ADJUSTING RAINFALL VALUES IN TOLIARA (MADAGASCAR) BY A PROBABILITY LAW

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ABSTRACT

The study carried out is based on Precipitation data for the study period from 1983 to 2012. The rainfall time series of Toliara (Madagascar) has been adjusted on probability laws to identify the recent law on precipitation. Four probability laws have been adapted on the amount of rain accumulated over the 30 years. The adjustment of the probability law on the pluviometer verifies that the rain height follows a generalized Pareto law.

Keywords: *Precipitation, Cumulus, normal Log law, Beta law, Gamma law, generalized Pareto law*

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1. INTRODUCTION

Since the adjustment of the distribution function of rain values (cumulative rain values) by a probability law is part of the objective of our study. Thus, some mathematical properties deserve to be underlined to be able to carry out this research well. In probability theory or in statistics, a probability density is a function which makes it possible to represent a probability law in the form of integrals. It is a data analysis tool used in fields as varied as the study of climatology. The hypothesis which is made in this study is that the local rain phenomenon follows a stochastic process which remains to be determined and for this, the intensity of precipitation is a random variable whose characteristics we will endeavor to determine.

2. METHODOLOGY

2.1. Lognormal law

Definition: A random variable X with values in $]0,+\infty[$follows the log-normal distribution of parameters $\mathcal{N}(m,\sigma)$ if $Y = log\ X$ follows the distribution$\mathcal{N}(m,\sigma)$.

The density of X is then:

$$f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{x}\left(-\frac{1}{2}\left(\frac{logx-m}{\sigma}\right)^2\right) & if\ x > 0 \\ 0 & if\ no \end{cases}$$

X then admits a hope and a variance:

$$E(X) = e^{m+\sigma^2/2}\ \ and\ \ V(X) = e^{m+\sigma^2}\left(e^{\sigma^2} - 1\right)$$

2.2. Beta Law

Definition: A random variable X with values in [0,1] is said to follow the beta law of parameters $r > 0, s > 0$ $(which\ we\ denote\ \rightarrow\ \beta\ (r, s))$ if it is absolutely continuous and admits for density:

$$f(x) = \begin{cases} \frac{1}{\beta(r,s)} x^{r-1}(1-x)^{s-1} & if \quad x \epsilon [0,1] \\ 0 & if\ \ no \end{cases}$$

Law $\beta\ (1\ /\ 2, 1\ /\ 2)$ is also called the law of the arc sinus.
X then admits an expectation and a variance:

$$E(X) = \frac{r}{r+s} \qquad and \qquad V(X) = \frac{rs}{(r+s)^2(r+s+1)}$$

2.3. Gamma law

Definition: We say that X follows the gamma distribution of parameters $p > 0, \lambda > 0$, which we denote by $X\ \rightarrow\ \Gamma\ (p, \lambda)$ if it is absolutely continuous, and admits for density:

$$f(x) = \begin{cases} \frac{\lambda}{\Gamma(p)} e^{\lambda x}(\lambda x)^{p-1} & if\ x > 0 \\ 0 & if\ no \end{cases}$$

X then admits an expectation and a variance:

$$E(X) = \frac{p}{\lambda} \qquad and \qquad V(X) = \frac{p}{\lambda^2}$$

2.4. Generalized Pareto law

Pareto's laws are very useful in Economics. They are also used for the properties of their distribution tails or survival functions which are decreasing of the “power” type, that is to say relatively slow.
Definition: A x continuous random variable follows a Pareto law of parameters $\alpha, x_0 \in \mathbb{R}_+^*$ if it admits for probability density the function:

$$\forall\ t\ \in\ \mathbb{R}, f_X(x) = \frac{\alpha x_0^\alpha}{t^{\alpha+1}} I_{|x_0:+\infty|}(t) = \begin{cases} 0 & if\ \ t < x_0 \\ \frac{\alpha x_0^\alpha}{t^{\alpha+1}} & if\ t > x_0 \end{cases}$$

3. RESULTS

3.1. Comparison of the function of the cumulative values of rain with the different probability laws

In the figure below, we have noticed that the rain data distribution function is an increasing function and whose values are positive. This curve shape prompts us to choose mathematical laws that seem to have the same variation for their distribution function to make the comparison, namely: the normal Log law; the Gamma law; The Beta law and the generalized law of Pareto.

