ANALYSIS OF THE METEOROLOGY DROUGHT IN TOLIARA (MADAGASCAR), STANDARDIZED PRECIPITATION INDEX APPROACH AND THE WAVELET FREQUENCY METHOD

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ABSTRACT

In the last thirty years, rainfall in southern Madagascar has an upward trend. We have used different methods to address the rainfall series. Rainfall data were used in this study to clarify the degree of severity of drought. The results show the existence of homogeneity breaks around the year 1996. It is noted that the period before that date qualified dry, the following is surplus period.

Keywords:*Precipitation, Temperature, Drought, SPI, Fourier analysis,wavelet*

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1. INTRODUCTION

In recent years, several studies have focused on global change, many of which have focused on droughts. According to certain scenarios of global change, the occurrence and impact of droughts are likely to increase in the coming years (Watson *et al.* 1997). So drought is difficult in detail, especially in Toliara, in the southwest of Madagascar. The objective of this article was to assess the vulnerability of global meteorological drought in southern Madagascar by reconstructing the historical occurrence of drought on several time scales and the categories of drought. For this, we used mathematical tools as a methodology.
Before seeing the drought, let's take a look at the evolution of precipitation and temperature. The study was devoted to assessing the future characteristics of droughts under climate change *(variation in precipitation and temperature)* using standardized precipitation indices.

2. METHODOLOGY

2.1. Standardized Precipitation Index *(SPI)*

2.1.1. Definition

The SPI is an index for measuring meteorological drought. It is a probability index based only on precipitation.

Table-1: Moisture and drought categories defined by the SPI values

Values SPI	**Humidity level And drought**
≥ +2.0	Extreme humidity
1.5 to 1.99	High humidity
1.0 to 1.49	Moderate humidity
-0.99 to 0.99	Close to normal
-1.49 to -1.0	Moderate drought
-1.99 to -1.5	Strong drought
≤ -2.0	Extreme drought

2.1.2. **Calculation method :**

The SPI was intended to determine the precipitation deficit for a given region to determine a period of time. The calculation of the SPI requires the adjustment of long series of precipitation data to the Gamma distribution which is the distribution that best represents the evolution of the rain series.
This Gamma distribution is defined by its probability density represented by:

$$g(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}$$

Or:

$\alpha > 0$: is the shape parameter of the distribution
$\beta > 0$: is the scale parameter of the distribution.
$x > 0$: : The height of monthly precipitation
$\Gamma(\alpha)$: represents the mathematical function Gamma and defined by:

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy$$

The parameters of the law (alpha and beta) are estimated by the maximum likelihood method *[MOSAAD, 2011]* , and this for each time scale.Thus, the estimation of these parameters is as follows:

$$\alpha = \frac{1}{4A}\left(1 + \sqrt{1 + \frac{4A}{3}}\right)$$

$$\beta = \frac{\bar{x}}{\alpha}$$

Or

$$A = \ln(\bar{x}) - \frac{\sum_{i=1}^{n} \ln(x_i)}{n}$$

n :*Number of observations over the period considered*
$\bar{x}$: *Historical average precipitation over the period considered*

These parameters are used to determine the Gamma distribution function for each precipitation observation and for each time period.The SPI is estimated by adjusting the distribution of the frequencies of precipitation in each station to the Gamma distribution of the probability of the density. The values of the parameters α and $\beta$the Gamma function are estimated for station and for each time scale (1.3, 9, 12, 24 months) [9].
Using the result parameters to find the accumulated probability of an observed precipitation episode, the cumulative probability (distribution function) becomes:

$$G(x) = \int_0^{x} g(x) dx$$

$$= \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^x x^{\alpha-1}e^{-\frac{x}{\beta}}dx$$

Since the Gamma function is undefined for $x = 0$and a series of precipitation can contain zeros, the cumulative probability becomes:

$$H(x) = q + (1-q)G(x)$$

Or:

$H(x)$: Corresponds to the cumulative probability when $x = 0$;

q : is the frequency of zero rain.

In this last calculation, the cumulative probability is only the variable Z of a standard normal distribution function characterized by an average value of zero and a unit variance: after this transformation, the calculated value gives the value of the SPI.

The SPI index is defined by :

$$Z = SPI = -\left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right) pour \ \ 0 < H(x) < 0.5$$

$$Z = SPI = +\left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right) pour \ \ 0.5 < H(x) < 1$$

Where:

$$t = \sqrt{ln\left[\frac{1}{(H(x))^2}\right]} pour \quad 0 < H(x) < 0.5$$

$$t = \sqrt{ln\left[\frac{1}{(1-H(x))^2}\right]} pour \ \ 0.5 < H(x) < 1$$

And

$c_0 = 2{,}515517$	$c_1 = 0{,}802853$	$c_2 = 0{,}010328$
$d_1 = 1{,}432788$	$d_2 = 0{,}189269$	$d_3 = 0{,}001308$

2.2. Wavelet

Given the nature of our signals, we used Morlet wavelets. This type of wavelet is well known because they are widely used for the analysis of datasets presenting oscillations and fluctuations, when one seeks to locate isolated structures in time in a noisy environment. Morlet's wavelet function consists of a plane wave modulated by a Gaussian bell. Morlet's mother wavelet is written:

$$\Psi_0(t) = \pi^{-1/4}\exp\left(-\frac{t^2}{2}\right)\exp(2\pi f_0 t i) \ \ avec \ \ f_0 = \frac{1}{\sqrt{2\log(2)}}Hz$$

3. RESULTS

3.1. **Synthetic results for the Time domain**

3.1.1. **Precipitation Analysis**

The databases used to reconstruct the precipitation analysis are the daily values measured in the far south of Madagascar. The first analyzes applied are descriptive statistics of the data, like the average. The analysis of interannual precipitation trends allows us to observe the increase in precipitation in southern Madagascar.

