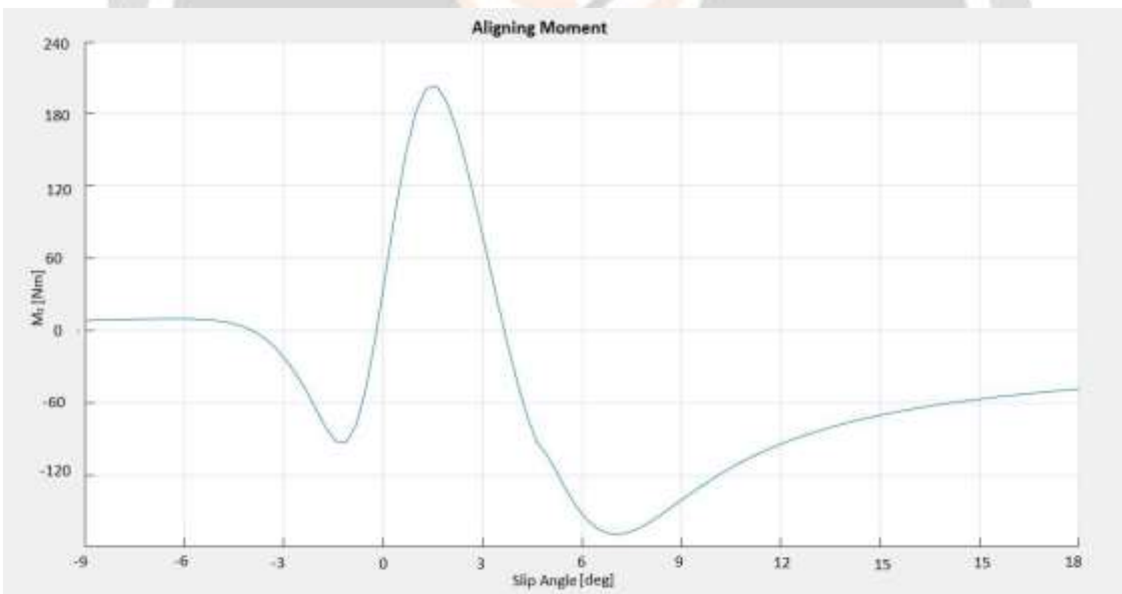


Fig.6-Aligning moment vs slip angle

Validation

Interpretation and analysis of data

We compared two different tires (shown as the blue and red lines) and found out-
It has a more longitudinal and lateral co-efficient of friction value for a given load (Fz).
Also, the slope of the curve (the red one) is more which shows that it is less load sensitive so it can attain more value than the other, so it has more grip. Tire 23 X 7 - 10 reaches more lateral force value for a dynamic camber at a lower slip angle range than other tested tires.