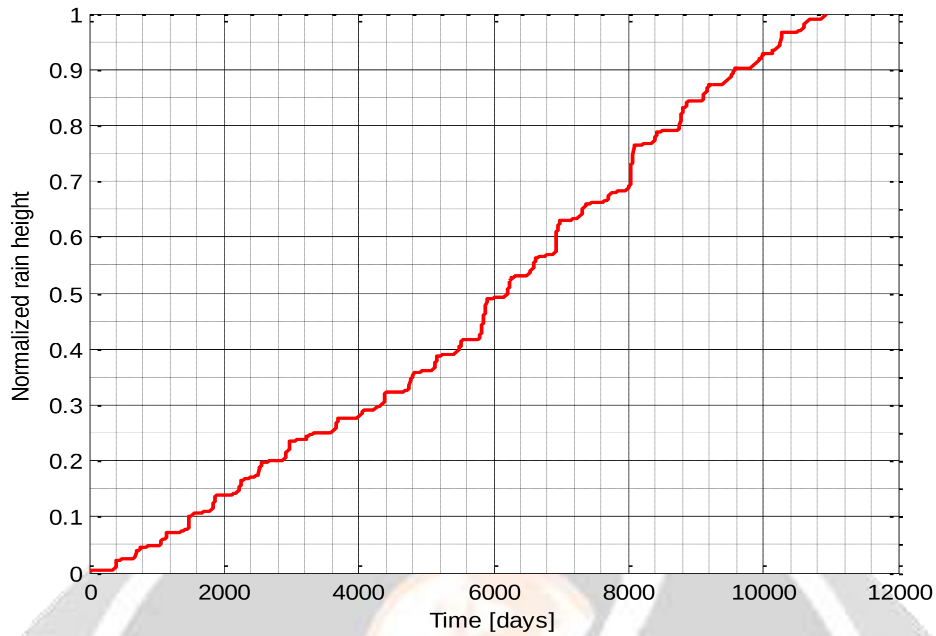

Fig-1: *Cumulated rainfall amounts from 1983 to 2012*

3.2. Normal Log Law

In probability and statistical theory, a random variable X is said to follow a log-normal law of parameters μ and σ if the variable $Y = ln(X)$follows a normal law of expectation μ and σ^2variance. This law is sometimes also called Galton's law. A variable can be modeled by a log-normal distribution if it is the result of the multiplication of a large number of small independent factors.