The *Figure 1* shows the cumulative amount of rainfall in the Toliara station. We note in this figure, there are some changes in rainfall amount; those changes are in the *5815 e nth s day* (equivalent to 1996) and 7997 umpteenth days (2002).
The *Figure 2* shows the annual rainfall amounts *(blue bar)* , the moving average of 3 years *(red curve)* and the total annual average rain having the 318,214mm value *(green curve)* .

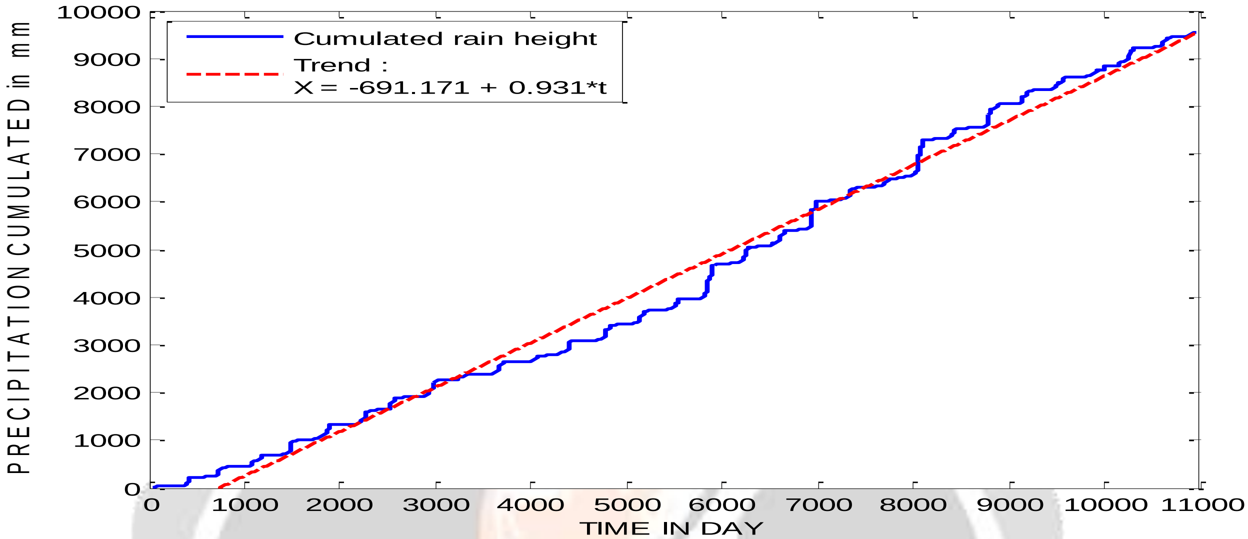

Fig-1: Cumulated rainfall amounts from 1983 to 2012 (TOLIARA station)

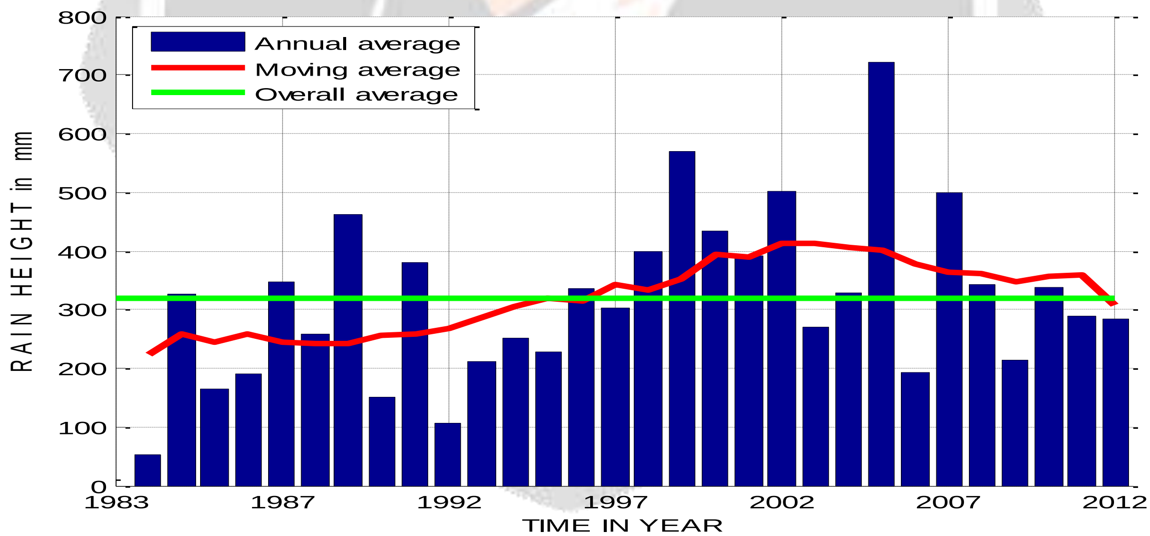

Fig-2: Time series of annual rainfall depths 1983 to 2012 (TOLIARA station)

Analysis of the temporal variability of annual rainfall over the different decades from 1983 to 2012 has shown that:

- ➢ The overall average showed that this change in rainfall started around 1996, and then that the moving average goes above the green curve.
- ➢ A heterogeneous distribution of rainfall intensities is observed on the whole in the Toliara station, whose period from 19983 to 1995 appears as the period of drought (because the filtered average is lower than the global average), but from l " 1996 to 2012, the precipitation depth is higher than the global annual average, which is a surplus period.

From Figures 1 and 2, we say that the rainfall will increase from 1996. But in 2002, the amount of precipitation was poor until 2012. So, we wonder if the drought will return after 2012.

