(μ, φ) – FUZZY LIE ALGEBRAS OVER A (μ, φ) – FUZZY FIELD

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ABSTRACT

The aim of this paper is to introduce and investigate the concept of (μ, φ) – fuzzy lie algebras over an (μ, φ) – fuzzy field and the relations belong to and quasi-coincidence with characterizations of a $(\in, \in V_q)$ – fuzzy lie algebras over a $(\in, \in V_q)$ – fuzzy field.

Keyword – fuzzy field, fuzzy Lie algebra, fuzzy point

1. INTRODUCTION

Zadeh [12] formulated the notion of fuzzy sets and after that many scholars developed fuzzy system of different algebraic structures. The ideas of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [10],has played a vital role in generating some different types of fuzzy subgroups. Using the belong-to relation ($\in$) and quasi- coincidence with relation (q) between fuzzy points and fuzzy sets, the concept of (μ, φ)- fuzzy subgroup was introduced by Bhakat and Das [4]. Akram [1] introduced (μ, φ)-fuzzy Lie subalgebras and investigated to some of its properties. Nanda[9] introduced fuzzy algebra over fuzzy field.It is natural to investigate similar types of generalization of the existing fuzzy subsystem.In[3],We introducd fuzzy Lie algebra over a fuzzy field and some properties were discussed.

In this paper, we introduce the concept of (μ, φ)- fuzzy Lie algebra over an (μ, φ)- fuzzy field and investigate some of its properties.

2. PRELIMINARIES

In this section we present some definition needed for our study. We denote a complete distributive lattice with the smallest element 0 and the largest element 1 by I. By a fuzzy subset of a nonempty set X, we mean a function from X to I.

2.1 Definition [5].

Let X be a field and let F be a fuzzy subset of X. Then F is called a fuzzy field of X if

(i) for all μ, φin X, F(μ-φ)-F(μ)$\wedge$ F(φ),

(ii) for all μ, $\varphi \neq 0$ in X, F($\mu\varphi^{-1}$) -F(μ)$\wedge$ F(φ).

2.2 Remarks It is seen that if F is a fuzzy field of X, then

$$F(0)\text{-}F(1)\text{-}F(\mu) = F(-\mu)=F(\mu^{-1}) \text{ for all } \mu \neq 0 \text{ in X.}$$

2.3 Definition

Let A be a fuzzy subset of a Lie algebra L. Then A is called a fuzzy Lie algebra of L over a fuzzy field F, if for all $x, y \in L, \mu \in X$,

(i) $A(x, y) \geq A(x) \wedge A(y)$,
(ii) $A(\mu x) \geq F(\mu) \wedge A(x)$,
(iii) $A([x, y]) \geq A(x) \wedge A(y)$.

3. THE RELATIONS BELONG TO AND QUASI-COINCIDENCE WITH

Let L be a Lie algebra over a field X, let $A: L \to [0,1]$ be a fuzzy set on L, and let $F: X \to [0,1]$ be a fuzzy set on X.The support of fuzzy set A is the crisp set that contains all elements of L that have nonzero membership grades in A.

3.1 Definition [10]

A fuzzy set $A: L \to [0,1]$ of the form

$$A(y) = \begin{cases} t \in (0,1], & if\ y = x, \\ 0 & if\ y \neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

For a fuzzy point x_t and a fuzzy set A in a set L, Pu and Liu [10] gave meaning to the symbol $x_t \mu A$ where $\mu \in \{\in, q, \in V_q\}$.

A fuzzy point x_t is said to belong to a fuzzy set A, written as $x_t \in A$, if $A(x) \geq t$. A fuzzy point x_t is said to be quasi-coincidence with a fuzzy set A, denoted by $x_t q A$, if $A(x) + t > 1$

For a fuzzy set $A: L \to [0,1]$ and $t \in (0,1]$, we denote $A_t = \{x \in L: x_t \in A\}$.

The following notations are used in this paper

1. $\in V_q$ means that either *belongto* or *quasi-coincident with,*
2. $\bar{\mu}$ means that μ does not hold.

Let min{*t*,*s*} be denoted *m(t,s)* and let max{*t*,*s*} be denoted by *M(t,s).* Take $I = [0,1]$ and $\wedge = min$, $\vee = max$ with respect to the usual order in definitions 2.1 and 2.3

3.2 Lemma A fuzzy subset F of a field X is a fuzzy field, if and only if it satisfies the following conditions:

(i) for all μ, φ *in X,* $\mu_t, \varphi_s \in F \Rightarrow (\mu - \varphi)_{m(t,s)} \in F$
(ii) for all $\mu, \varphi \neq 0$ *in X,* $\mu_t, \varphi_s \in F \Rightarrow (\mu \varphi^{-1})_{m(t,s)} \in F$,
for all t,s $\in (0,1]$.