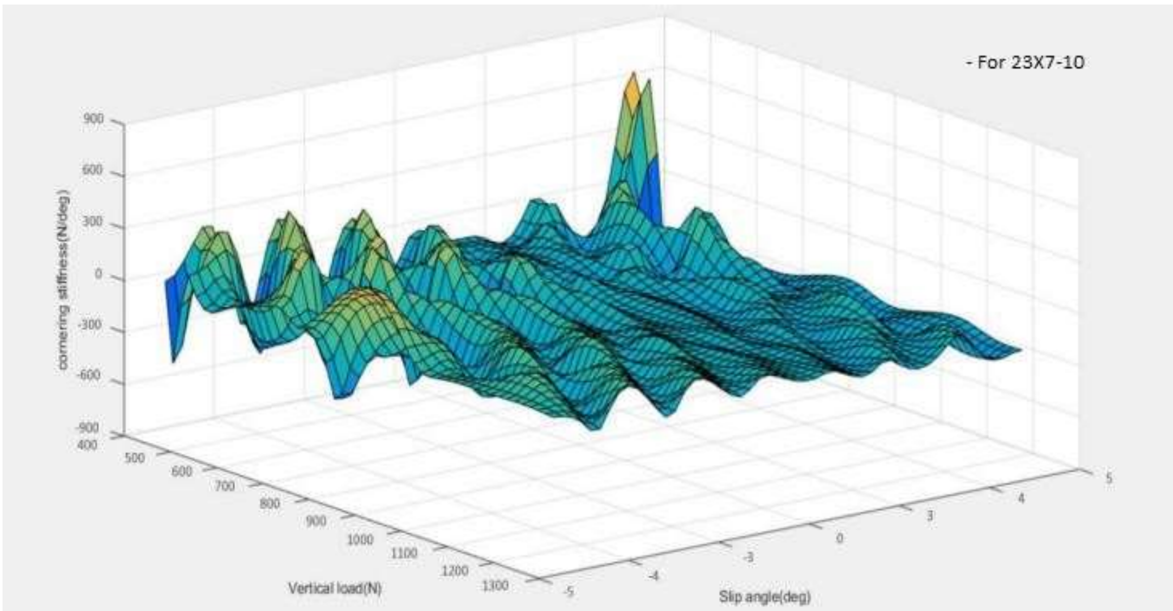


Fig.7 - for 23x7-10 tire

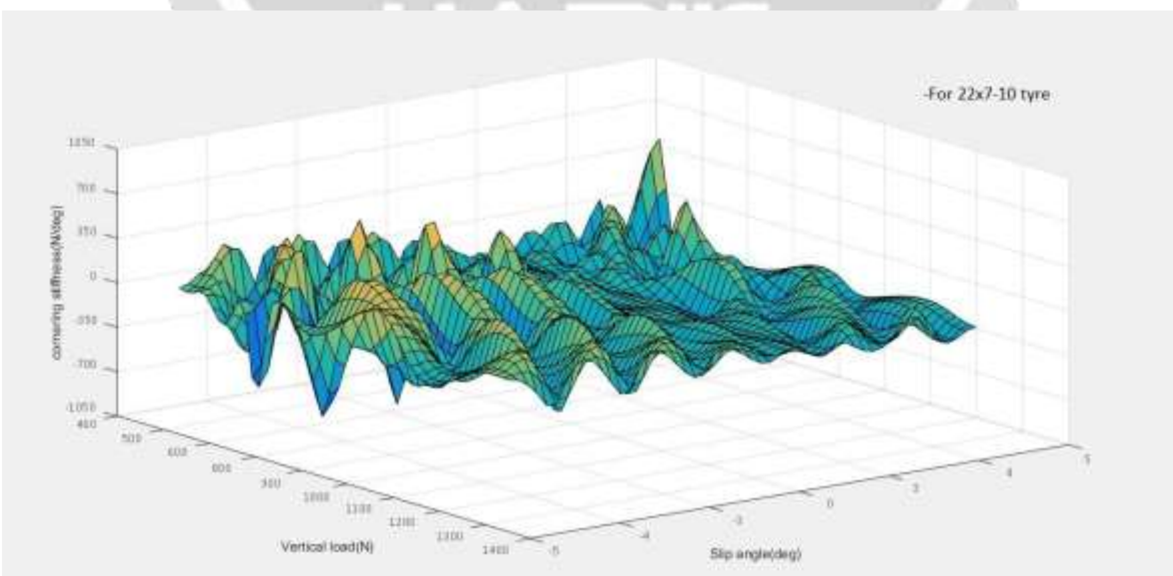


Fig.8 - For 22x7-10 tire

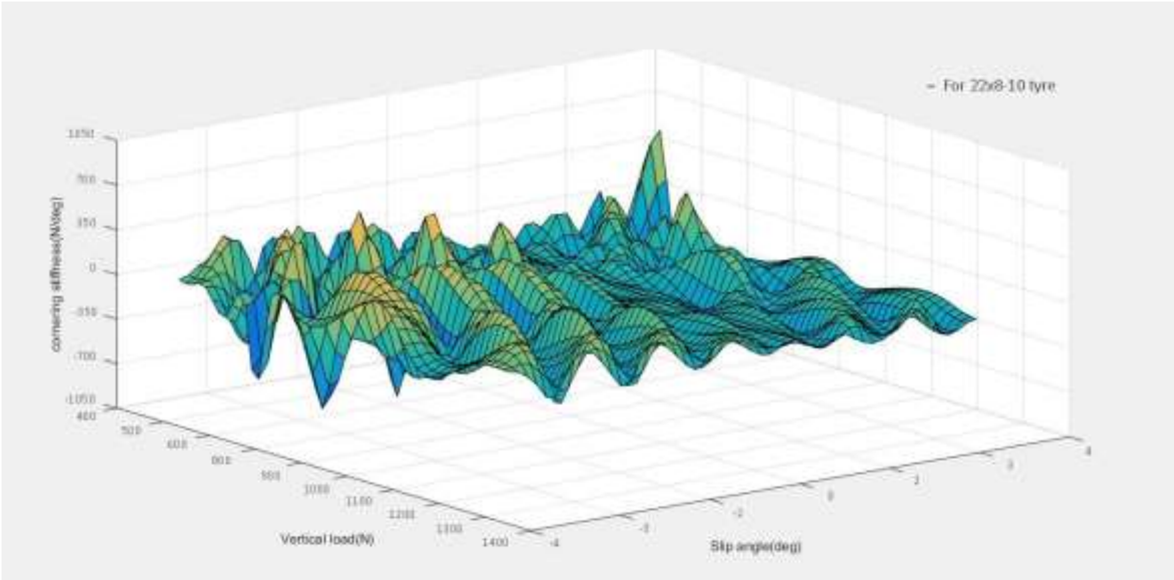


Fig.9 - For 22x8-10 tire

From Figures 7,8 & 9 of cornering stiffness, it can be shown tire 23 x 7-10 has more cornering stiffness value. Also, from the slopes and breaks the other tires are more controllable but less responsive because these can't attain values at the required rate so tire behavior will be not as good as tire 23 x 7 - 10.

For achieving tripoding conditions while traversing a corner, load transfer from the inner wheel should be equal to its weight. So, load transfer due to inertial force of sprung mass, unsprung mass, and due to roll moment are calculated for the inner wheel. Different tires diameter is taken into account for achieving the tripoding condition i.e., net load transfer is equal to weight on that wheel(inner). And it was found that tire of diameter 23" suits the above conditions. So, 23" diameter tires are selected for rear considering tripoding, traction and effective power transmission considering rear-wheel drive and spool setup.

rear RC ht(m)	FfsF(N)	FrsF(N)	Muf kg	mur kg(22	Mur kg(23	FUF 22 N	FfuF21	Fruf22N	Fruf 23 N	Ff 22 N	Ff 21 N	Frf 23 N	Fruf 22 N
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	120.96857	119.625544	64.54983312	60.33713884
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	120.59892	119.25589	63.95872494	59.74603065
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	120.22342	118.880394	63.35968453	59.14699025
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	119.842	118.498975	62.7526023	58.53990801
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	119.45457	118.111547	62.13736625	57.92467196
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	119.06105	117.718027	61.51386193	57.30116764
0.12917	**107.41**	**63.86**	**22.6**	20.8	**22.8**	**29.54659**	**28.204**	28.85821714	**33.07091**	**118.66**	**117.32**	**60.8819724**	56.66927809
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	118.25538	116.912358	60.24157805	56.02888376
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	117.84305	116.500027	59.59255676	55.37986247