3.1.2. **Chronic Temperature Analysis**

It is an essential parameter which conditions all physiological activities and chemical reactions (role of temperature in the reactivation of cambium and the breaking of dormancy, in the inhibition of photosynthesis ...)
The reference temperature for calculating st anomaly global temperature was taken as the average annual global temperatures over thirty years.
We carried out a small animation highlighting the anomaly *(difference compared to an average)* of the temperatures over the period 1983-2012, based on the average of this period. The negative anomaly means that the temperature for the period concerned is below average.
The diagnosis on the evolution of the temperature in Toliara is carried out starting from the data having measurements for 30 years. Analysis of the trends reveals a significant increase in average temperatures.

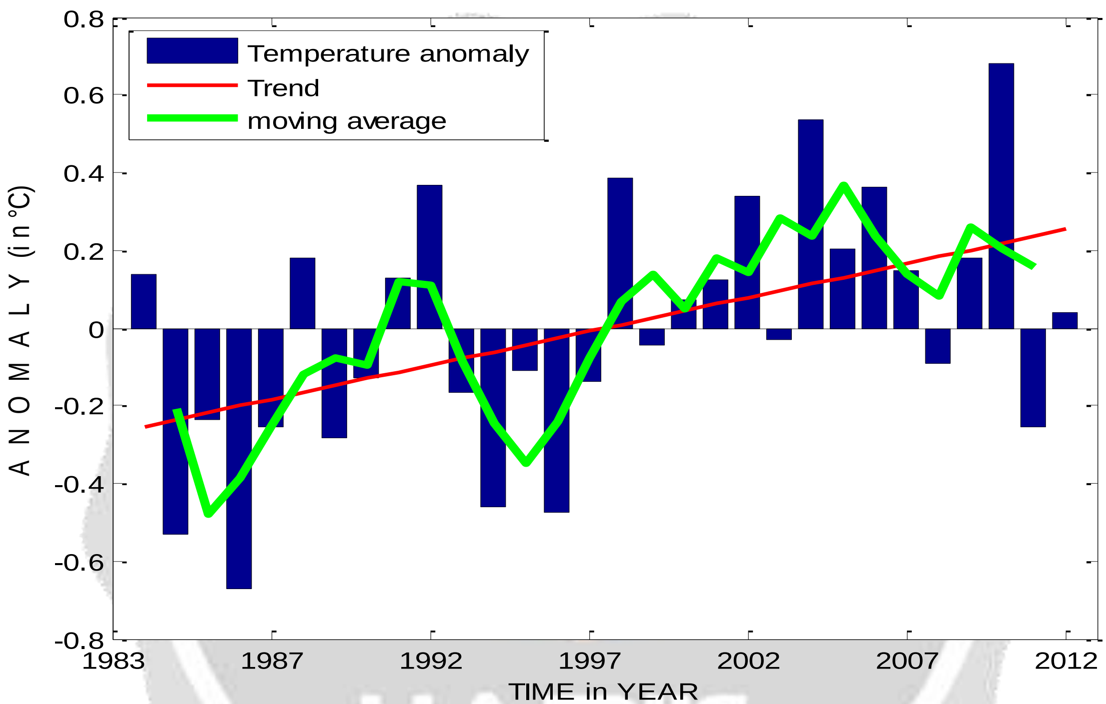

Fig-3: Average temperature anomalies in Toliara compared to the average for the period 1983-2012

The *Figure 3* shows the change in mean annual temperature (the average difference over the period 1983-2012). Even if we must remain cautious on the exact figure of this trend, global warming is a reality in Toliara.
This figure of temperature anomalies makes it possible to better take the measure of the phenomenon, which extends from 1983 to 2012, and shows that until 1997, the hot and cold anomalies succeeded each other by varying around 0. Since 1998, things have changed. We observe that an increase in temperatures of up to 0.7 ° C.

3.1.3. **Occurrence and analysis of drought**

a. Drought analysis using drought indices

The SPI is determined for each of the accumulated precipitation values, calculated for the 30 periods considered (1983 to 2012) of each year of the historical series. The calculation formulas and the values found from the SPI are programmed in Matlab software at each study station.
b. SPI index for the consecutive month
SPI values were calculated for the total period and also for a specific month. The *Figures 4 to 6* illustrate the SPI values based on 1, 3 and 6 months to the pitch of respective time. Appearance of dryness is defined when SPI is negative and its intensity is - 0.99 or less. Several episodes of drought have been detected.

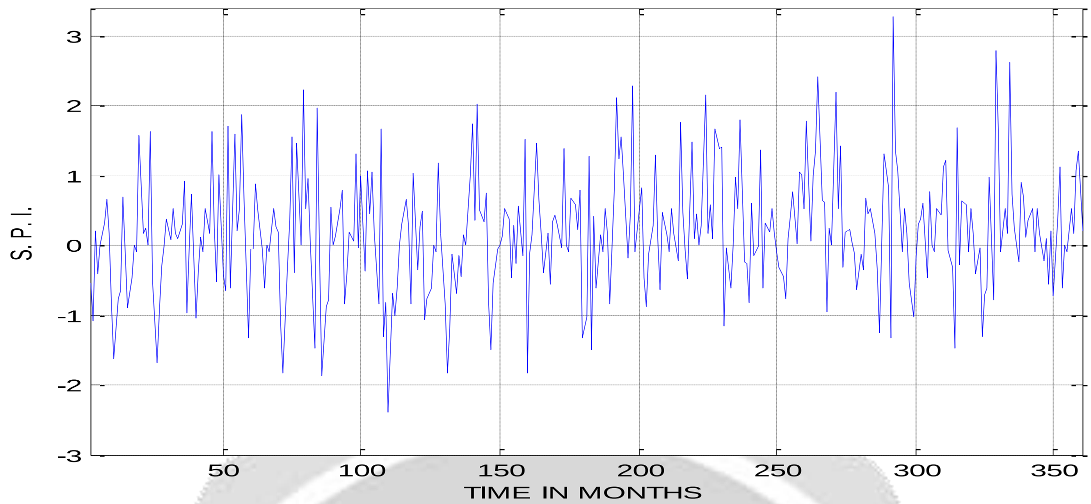

Fig-4: Time series of SPI-1 in Toliara

In the graphs of *Figures 4 to 6*,the SPI reaches values below the limit of - 0.99. It is thus possible to identify the probability of encountering dry sequences to determine the date of their start, their end and therefore their duration.