Proof. (i)

Given, for all $\mu, \varphi \in X$, $\mu_t, \varphi_s \in F$
To prove. $(\mu - \varphi)_{m(t,s)} \in F$
By definition 2.1
. $F(\mu_t - \varphi_s) - F(\mu_t) \wedge F(\varphi_s)$
$F(\mu_t - \varphi_s) = F(\mu_t) \wedge F(\varphi_s,), F(\mu_t - \varphi_s) = F(\mu_t \wedge \varphi_s)$
$F(\mu_t - \varphi_s) = \min(t, s) \in F, (\mu - \varphi)_{m(t,s)} \in F$
Conversely, given $(\mu - \varphi)_{m(t,s)} \in F$,
To prove, $\mu_t, \varphi_s \in F$

$Now,\ F(\mu_t - \varphi_s) = F(\mu_t \wedge \varphi_s), F(\mu_t - \varphi_s) = F(\mu_t) \wedge F(\varphi_s),$

$F(\mu_t - \varphi_s) - F(\mu_t) \wedge F(\varphi_s)$ Therefore, $\mu_t, \varphi_s \in F$, for all μ in X.

(ii)Given, for all $\mu, \varphi \neq 0\ in\ X, \mu_t, \varphi_s \ \in F$

To prove. $(\mu\varphi^{-1})_{m(t,s)} \in F$

By definition 2.1.

$F(\mu_t \varphi_s^{-1})$- $F(\mu_t) \wedge F(\varphi_s)$, $F(\mu_t \varphi_s^{-1})$= $F(\mu_t) \wedge F(\varphi_s)$

$F(\mu_t \varphi_s^{-1})$= $F(\mu_t \wedge \varphi_s)$

$F(\mu_t \varphi_s^{-1})$= min (t,s) $\in F$

Therefore, $(\mu\varphi^{-1})_{m(t,s)} \in F$,

Conversely,

Given, $(\mu\varphi^{-1})_{m(t,s)} \in F$

$F(\mu_t \varphi_s^{-1})$= min (t,s) $\in F$

$F(\mu_t \varphi_s^{-1})$= $F(\mu_t) \wedge F(\varphi_s)$

$F(\mu_t \varphi_s^{-1})$- $F(\mu_t) \wedge F(\varphi_s) \in F$,

Therefore, $\mu_t, \varphi_s \ \in F$, for all $\mu, \varphi \neq 0\ in\ X$

3.3 Lemma Let L be a Lie algebra over a field X. Then a fuzzy subset A of Lie algebra L is a fuzzy Lie algebra over a field F of X if and only if it satisfies the following conditions:

(i) $x_t, y_s \epsilon A \ \Rightarrow (x - y)_{m(t,s)} \epsilon A$
(ii) $x_t \epsilon A, \mu_r \epsilon F \ \Rightarrow (\mu x)_{m(r,t)} \epsilon A$
(iii) $x_t, y_s \in A \ \Rightarrow ([x, y])_{m(t,s)} \in A$,

For all x,y ϵ L, for all μϵ X, for all t, s, r (0,1].

Proof.

Given $x_t, y_s \epsilon A$

From Definition.2.2. $A(x - y) - A(x) \wedge A(y)$

$A(x_t - y_s) - A(x_t) \wedge A(y_s)$, $A(x_t - y_s) = A(x_t \wedge y_s)$

$A(x_t - y_s) = \min(t, s)\ \epsilon A$, $(x - y)_{m(t,s)} \epsilon A$

Therefore, $(x - y)_{m(t,s)} \epsilon A$

Conversely, $(x - y)_{m(t,s)} \epsilon A$, $A(x_t - y_s) = \min(t, s)\ \epsilon A$

$A(x_t - y_s) = A(x_t \wedge y_s)$, $A(x_t - y_s) - A(x_t) \wedge A(y_s)\ \epsilon A$

Therefore, $x_t, y_s \epsilon A$

(ii)Given, $x_t \epsilon A$, $\mu_r \epsilon F$

To prove, $(\mu x)_{m(r,t)} \epsilon A$,

$A(\mu x) - F(\mu) \wedge A(x),\ A(\mu_r x_t) - F(\mu_r) \wedge A(x_t),$

$A(\mu_r x_t) = F(\mu_r) \wedge A(x_t)$, $A(\mu_r x_t) = F(\mu_r) \wedge A(x_t) \in A$

$A(\mu_r x_t) = \min(r, t) \in A$

Therefore, $(\mu x)_{m(r,t)} \epsilon A$

Conversely, to prove, $x_t \epsilon A$, $\mu_r \epsilon F$

Given, $(\mu x)_{m(r,t)} \epsilon A$, $A(\mu_r x_t) = \min(r, t) \in A$

$A(\mu_r x_t) = F(\mu_r) \wedge A(x_t) \in A, A(\mu_r x_t) - F(\mu_r) \wedge A(x_t)$