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	117.42427	116.081241	58.9347836	54.72208931
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	116.99893	115.655905	58.26813086	54.05543658
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	116.56695	115.223919	57.59246801	53.37977373
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	116.12821	114.785183	56.90766154	52.69496726
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	115.68262	114.339595	56.21357495	52.00088066
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	115.23008	113.887049	55.51006863	51.29737434
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	114.77046	113.427436	54.79699979	50.58430551
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	114.30367	112.960646	54.07422239	49.86152811
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	113.82959	112.486565	53.34158702	49.12889273
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	113.3481	112.005078	52.5989408	48.38624652
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	112.85909	111.516064	51.84612736	47.63343307
0.12917	107.40517	63.859707	22.6	20.8	22.8	29.54658808	28.203561	28.85821714	33.07091143	112.36243	111.019403	51.08298663	46.87029235

Fig.10 - Calculations for tripoding at required load transfer considering different diameters of tire i.e., 22" and 23" tires

SENSOR1(FR)	SENSOR2(FL)	SENSOR3(RL)	SENSOR(RR)
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=49.92	FL=47.86	RL=42.29	RR=45.78
FR=50.04	FL=47.98	RL=40.36	RR=45.87
FR=50.54	FL=48.46	RL=38.28	RR=46.20
FR=51.04	FL=48.94	RL=36.24	RR=46.53
FR=51.42	FL=49.30	RL=35.78	RR=46.78
FR=51.79	FL=49.66	RL=33.18	RR=47.03
FR=52.04	FL=49.90	RL=30.67	RR=47.20
FR=52.42	FL=50.26	RL=27.81	RR=47.45
FR=52.79	FL=50.61	RL=26.24	RR=47.70
FR=53.41	FL=51.21	RL=24.44	RR=48.11
FR=53.66	FL=51.45	RL=20.59	RR=48.28
FR=53.91	FL=51.69	RL=17.75	RR=48.45
FR=54.41	FL=52.17	RL=14.05	RR=48.78
FR=54.66	FL=52.41	RL=11.21	RR=48.95
FR=54.91	FL=52.65	RL=9.36	RR=49.11
FR=55.16	FL=52.89	RL=7.15	RR=49.28
FR=55.41	FL=53.13	RL=8.59	RR=49.45
FR=55.66	FL=53.37	RL=13.67	RR=49.61
FR=55.91	FL=53.61	RL=18.75	RR=49.78
FR=56.16	FL=53.85	RL=28.82	RR=49.94