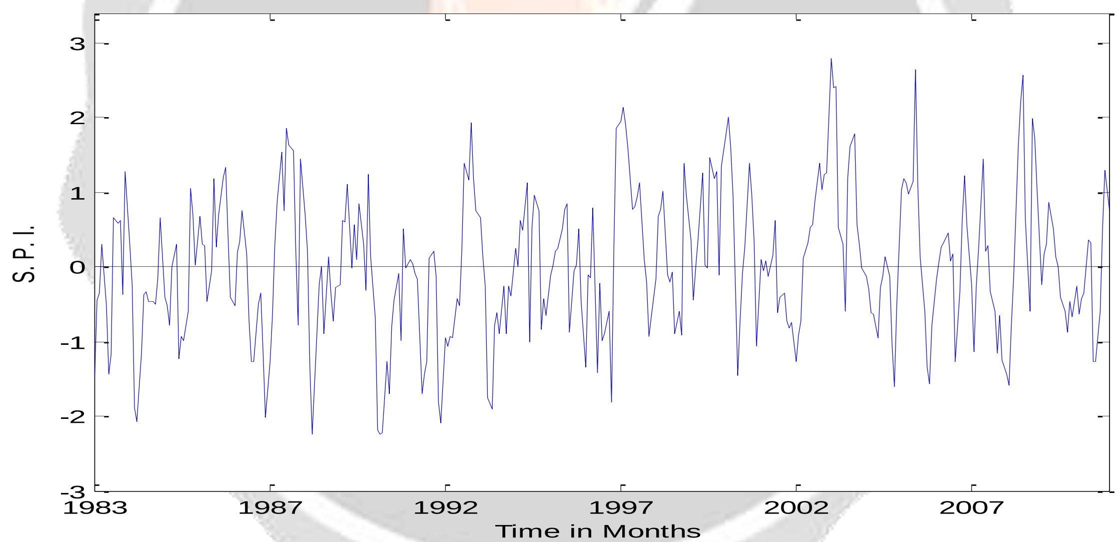

Fig-5: Trend and Time Series of SPI-3 at Toliara

If we place ourselves in the short term *(SPI-1)* , we see that the values of SPI vary enormously and are sometimes positive, sometimes negative. But under these conditions, each new rain measured has a considerable influence on the cumulative quarterly precipitation. In the medium and long term, it is obvious that each new rainy event has less impact on the cumulative totals and therefore the SPI values are less fluctuating.

Figure 7 illustrates the cumulative magnitude of negative SPI values based on the 1, 3, 6 and 12 month scale. The figure can be used as a guide for selecting the driest years and also to compare between different droughts. As the figure shows, several years (*like 1983 and 1992*) exposed to drought.