Therefore, $x_t \epsilon A$, $\mu_r \epsilon F$

(iii)Consider $x_t, y_s \in A$

To prove, $([x, y])_{m(t,s)} \in A$,

$A\big([x_t, y_s]\big) - A(x_t) \wedge A(y_s)$, $\quad A\big([x_t, y_s]\big) = A(x_t) \wedge A(y_s)$

$A[x_t], A[y_s] = A(x_t) \wedge A(y_s), \big([x_t, y_s]\big) \epsilon A = m(t, s) \epsilon A$

Therefore, $([x, y])_{m(t,s)} \in A$

Conversely,

Consider, $([x, y])_{m(t,s)} \in A$, to prove $x_t, y_s \in A$

$([x,y])_{m(t,s)} \epsilon A$, $([x_{t,}y_s\,])\epsilon A = m(t,s)\epsilon A$,
$A[x_t], A[y_s] = A(x_t) \wedge A(y_s)$, $A([x_{t,}y_s\,]) - A(x_t) \wedge A(y_s)$
Therefore, $x_{t,}y_s \,\epsilon\, A$.

4. (μ,φ)FUZZY LIE ALGEBRAS OVER AN (μ,φ) – FUZZY FIELD

let μ and φ denote any one of $\in, q, \in Vq$ unless otherwise specified.

4.1 Definition

Let X be a field and let $F: L \to [0,1]$ be a fuzzy subset of X. Then F is called(μ,φ)- fuzzy field of X, if it satisfies the following conditions:

(i) For all μ,φ in X,$\mu_t \alpha F, \varphi_s \alpha F \Rightarrow (\mu-\varphi)_{m(t,s)} \varphi F$,

(ii) For all $\mu,\varphi \neq 0$ in X, $\mu_t \alpha F, \varphi_s \alpha F \Rightarrow (\mu\varphi^{-1})_{m(t,s)}\, \varphi F$

for all t,s ϵ (0,1].

4.2 Definition

Let L be a Lie algebra over a field X, and let $F: L \to [0,1]$ be an (μ,φ)- fuzzy field of X. Then a fuzzy subset $A: L \to [0,1]$ is called an (μ,φ)- fuzzy Lie algebra of L over an (μ,φ)- fuzzy field F of X, if it satisfies the following conditions:

(i) $x_t \alpha A, y_s \alpha A \Rightarrow (x-y)_{m(t,s)} \varphi A$

(ii) $x_t \alpha A, \mu_r \alpha F \Rightarrow (\mu x)_{m(r,t)} \varphi A$

(iii) $x_t \alpha A, y_s \alpha A \Rightarrow ([xy])_{m(t,s)} \varphi A$

For all x, y $\epsilon\, L$, for all $\mu \,\epsilon\, X$, for all t, s, r ϵ(0, 1].

4.3 Example

In the real vector space $\mathbb{R}^3$, define [x, y] = $x \times y$, where $'\times'$ is cross product of vectors for all $x, y \,\epsilon\, \mathbb{R}^3$. Then $\mathbb{R}^3$ is a Lie algebra over the field $\mathbb{R}$.

$$A(a,b,c) = \begin{cases} 1 & if\ a=b=c=0, \\ 0.5 & if\ a\neq 0, b=0, c=0, \\ 0 & otherwise \end{cases}$$

and define $F: R \to [0,1]$ for all $\mu \,\epsilon\, R$, by

$$F(\mu) = \begin{cases} 1 & if\ \mu \in Q \\ 0.5 & if\ \mu \in Q(\sqrt{2}) - Q, \\ 0 & if\ \mu \in R - Q(\sqrt{2}). \end{cases}$$

(i) Then by actual computation, it follows that F is an (ϵ,ϵ) -fuzzy field of R and A is an (ϵ,ϵ) fuzzy Lie algebra of R^3 over the (ϵ,ϵ)- fuzzy field F of R. also it can be verified that A is an $(\epsilon, \epsilon\, V_q)$ – fuzzy Lie algebra of R^3 over an $(\epsilon, \epsilon\, V_q)$ - fuzzy field F of R.

(ii) Let $x = (1,0,0), y = (2,0,0), t = 0.4,\ s = 0.3.$ Then $A(x-y) = 0.5\ and\ m(t,s) = 0.3.$ $A(x-y) + m(t,s) < 1$. So $(x-y)_{m(t,s)} \overline{q}\, A$. Hence A is not an (ϵ, q) – fuzzy Lie algebra.

(iii) Let $x = (0,0,0), y = (2,0,0)$, be elements in R^3 and t = 0.4, s = 0.6. Then $x_t qA$ and $y_s qA$. But $A(x-y) + m(t,s) = 0.5 + 0.4 < 1$. This shows that $(x-y)_{m(t,s)} \overline{q}\, A$. Hence A is not a *(q,q)*- fuzzy Lie algebra.