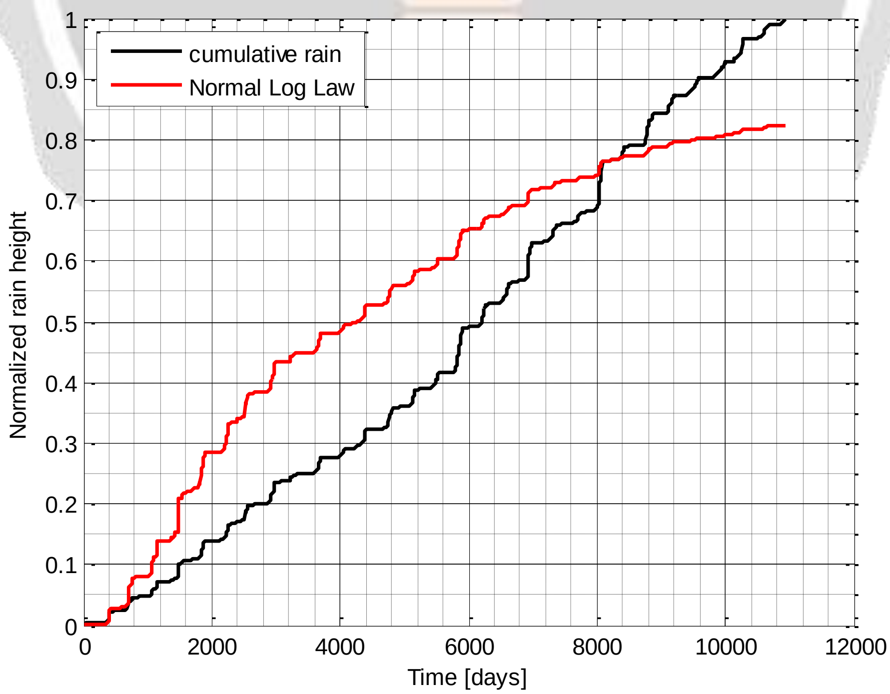

Fig-2: Adjustment of the normal log law on the height of cumulative rainfalls

For these curves *(Figure 2)*, the normal log law has parameters $\mu = -1{,}222$ and $\sigma = 1{,}3126$.
The normal Log distribution does not quite follow the variation of the cumulative rainfall curve. And we observe that the cumulative curve of the normal log law deviates from the cumulative rain function.

3.3. Gamma law

In probability theory and statistics, a Gamma distribution, or Gamma law *(or, which corresponds to the capital (gamma) in Greek)*, is a type of probability law of positive real random variables. The family of Gamma distributions includes among others the exponential laws, the laws of sums of independent random variables following the same exponential law, as well as the law of χ^2. It therefore makes it possible to model a wide variety of phenomena for positive quantities.

For our case the gamma law has parameters $k = 1{,}2476$ and $\theta = 0{,}3707$. The curve of the cumulative function of the gamma law *(Figure 3)* cannot follow the curve of the cumulative amount of rain, and it deviates. This means that the gamma law can no longer follow the trend of the data.
This situation increased the value of the deviation from the curve of the cumulative function.

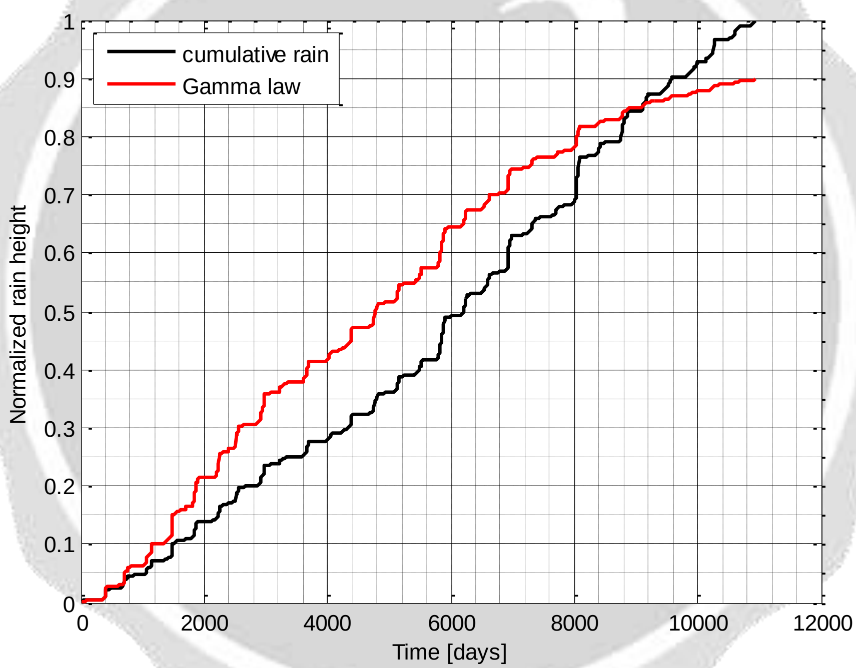

Fig-3: Adjustment of the Gamma law on the height of the cumulative rains

3.4. Beta Law

It is the law of a variable $X;\ 0 \leq X \leq 1$depending on two real parameters $n\ and\ p$. These laws are used in BAYESIAN statistics to represent the priori distribution of the probability of an event. It is good to note that the cumulative function " F " of the heights of rains is presented by its normalized values, that means that its final value is equal to 1. The *Figure 4* illustrates the adjustment of the beta distribution on the cumulative rainfall height Toliara. It is observed that these two curves are almost confused, that is to say the difference between these curves is very small. For our case, the Beta law has parameters $n = 0{,}74118\ et\ p = 0{,}8449$.