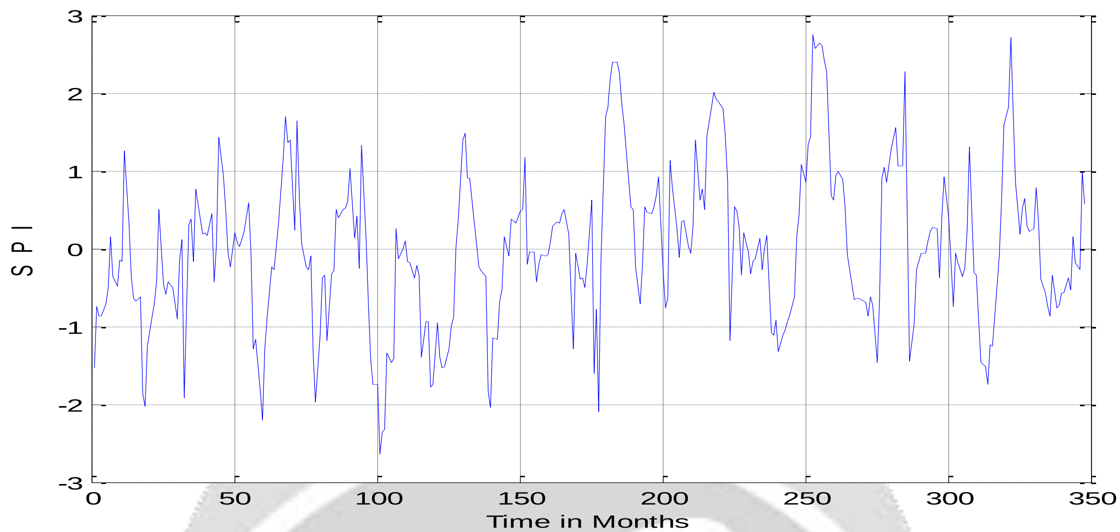

Fig-6: Trend and time series of SPI-6 at Toliara

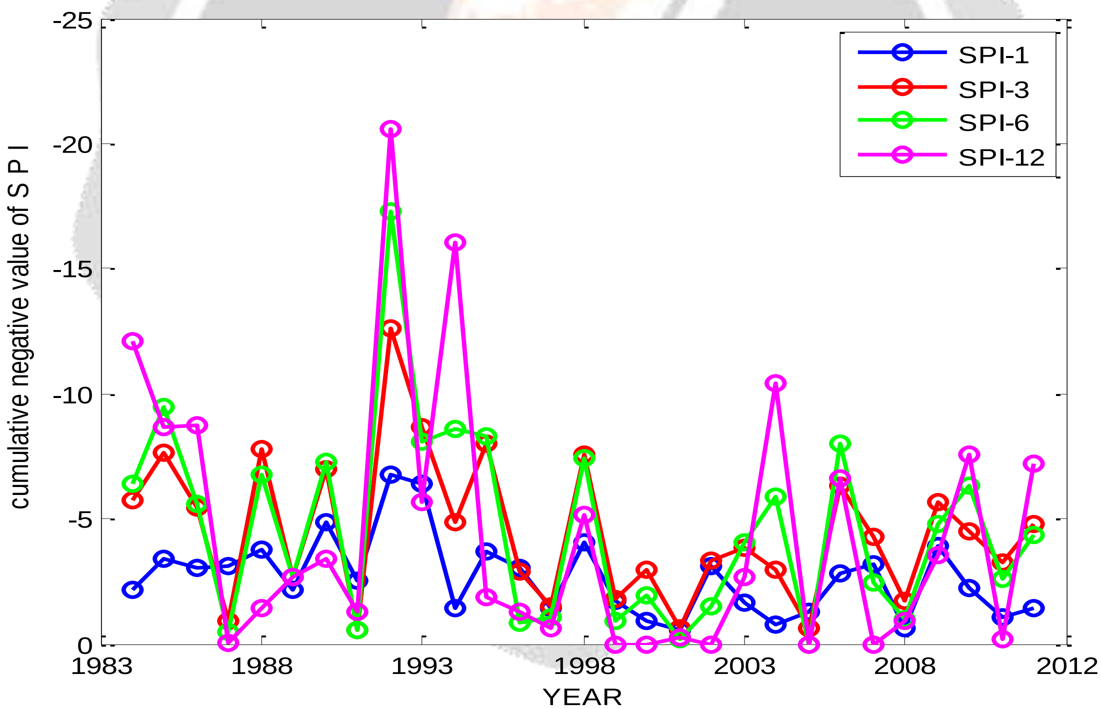

Fig-7: Cumulative negative value of SPI at different timescales (Toliara Station)

c. SPI index for a month January, February and March

Figures *8 to 10* characterize the value of SPI-1 for the months of January, February and March. The results show that drought occurred in summer although there is a considerable increase in winter precipitation. The most extremely drought event in southern Madagascar was during the winter.

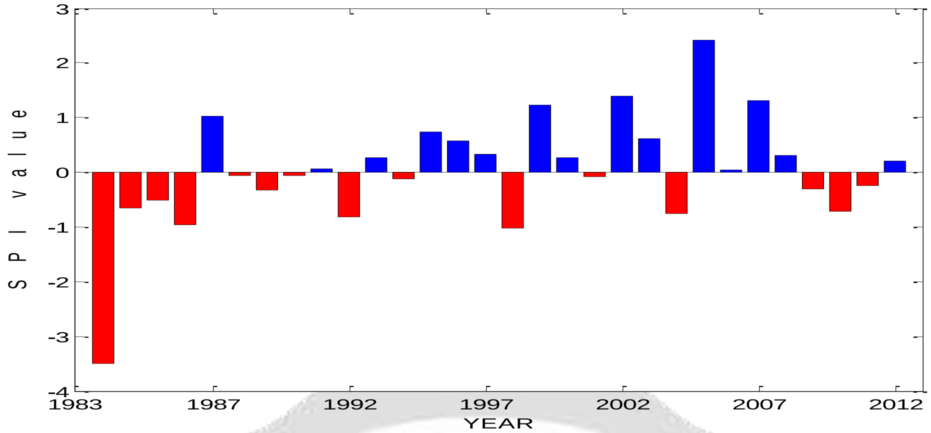

Fig-8: Values of the drought severity index on values of one month SPI _1_Janvier (Toliara Station)

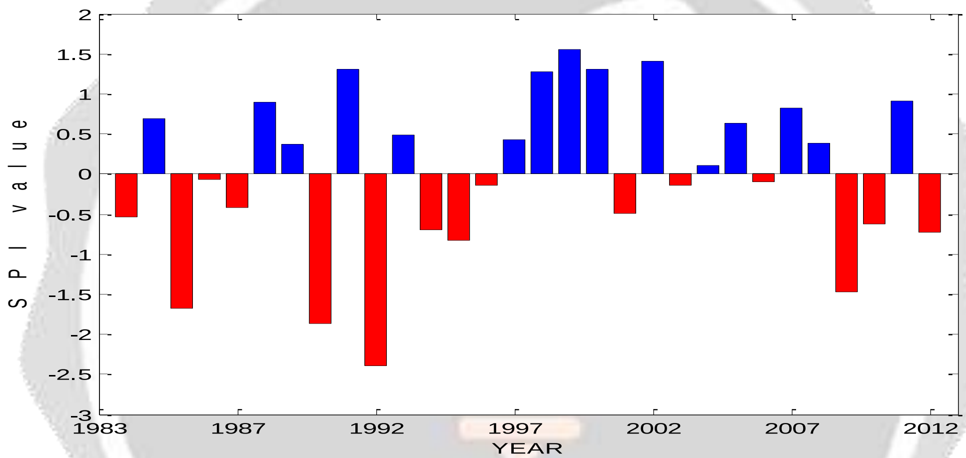

Fig-9: Values of the drought severity index on values of one month SPI _1_February (Toliara station)