4.4 Theorem Let X be a fuzzy field. Then fuzzy subset F: X $\to$ [0,1] is a fuzzy field if and only if F is an(ϵ,ϵ)– fuzzy field of X.

Proof.

The result follows immediately from Lemma 3.2.

4.5 Theorem Let L be a Lie algebra over a field X. Then a fuzzy subset A of L is a fuzzy Lie algebra over a fuzzy field F of X if and only if A is an (ϵ,ϵ)-fuzzy Lie algebra of L over an (ϵ,ϵ)- fuzzy field F of X.

Proof.

The result follows immediately from Lemma 3.2. and 3.3.

4.6 Theorem Let X be a fuzzy field and let fuzzy subset F: X → [0,1] be afuzzy subset of X. Then F is an $(\epsilon, \epsilon \, V_q)$– fuzzy field of X if and only if

(i) *For all* μ, φ *in* $X, F(\mu - \varphi) \geq m(F(\mu), F(\varphi), 0.5)$,

(ii) *For all* $\mu, \varphi \neq 0$ *in* $X, F(\mu\varphi^{-1}) \geq m(F(\mu), F(\varphi), 0.5)$.

Proof.

Suppose that F is an $(\epsilon, \epsilon \, V_q)$- fuzzy field of X. It is clear that

$m(F(\mu), F(\varphi), 0.5) = m\big(m(F(\mu), F(\varphi), 0.5)\big)$

We consider two possibilities.

Case 1.

Let $m\big(F(\mu), F(\varphi)\big) < 0.5. then, m(F(\mu), F(\varphi), 0.5) = m\big(F(\mu), F(\varphi)\big)$. If possible, let $F(\mu - \varphi) < m(F(\mu), F(\varphi), 0.5) = m(F(\mu), F(\varphi))$.Let r, s∈ (0,1] be such that $F(\mu - \varphi) < r < s < m(F(\mu), F(\varphi))$.

Then $\big(F(\mu) < r, F(\varphi)\big) < s$ and so $\mu_r \in F$ and $\varphi_s \in F$. Also $F(\mu - \varphi) < m(r,s)$ shows that $(\mu - \varphi)_{m(r,s)} \overline{\in} F$ and $F(\mu - \varphi) + m(r,s) < m(r,s) + m(r,s) < 1$ shows that $(\mu - \varphi)_{m(r,s)} \bar{q}$ F.

Therefore $(\mu - \varphi)_{m(r,s)} \overline{\in Vq} F$, a Contradiction.

Case 2.

Let $m\big(F(\mu), F(\varphi)\big) - 0.5$. Then $m(F(\mu), F(\varphi), 0.5) =$ 0.5. If possible, let $F(\mu - \varphi) <$ 0.5. Then $\mu_{0.5} \in F, \varphi_{0.5} \in F$, but $(\mu - \varphi)_{m(r,s)} \overline{\in Vq} F$, a contradiction. Therefore, it follows that $F(\mu - \varphi) < m(F(\mu), F(\varphi), 0.5)$. Similarly, (ii) is proved.

Conversely,

Suppose that conditions (i) and (ii) are satisfied by a fuzzy set F of X.

Let $\mu_r \in F$, $\varphi_s \in F$, for $\mu, \varphi \in X$ and r, s∈ (0,1]. Then $F(\mu) - r$, $F(\varphi) - s$ and so $F(\mu), F(\varphi) - m(r,s)$. Since F satisfies condition (i),

$F(\mu - \varphi) - m(F(\mu), F(\varphi), 0.5) - m(r, s, 0.5)$.

Now Consider the possibilities $m(r,s) - 0.5$ or $m(r,s) > 0.5$. If $m(r,s) - 0.5$, then $m(r,s,0.5) = m(r,s)$ and $F(\mu - \varphi) - m(r,s)$ and $(\mu - \varphi)_{m(r,s)} \in F$. Then $m(r,s) > 0.5$, then $m(r,s,0.5) = 0.5$ and $F(\mu - \varphi) - 0.5$. So, $F(\mu - \varphi) + m(r,s) - 0.5 + m(r,s) > 0.5 + 0.5 = 1$ and hence $(\mu - \varphi)_{m(r,s)} qF$. Therefore, it follows that if $\mu_r \in F$, $\varphi_s \in F$, then $(\mu - \varphi)_{m(r,s)} \in VqF$. Similarly, if$\mu_r \in F$, $\varphi_s \in F$ for all $\mu, \varphi \neq 0$ in X, then $(\mu\varphi^{-1})_{m(r,s)} \in VqF$. Hence F is an $(\in, \in Vq)$- fuzzy field of X.