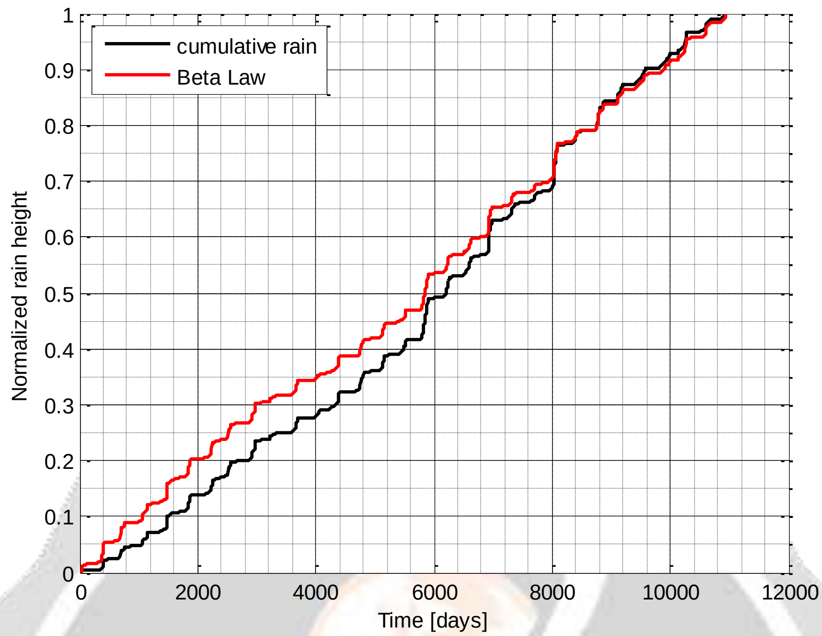

Fig-4: Adjustment of the Beta law on the height of cumulative rainfalls

3.5. General law of PARETO

The Pareto distribution is a special type of power law that has its applications in the physical sciences. The parameters of the generalized Pareto law are determined by the values:

$$k = -0{,}9702 < 0 \,; \qquad \boldsymbol{x_m} = 0{,}9702 \ and \ \theta = 0.$$

For $k < 0$, the generalized Pareto distribution behaves like the BETA law for finite size. These two curves *(Figure 5)* show that the Pareto distribution follows the shape of the cumulative function well. The law also manages to track the cumulative amount of rain. So the law of rain height in Toliara is follows the generalized law of Pareto.

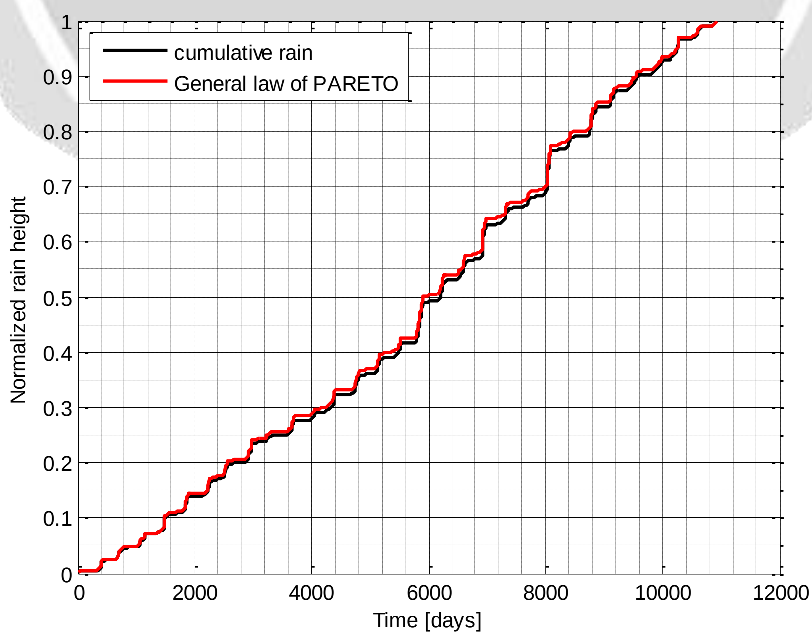

Fig-5: Adjustment of the generalized law of Pareto on the height of the cumulative rains