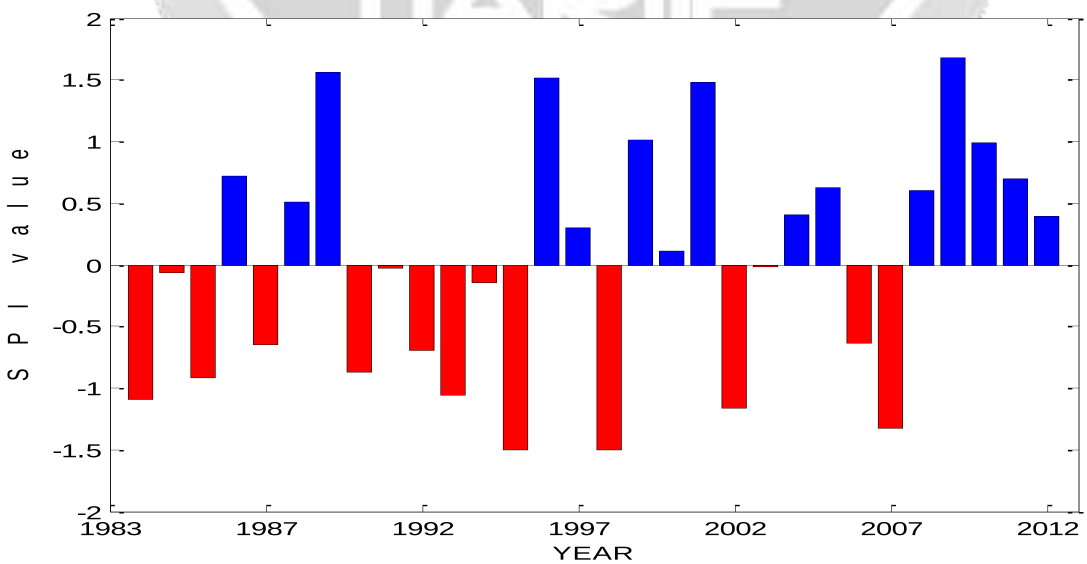

Fig-10: Values of the drought severity index on values of one month SPI _1_Mars (Toliara Station)

d. Probability of occurrence of droughts

We analyzed the presence of the different categories of drought on a 1, 3, 6 and 12 month time scale. The objective is to identify episodes of drought on a comparable time scale according to their frequencies of occurrence. The *Figure 11* shows the percentages of drought occurrence expressed in the time scale for each category of drought. Each percentage is obtained by reporting the drought occurring in each time scale and the total occurrence of drought on the same time scale. And we have found that the quality of drought in Toliara is close to normal.

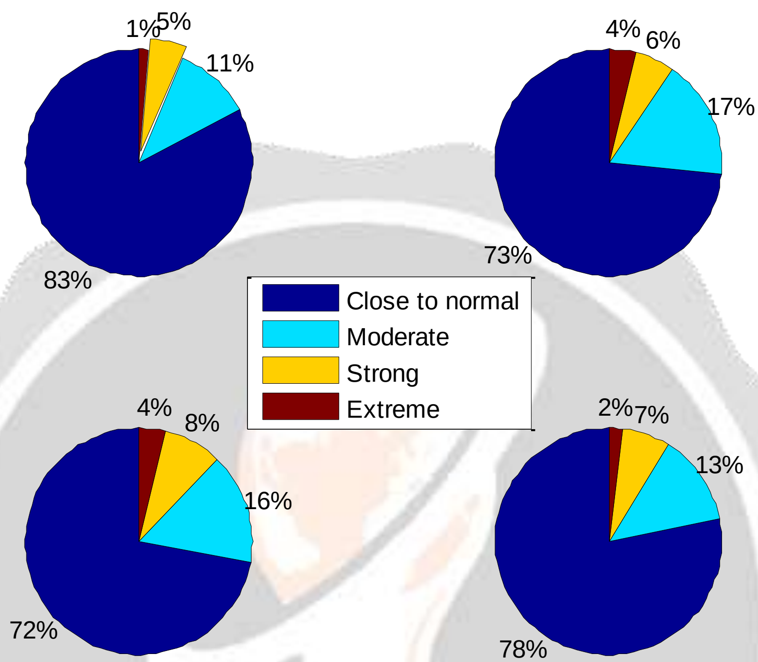

Fig-11: Probability of occurrence of drought in Toliara for each category on a different time scale. a) SPI-1, b) SPI-3, c) SPI-6 and d) SPI-12

3.2. **Analysis of the frequency variability of the severity of drought (wavelet)**

The *Figure 12* shows the massive extension of drought in Toliara. We observe that this figure indicates changes in the intensity of droughts.