4.7 Theorem Let L be a Lie algebra over a filed X. Then a fuzzy subset A of L is an (∈, ∈ Vq)-fuzzy Lie algebra of L over an (∈, ∈ Vq)- fuzzy field F of X if and only if

(i) *For all* x, y∈ $L, A(x - y) - m(A(x), A(y), 0.5)$,

(ii) *For all* x∈ $L, \mu \in X, A(\mu x) - m(F(\mu), A(x), 0.5)$,

(iii) *For all* x, y∈ $L, A([xy]) - m(A(x), A(y), 0.5)$.

Proof.

Suppose that A is an (∈, ∈ Vq)- fuzzy Lie algebra of L over an (∈, ∈ Vq)- fuzzy field F of X. It is clear that $m(F(\mu), A(x), 0.5) = m\big(m(F(\mu), A(x), 0.5)\big)$. We consider two possibilities.

Case 1.

Let $m(F(\mu), A(x)) < 0.5$. Then, $m(F(\mu), A(x), 0.5) = m\big(F(\mu), A(x)\big)$. If possible , let $A(\mu x) < m(F(\mu), A(x), 0.5) = m\big(F(\mu), A(x)\big)$. Let $t \in (0,1]$ be such that $A(\mu x) < t < m\big(F(\mu), A(x)\big)$. Then, $F(\mu) > t \; and \; A(x) > t$. So, $\mu_t \in F$ and $x_t \in A$. But $A(\mu x) < t$ and $A(\mu x) + t < t + t < 2m\big(F(\mu), A(x)\big) < 1$. This shows that $(\mu x)_t \overline{\in Vq} A$, a contradiction.

Case 2.

Let $m(F(\mu), A(x)) - 0.5$). If possible, let $A(\mu x) < m(F(\mu), A(x)) < 0.5$)= 0.5. Then we have $\mu_{0.5} \in F$ and $x_{0.5} \in A$, but $(\mu x)_{0.5} \overline{\in Vq} A$, a contraction. Therefore, it follows that $A(\mu x) - m(F(\mu), A(x), 0.5)$. Thus, (ii) proved. Similarly, (i) and (ii) are proved.

Conversely, suppose that the condition (i), (ii) are satisfied by a fuzzy set A of L. Let $x_t \in A,\ y_s \in A$, for $x, y \in L$ and $t, s \in (0,1]$. Then, $A(x) - t, A(y) - s$ and so $m(A(x), A(y) - m(t,s)$. Since A Satisfies condition (iii),

$A([x,y]) - m(A(x), A(y), 0.5) - m(t, s, 0.5)$.

Now consider the possibilities $m(t,s) - 0.5$ *or* $m(t,s) > 0.5$. If $m(t,s) - 0.5$, then, $m(t,s,0.5) = m(t,s)$ and $A([x,y]) - m(t,s)$, and so$([x,y])_{m(t,s)} > 0.5$, then, $m(t,s,0.5) = 0.5$ and $A([x,y]) - 0.5$. So $A([x,y]) + m(t,s) - 0.5 + m(t,s) > 0.5 + 0.5 = 1$ and hence $([x,y])_{m(t,s)} qA$. Therefore, it follows that if $x_t \in A, y_s \in A$, then $([x,y])_{m(t,s)} \in VqA$. Similarly, if $x_t \in A, y_s \in A$, then $(x - y)_{m(t,s)} \in VqA$ and if $\mu_r \in F,\ x_t \in A$, then $(\mu x)_{m(r,t)} \in VqA$. Hence, A is an $(\in, \in \mathrm{Vq})$- fuzzy Lie algebra of L over an $(\in, \in \mathrm{Vq})$- fuzzy field F of X.

4.8 Proposition Let L be a Lie algebra over a field X. Then every $(\in, \in)$- fuzzy Lie algebra of L over an $(\in, \in)$-fuzzy field of X is an $(\in, \in \mathrm{Vq})$-fuzzy Lie algebra of L over an $(\in, \in \mathrm{Vq})$-fuzzy field of X.

Proof.