3.6. Comparison of the four different laws

The estimate generally represents only 95% of the distribution.

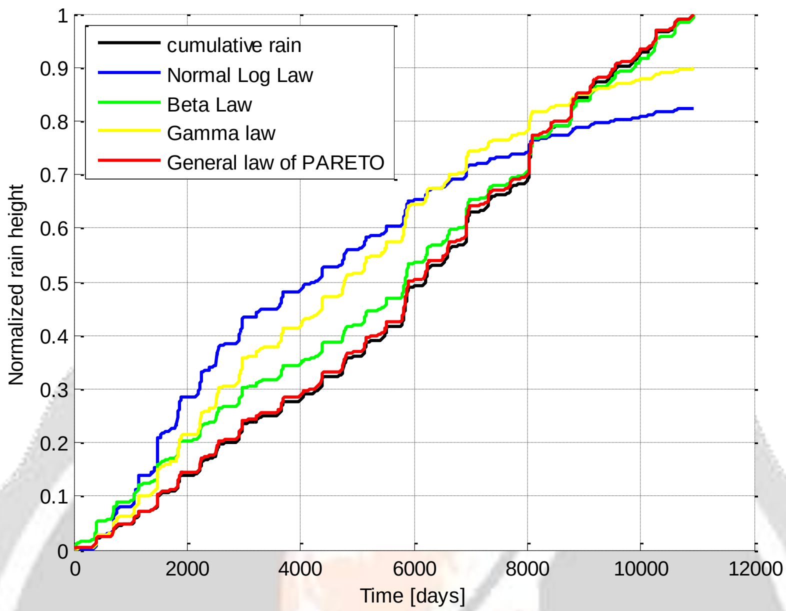

Fig-6: Comparison of the various adjustments of the probability distribution of the height of the accumulated rains

The *Figure 6* shows the comparison of the various adjustments of the probability distribution on the rain height. We notice that the generalized law of Pareto is the most similar law of this distribution. It manages to follow the asymptotic behavior of the distribution curve. There is a divergence at the end of the curve for the normal log law and the gamma law.

4. CONCLUSION

The adjustment of the cumulative values of rain by different probability laws, namely: log-normal law, gamma law, beta law, generalized Pareto law, allowed us to note that among these four laws mentioned above, it is the generalized law of Pareto which is very close to the distribution of the cumulative heights of rain.

5. ACKNOWLEDGEMENT

Our deep gratitude to RATIARISON Adolphe Andriamanga, Emeritus Professor at the Faculty of Sciences of the University of Antananarivo, Madagascar and his team, for allowing us to conduct our investigations in their DYACO laboratory

6. REFERENCES

[1] Embrechts, P., C. Klüppelberg, and T. Mikosch, 1997. Modelling Extremal Events for Insurance and Finance. New York: Springer.
[2] Evans, M., N. Hastings, and B. Peacock, 1993. Statistical Distributions. 2nd ed., Hoboken, NJ: John Wiley & Sons, Inc., pp. 102–105.
[3] Hahn, Gerald J., and S. S. Shapiro, 1994. Statistical Models in Engineering. Hoboken, NJ: John Wiley & Sons, Inc., p. 88.
[4] Hahn, Gerald J., and S. S. Shapiro, 1994. Statistical Models in Engineering. Hoboken, NJ: John Wiley & Sons, Inc., p. 95.
[5] Kotz, S., and S. Nadarajah, 2000. Extreme Value Distributions: Theory and Applications. London: Imperial College Press.