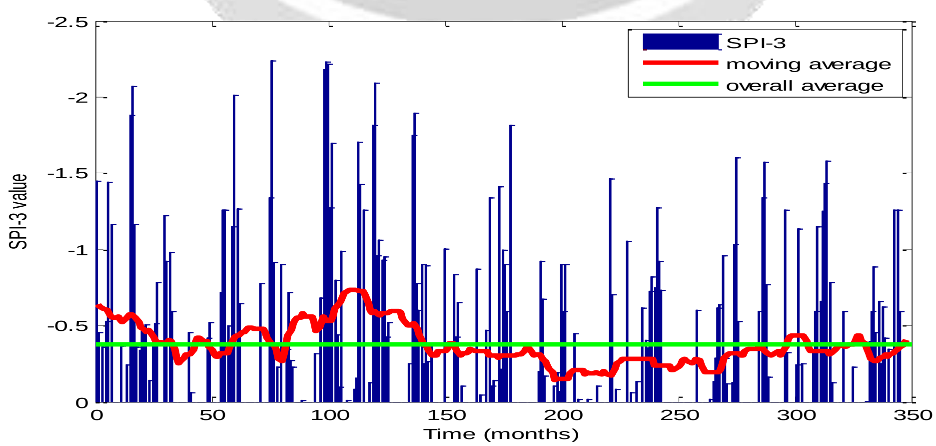

Fig-12: Time series of drought in Toliara from 1983-2012

Figure 12 is obtained from the SPI-3 index *(that is to say, negative values of SPI-3 are used to interpret as a drought phenomenon)* and this curve shows 51% of this index. Examination of this figure shows us the random variation of drought in Toliara. It is observed that drought decreasing trend, that is to say, the moving average is above the globalaverage in the period from January 1983 to August 1994 *(corresponding to 1 st to 144 th month of the period 'study)* and from the 148th month *(corresponds to December 1994)*, the moving average is below the overall average. This means that there is a decrease in the intensity of drought in Toliara.
The *Figure 13* shows the Fourier analysis of the time series of drought. We observe a maximum peak which gives us a pseudo-period of 18.95 months. This means that drought is repeated every 18.95 months in the general case.

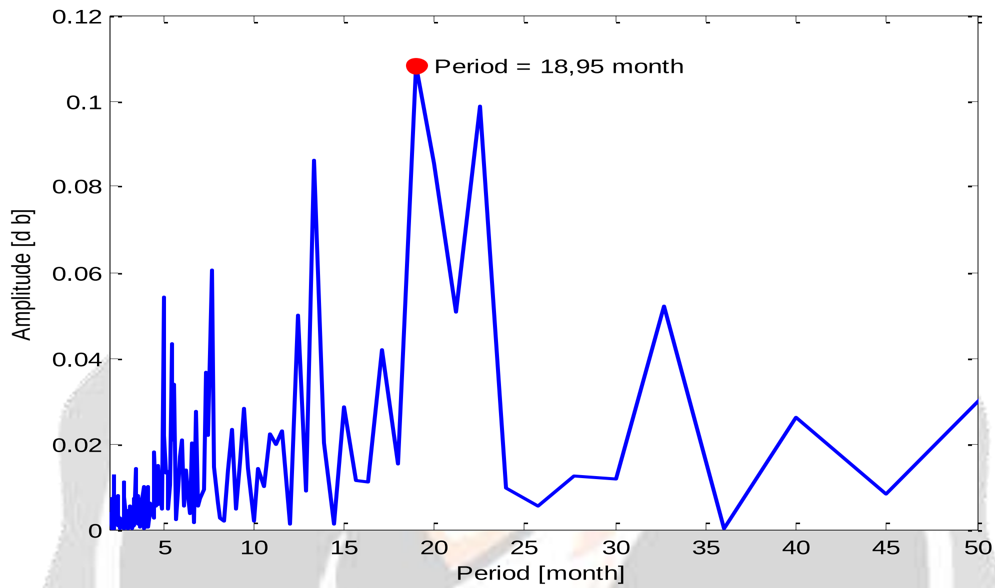

Fig-13: Spectrogram of the monthly drought chronicle

The Figure *14* shows the analysis by the maximum entropy method *(MEM).* This analysis consists of breaking down the drought signal into its different components. In addition, tests of statistical significance were carried out (at the 5% threshold, *curve in red)* on the cycles obtained.
This figure then shows the evolution of the spectral signatures of the drought in Toliara. And we mainly identify three peaks: the first is between 24 and 14 months, the second between 8 and 6 months and the third peak is between 5 and 4 months. These three periods mark different events that occur within the data. To determine which type of drought corresponds to these peaks, we used the results obtained by the Continuous Wavelet Transformation *(Figure 15).*

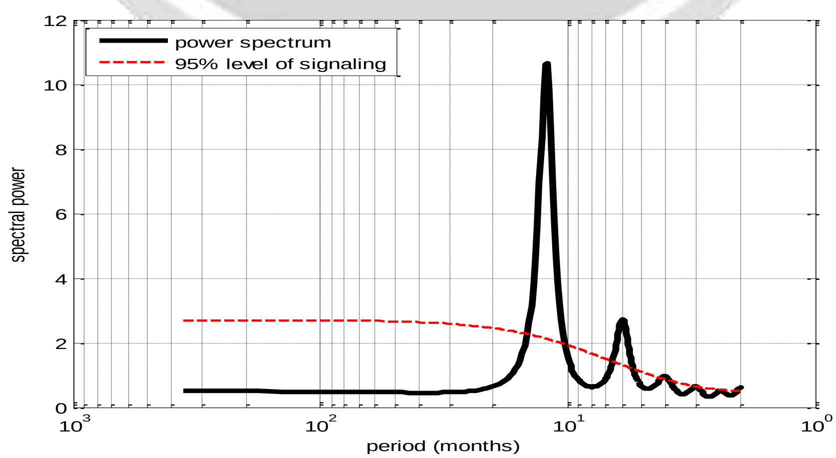

Fig-14:Power spectrum of drought

Next, the series are examined by Morlet's wavelet method. In Figure*15*, the various frequencies have been "mapped" over time, the statistical confidence threshold being 95%. The ordinate represents the periodicity (= *1 / f)* measured in months; the abscissa is the time axis *(from 1983 to 2012)* ; the colored gradients in the graph represent the power of the wavelets *(in dB)* , the power zones statistically significant are surrounded by a black line. The graph area with lighter colors is outside the "influence cone" and these values have no statistical significance.