Suppose A is an $(\in, \in)$fuzzy Lie algebra of L over an $(\in, \in)$ fuzzy field F of X. Let $\mu, \varphi \in X, r, s \in (0,1]$. Since F is an $(\in, \in)$-fuzzy field of X, $\mu_r \in F, \varphi_s \in F \Rightarrow (\mu - \varphi)_{m(r,s)} \in F$, then $\mathrm{F}(\mu - \varphi) - m(r,s)$ shows that $(\mu - \varphi)_{m(r,s)} \in VqF$. Similarly, $(\mu\varphi^{-1})_{m(r,s)} \in VqF$for all $\mu, \varphi \neq 0$ in X. So F is an $(\in, \in \mathrm{Vq})$-fuzzy field of X. Since A is an $(\in, \in)$ - fuzzy Lie algebra, for $x, y \in \mathrm{L}, \mathrm{t}, \mathrm{s} \in (0,1],\ x_t \in A,\ y_s \in A \Rightarrow ([x,y])_{m(t,s)} \in \mathrm{A}$. Thus, $A([x,y]) - m(t,s)$. Then by definition $([x,y])_{m(t,s)} \in \mathrm{VqA}$ and $x_t \in A,\ \mu_s \in F \Rightarrow (\mu x)_{m(t,s)} \in \mathrm{VqA}$. Hence A is an $(\in, \in \mathrm{Vq})$-fuzzy Lie algebra of L over an $(\in, \in \mathrm{Vq})$-fuzzy field F of X.

4.9 Remark The converse of this proposition may not be true as seen in the following example.

4.10 Example

Let L=R^3 and [x,y]=x-y, where '-' is cross product for all $x, y \in \mathrm{L}$. Then L is a Lie algebra over the field R. Define $A: R^3 \to [0,1]$ for all $x = (a,b,c) \in R^3$ by

$$A(a,b,c) = \begin{cases} 0.6 & if\ a = b = c = 0, \\ 0.8 & if\ a \neq 0, b = 0, c = 0, \\ 0.5 & otherwise \end{cases}$$

And define $F: R \to [0,1]$ for all $\alpha \in R$ by

$$F(\mu) = \begin{cases} 0.6 & if\ \mu \in \mathbb{Q}, \\ 0.8 & if\ \mu \in \mathbb{Q}(\sqrt{2}) - \mathbb{Q}, \\ 0.5 & if\ \mu \in \mathbb{R} - \mathbb{Q}(\sqrt{2}) \end{cases}$$

Then by Theorem 4.7, it follows that A is an $(\in, \in \mathrm{Vq})$-fuzzy Lie algebra of R^3 over an $(\in, \in \mathrm{Vq})$-fuzzy field F of R.

But this is not an $(\in, \in)$fuzzy Lie algebra of R^3 over an $(\in, \in)$-fuzzy field of R. Let $x = (1,0,0)$. Then $A(1,0,0) = 0.8 > 0.65 > 0.62$. So $x_{0.65} \in A$ and$x_{0.62} \in A$. But $(x - x)_{m(0.65,0.62)} = 0_{0.65} \overline{\in} A$. It is clear that $A(0) + 0.62 = 0.6 + 0.62 > 1$ and so $0_{0.65} \in \mathrm{VqA}$. Therefore A is not an $(\in, \in)$-fuzzy Lie algebra of R^3 over an $(\in, \in)$-fuzzy field F of R.

4.11 Theorem Let A be an $(\in, \in \mathrm{Vq})$-fuzzy Lie algebra of L over an $(\in, \in \mathrm{Vq})$-fuzzy field of F of X such that $\mathrm{M}(\mathrm{A}(\mathrm{x}), \mathrm{F}(\mu)) > 0.5$ for all $\mathrm{x} \in \mathrm{L}$ and for all $\mu \in \mathrm{X}$. Then A is an $(\in, \in)$- fuzzy Lie algebra of L over an $(\in, \in)$-fuzzy field of X.

Proof.

Suppose that A is an $(\in, \in Vq)$-fuzzy Lie algebra of L over an $(\in, \in Vq)$-fuzzy filed F of X. Let $\mu, \varphi \in X$ and $t, s \in (0,1]$ be such that$\mu_t \in F, \varphi_s \in F$. Then, $F(\mu) - t, F(\varphi) - s$ and so $m(F(\mu), F(\varphi)) - m(t,s)$. It follows from Theorem 4.6 that $F(\mu - \varphi) - m(F(\mu), F(\varphi), 0.5)$. Given that $M(A(x), F(\mu)) < 0.5$ for all $\mu \in X$, for all $x \in L$,

Then we have $m(F(\mu), F(\varphi)) < 0.5$.

So $F(\mu - \varphi) - m(F(\mu), F(\varphi)) - m(t,s)$.

Therefore, $(\mu - \varphi)_{m(t,s)} \in F$.

Similarly, , $(\mu\varphi^{-1})_{m(t,s)} \in F$ for all $\mu, \varphi \neq 0$ in X.