Fig.11- Travel of each of the air suspensions obtained during left with the help of ultrasonic sensors

From Fig.11 we inferred that tripoding condition is achieved as the vehicle while traversing a left corner, the spring travel of rear inner wheel is least as the load from this wheel is transferred to the front right wheel (diagonally) and rear right wheel (laterally). As a result, the rear inner wheel(left) loses contact patch for some time and the criteria for selection of a 23x7-10 tire for the rear is validated.

ROLL=0.00	CAMBER=0.00	TOE=0.00	CASTOR=1.88	YAW RATE=0.00	SLIP ANGLE=0.00	SLIP ANGLE=0.00
ROLL=0.10	CAMBER=-0.11	TOE=0.00	CASTOR=1.85	YAW RATE=0.89	SLIP ANGLE=0.43	SLIP ANGLE=0.56
ROLL=0.20	CAMBER=-0.23	TOE=0.00	CASTOR=1.82	YAW RATE=1.42	SLIP ANGLE=0.72	SLIP ANGLE=0.89
ROLL=0.30	CAMBER=-0.35	TOE=0.00	CASTOR=1.80	YAW RATE=1.86	SLIP ANGLE=0.96	SLIP ANGLE=1.07
ROLL=0.40	CAMBER=-0.47	TOE=0.01	CASTOR=1.77	YAW RATE=2.31	SLIP ANGLE=1.09	SLIP ANGLE=1.28
ROLL=0.50	CAMBER=-0.59	TOE=0.01	CASTOR=1.74	YAW RATE=2.94	SLIP ANGLE=1.47	SLIP ANGLE=1.59
ROLL=0.60	CAMBER=-0.71	TOE=0.02	CASTOR=1.71	YAW RATE=3.55	SLIP ANGLE=1.72	SLIP ANGLE=1.92
ROLL=0.70	CAMBER=-0.84	TOE=0.03	CASTOR=1.69	YAW RATE=4.12	SLIP ANGLE=1.83	SLIP ANGLE=2.16
ROLL=0.80	CAMBER=-0.96	TOE=0.03	CASTOR=1.66	YAW RATE=4.72	SLIP ANGLE=2.14	SLIP ANGLE=2.54
ROLL=0.90	CAMBER=-1.08	TOE=0.03	CASTOR=1.63	YAW RATE=5.15	SLIP ANGLE=2.41	SLIP ANGLE=2.93
ROLL=1.00	CAMBER=-1.20	TOE=0.04	CASTOR=1.60	YAW RATE=5.59	SLIP ANGLE=2.89	SLIP ANGLE=3.40

Fig.12 - Validation of slip angle using Accelerometer

Fig 12 shows the variation of slip angles with roll, camber toe, castor angles and yaw rate which are obtained during the testing phase through the installed accelerometer. These data of slip angles so obtained are in close comparison with the values obtained during tire testing and analysis. Hence, the chosen front tire of 23x7-10 is validated with desired slip characteristics.

Estimated vs Achieved

So, the validated parameters and behaviors are found to be nearly equal to the estimated value- We are able to achieve tripoding with this chosen set of tires.
Also estimated slip angles of 3.46 and 2.79 were validated with minimum errors.

Conclusion

The aim of this report is to create a complete and simplified model for tire validation that identifies how different tire parameters affect the balancing, handling properties, and ride comfort. The experimental evaluation in real-time embedded estimation processes yielded good estimation close to the measurements.

References

[1] Vehicle data acquisition using CAN | Optimum G | April 11, 2017 | Tech Tips
[2] Advanced Tire Mechanics by Yukio Nakajima
[3] The University of Windsor | Faculty of Mechanical, Materials and Automotive Engineering | 92-420 Capstone II | 2007 Baja Project- Suspension
[4] Race Car Vehicle Dynamics book- SAE International William F. Milliken Jr.
[5] Tune to Win: The Art and Science of Race Car Development and Tuning Book| Carroll Smith