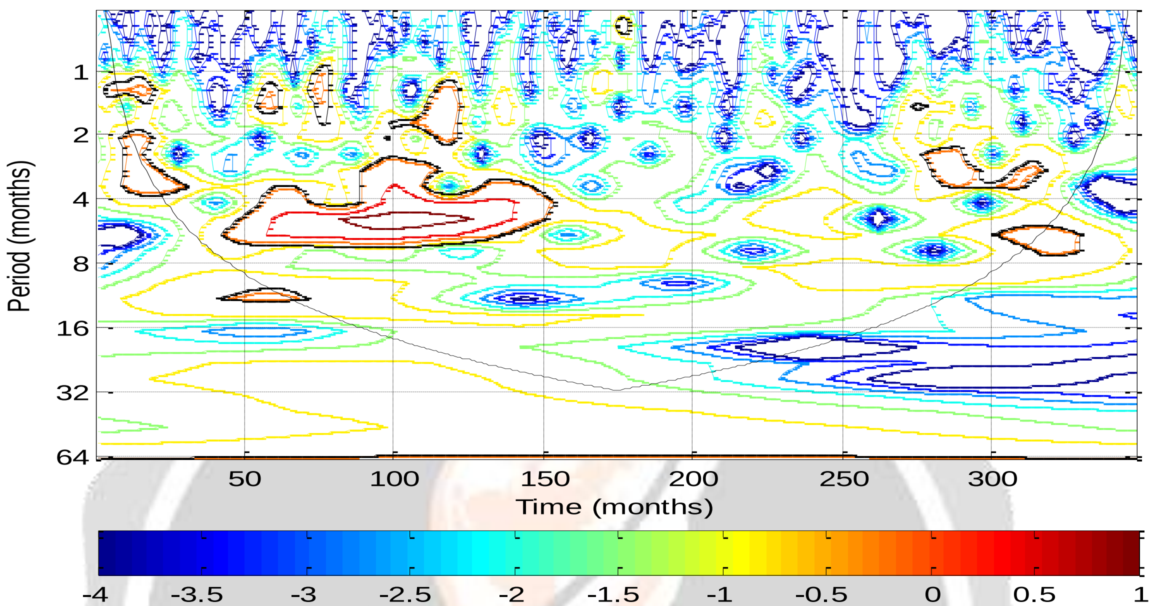

Fig-15: Wavelet decomposition of the drought in Toliara

We immediately observe the presence of two cycles inside the significant zone, the first cycle is from August 1986 to September 1995 and the second is started from February 2006 until May 2009.

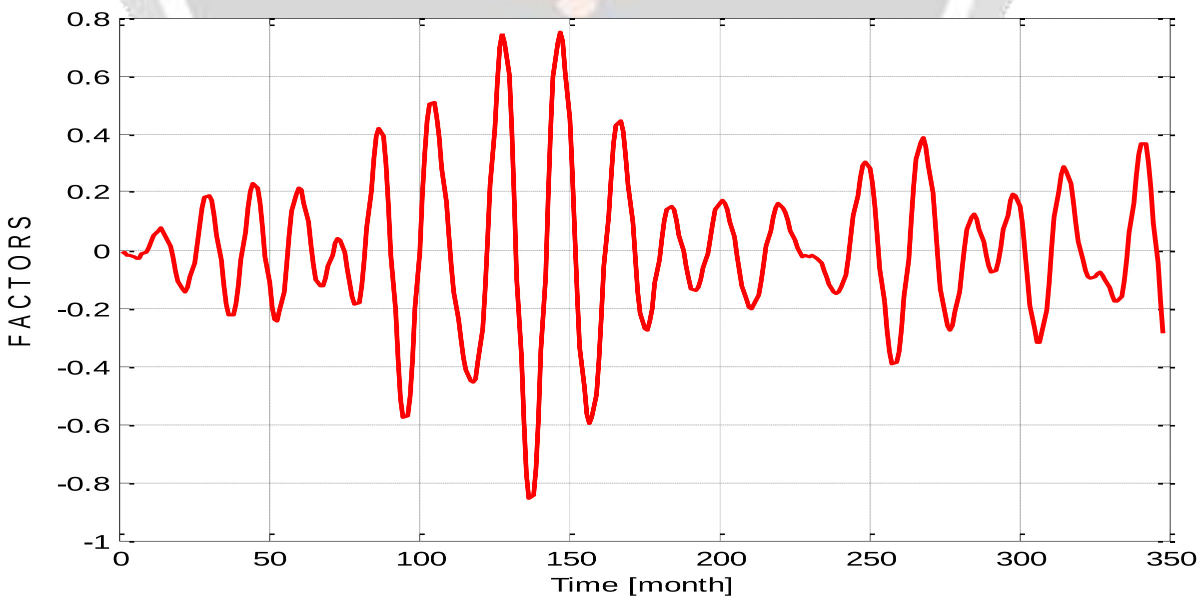

Fig-16: Wavelet coefficients for Drought

The figure *16* has allowed us to temporarily locate drought 24 - 14 months in Toliara. Pseudoperiodic oscillations are observed over time intervals. We have deduced that it is in these intervals that the 24 - 14 month drought appears.

4. CONCLUSION

Two approaches *(time and frequency analysis)* were used for analyzing the drought that is being produced in Toliara . The index of standardized precipitation shows that the frequency of occurrence of dry successive years is relatively high: about thirty years of study , from 1983 to 1996 experienced widespread drought in the stations of studied, making it the driest decade.
The rapid Fourier transformation allowed us to see that the drought time series shows a cycle of periodicity of 22.5 months. The maximum entropy method shows the peaks 26.91 to 14.58 months and 3.139 to 2.713 months as well as the anomaly in the SPI time series. Morlet's wavelet transformation enabled us to locate the period of onset of drought during the period from 1983 to 1995.
These studies constitute important information that has been used to describe and construct a mathematical modeling of the dynamics of this precipitation time series.

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