Therefore, F is an $(\in,\in)$- fuzzy field of X.
Let $x, y \in L$ and $t_1,t_2 \in (0,1]$ be such that $x_{t1} \in A, y_{t2} \in A$. Then $A(x) - t_1, A(y) - t_2$ and so $m(A(x), A(y) - m(t_1, t_2)$.from Theorem 4.7 $A(x-y) - m(A(x), A(y), 0.5)$ and from the given condition we get $m(A(x), A(y)) < 0.5$. Therefore, $A(x-y) - m(t_1, t_2)$ and so $(x-y)_{m(t1,t2)} \in A$.
Let $x \in L, \mu \in X, s, t \in (0,1]$ be such that $\mu_s \in F, x_t \in A$. Then $F(\mu) - s, A(x) - t$ and so $m(F(\mu), A(x) - m(s,t)$. by theorem 4.7,

$$A(\mu x) - m(A(x), F(\mu), 0.5) = m(A(x), F(\mu) - m(s,t).$$

So, $(\mu x)_{m(s,t)} \in A$. Let $x_{t1} \in A$, $y_{t2} \in A \Rightarrow ([x,y])_{m(t1,t2)} \in A$. Therefore, A is an $(\in,\in)$-fuzzyLie algebra of L over an $(\in,\in)$-fuzzy field F of X.

4.12 Proposition If A is an $(\in, \in \mathrm{V}q)$-fuzzy Lie algebra of L over an$(\in, \in \mathrm{V}q)$-fuzzy field F then.

(1) $A(0) - m(A(x), 0.5)$,
(2) $A(-x) - m(A(x), 0.5)$,
(3) $A(x+y) - m(A(x), A(y), 0.5)$.

Proof.
Let $x \in L, y \in L$,then, from Theorem 4.7, the following hold.

(1) $A(0) = A([x,x]) - m(A(x), 0.5)$. so $A(0) - m(A(x), 0.5)$.
(2) $A(-x) = A(0-x) - m(A(0), A(x), 0.5) = m(m(A(x), 0.5, A(0)) = m(A(x), 0.5)$. Therefore $A(x) - m(A(x), 0.5)$.
(3) $A(x+y) = A(x— y - m(A(x), A(-y), 0.5) - m(A(x), m(A(y), 0.5), 0.5) = m(A(x), A(y), 0.5)$. Therefore, $A(x+y) - m(A(x), A(y), 0.5)$.

4.13 Theorem Let A be an $(\in, \in \mathrm{V}q)$-fuzzy Lie algebra of L over an $(\in, \in \mathrm{V}q)$-fuzzy field F of X. Then, $t \in (0,0.5]$, A_t is a Lie algebra over F_t when F_t contains at least two elements.
Proof.
For $t \in (0,0.5]$, suppose F_t contains at least two elements.
Let $\mu, \varphi \in F_t$. Then $\mu_t \in F$, $\varphi_t \in F$ and so $F(\mu) - t, F(\varphi) - t$. This shows that $m(F(\mu), F(\varphi)) - t$ and so $m(F(\mu), F(\varphi), 0.5) - m(t, 0.5)$.Therefore,
$(\mu - \varphi) - m(F(\mu), F(\varphi), 0.5) - m(t, 0.5) = t$
And hence $(\mu - \varphi)_t \in F$. Thus, $\mu - \varphi \in F_t$. Similarly, $\mu\varphi^{-1} \in F_t$ for all $\mu, \varphi \neq 0$ in F_t. Therefore F_t is subfield of X.
Suppose $x, y \in A$.Then $A(x) - t, A(y) - t$ and $m(A(x), A(y), 0.5) - m(t, 0.5) = t$. So $A(x+y) - m(A(x), A(y), 0.5) - t$ and hence $(x+y) \in A_t$. Let $\mu \in F_t$, $x \in F_t$. Then $F(\mu) - t$, $A(x) - t$ and $m(F(\mu), A(x)) - t$. Thus $m(F(\mu), A(x), 0.5) - t$. and so $A(x) - m(F(\mu), A(x), 0.5) - t$. Hence $\mu x \in A_t$
Similarly, for $x, y \in A_t$, $[x,y] \in A_t$. Therefore, A_t is a Lie subalgebra over the field F_t.
Let $f: L \to L^{\prime}$ be a function. If A and B are fuzzy subsets of L and $L^{\prime}$, respectively, then $f(A)$ and $f^{-1}(B)$ are defined using Zadeh's extension principle (7).
If α is one of the $\{\in, q, \in Vq\}$, it follows that $x_t \alpha f^{-1}(B)$ if and only if $(f(x))_t \alpha(B)$ for all $x \in L$ and for all $t \in (0,1]$.
4.14 Theorem *Let L and L' be Lie algebras over a field X and let $f: L \to L^{\prime}$ be a homomorphism. If B is an $(\in, \in Vq)$-fuzzy Lie algebra of L' over an $(\in, \in Vq)$-fuzzy field of X, then $f^{-1}(B)$ is an $(\in, \in Vq)$-fuzzy Lie algebra of L over the $(\in, \in Vq)$-fuzzy field F of X.*

Proof.
Let $x, y \in L$ *and* $t, s \in (0,1]$ be such that $x_t \in f^{-1}(B)$ and $y_s \in f^{-1}(B)$. Then $(f(x))_t \in B$, $(f(y))_s \in B$. Since B is an $(\in, \in Vq)$-fuzzy lie algebra over L' over an $(\in, \in Vq)$-fuzzy field of X,

$$\left(f(x-y)\right)_{m(t,s)} = \left(f(x) - f(y)\right)_{m(t,s)} \in VqB.$$

So we have

$\left((x-y)\right)_{m(t,s)} \in Vqf^{-1}(B)$. Similarly, $([x,y])_{m(t,s)} \in \ Vqf^{-1}(B)$.

Let $\mu \in X, x \in L$ and $r, t \in (0,1]$ be such that $\mu_r \in F \ and \ x_t \ \in f^{-1}(B)$. Then $(f(x))_t \in B$ and so

$$\left(f(\mu x)\right)_{m(r,t)} = \left(\mu f(x)\right)_{m(r,t)} \in VqB$$

and hence $(\mu x)_{m(r,t)} \in \ Vqf^{-1}(B)$.

Therefore, $f^{-1}(B)$ is an $(\in, \in$ Vq)-fuzzy Lie algebra of L over the $(\in, \in$ Vq)-fuzzy field F of X.

4.15 Definition

A fuzzy set δ of a set Y is said to process Sup Property if for every nonempty subset S of Y, there exists $x_0 \in S$ such that

$$\delta(x)_0 = Sup(\delta(x)|x \in S)$$

4.16 Theorem Let L and L' be Lie algebras over a field X and let f: L → L' be a onto homomorphism. Let A be an $(\in, \in$ Vq)-fuzzy Lie algebra of L' over an $(\in, \in$ Vq)-fuzzy field of X, whish satisfies the sup property. Then f(A) is an $(\in, \in$ Vq)-fuzzy Lie algebra of L' over the $(\in, \in$ Vq)-fuzzy field F of X.

Proof.

Let $a, b \in L' and \ t, s \in (0,1]$ be such that $a_t \in f\ (A) and \ b_s \in f\ (A)$. Then $f(A)(a) - t \ and \ f(A)(b) - s$ and so

$$Sup\{(A(z)|z \in f^{-1}\,(a) - t \text{ and } Sup\{(A(w)|w \in f^{-1}\,(b) - s.$$

Since f is onto, $f^{-1}\,(a)$ and $f^{-1}\,(b)$ are nonempty subsets of L and by the sup property of A, there exists $x \in f^{-1}\,(a)$ and $y \in f^{-1}\,(b)$ such that

$$A(x) = \sup\{(A(z)|z \in \ f^{-1}(a)\} \text{ and } A(y) = \sup\{(A(w)|w \in \ f^{-1}(b)\}$$

Then $x_t \ \in A \ and \ y_s \in A$. Since A is an $(\in, \in$ Vq)-fuzzy Lie algebra of L over the $(\in, \in$ Vq)-fuzzy field F of X, we have $([x,y])_{m(t,s)} \in VqA$ and so $A([x,y]) - m(t,s)$ or $A([x,y]) + m(t,s) > 1$. Now $f(x) = a, f(y) = b$ and so $[x,y] \in \ f^{-1}([a,b])$.

$$f(A)([a,b]) = \sup(A(z)|\ z \ \in f^{-1}([a,b])) \geq A([x,y])$$

And so $f(A)([a,b]) \geq m(t,s) \ or \ f(A)([a,b]) + m(t,s) > 1$. Thus $([a,b])_{m(t,s)} \in Vqf(A)$. Also $(x-y)_{m(t,s)} \in VqA$ shows that $(a-b)_{m(t,s)} \in Vqf(A)$.

Let $\mu \in X, b \in L' and \ r, s \in (0,1]$ be such that $\mu_r \in F \ and \ b_s \in f(A)$. Then it follows that $\mu_r \in F \ and \ y_s \in A$. So $(\mu y)_{m(r,s)} \in VqA$. Thus $A(\mu y) \geq m(r,s)$ or $A(\mu y) + m(r,s) > 1$.

But $f(A)(\mu b) = Sup\{A(w)|w \in f^{-1}(\mu b)\} \geq A(\mu y)$. This shows that $(\mu b)_{m(r,s)} \in Vqf(A)$.

Therefore, *f(A)* is an $(\in, \in$ Vq)-fuzzy Lie algebra of L'over the $(\in, \in$ Vq)-fuzzy field F of X.

CONCLUSION

In this paper, WE conclude and investigateD the concept of (μ, φ) – fuzzy lie algebras over an (μ, φ) – fuzzy field and the relations belong to and quasi-coincidence with characterizations of a $(\in, \in V_q)$ – fuzzy lie algebras over a $(\in, \in V_q)$ – fuzzy field. This concept can further be generalized to interval valued fuzzy field for new results in our future work.

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