APPLICATIONS OF OPTIMIZATION FOR PRODUCTION COST PROBLEM IN BUSINESS

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ABSTRACT

In recent years, optimization methods have been widely and effectively applied in economics, engineering, transportation, information technology, and many other scientific disciplines. This article will systematize a real-life model that needs the help of Mathematics to solve production cost problems in business to clarify the relationship between Mathematics and practice.

Keyword: *Optimization, optimal solution, mathematical model.*

1. INTRODUCTION

Currently, the direction of scientific research to serve practice and solve problems arising in practice is of great interest. One of the most interested directions is the field of mathematical optimization [1-9]. When planning production and design based on optimization principles, it will save costs in terms of capital, raw materials, time and labor while increasing efficiency, productivity and quality of work. Therefore, the problem is to model real-world problems into optimization problems. Then, the results of the optimization problem will give us the most reasonable production plan in practice.

2. OPTIMIZATION PROBLEM

The general optimization problem is stated as follows:
Maximum (minimum) the function $f(x)$ with conditions:

$$g_i(x) \le b_i,\ i = \overline{1,m} \quad (1)$$

$$x \in X \subset R^n. \quad (2)$$

Inequalities in the system (1) can be replaced by equality or inequality in the opposite direction.
Then, the function $f(x)$ is called the objective function, the functions $g_i(x)$ are called the binding function. Each inequality (or equality) in the system (1) is called a binding.
Domain D = $\left\{x \in X \,\middle|\, g_i(x) \le b_i, i = \overline{1,m}\right\}$ is called the bound domain (or acceptable domain). Each $x \in D$ is called an alternative (or an acceptable solution). An option $x^* \in D$ is the maximum (or minimum) of the objective function, it means:

$$f(x^*) \ge f(x), \forall x \in D \text{ (with the problem of maximization)}$$

$$f(x^*) \le f(x), \forall x \in D \text{ (with the problem of minimization)}$$

is called the optimal solution. Then the value of $f(x^*)$ is called the optimal value of the problem. Solving an optimization problem is finding the optimal solution x^*.

Optimization problems (are known as mathematical programming) are divided into several categories: Linear programming (the objective function and the constraint functions are linear), nonlinear programming (the objective function and the constraint functions, at least one function is nonlinear), dynamical programming (Objects are considered as multi-stage processes) …. Optimization theory has given many methods to find the optimal solution depending on each problem. However, Linear programming is a problem that is fully studied in both theory and practice because: simple linear model to be able to apply, many other programming problems (original programming, nonlinear programming) can be approximated with high accuracy by a series of linear programming problems.

3. MATHEMATICAL MODELING PROBLEM

The modeling process of a real-world system consists of four steps:

Step 1: Build a qualitative model for the problem. In this step, we often state the model in words, in diagrams and give the conditions to be satisfied and the goals to be achieved.

Step 2: Describe the qualitative model through mathematical language. Specifically, it is necessary to determine the objective function (the most important) and express the conditions and constraints in the form of equations and inequalities.

Step 3: Use appropriate mathematical tools to solve the problem given in step 2. Sometimes, the actual problems are large, so when solving, it is necessary to program the algorithm in an appropriate programming language. appropriate, let the computer run and output the results.

Step 4: Analyse and verify the results in step 3, then consider whether to apply the results of the model in practice.

4. MODELS OF OPTIMIZATION FOR PRODUCTION COST PROBLEM IN BUSINESS

In reality, when producing a product, many factories always want to use the lowest cost of raw materials and labor but get the most profit. So they need to plan their production so that they can achieve this goal. For more clarity, we consider the following real problem:

A factory produces two types of products X and Y by the ingredients A, B, C. Know that:

- To produce one unit of product X have to use b_1 unit of the ingredient A and c_1 unit of the ingredient B and d_1 unit of the ingredient C.
- To produce one unit of product Y have to use b_2 unit of the ingredient A, c_2 unit of the ingredient B and d_2 unit of the ingredient C.

They already have a stockpile of ingredients A, B, C respectively *b*, *c* and *d*. Know that: the profit of one unit of product X is a_1 million, the profit of one unit of product Y is a_2 million. Make a plan so that with those prepared ingredients, the factory will earn the most profit.

Put x_1, x_2 are respectively the quantities of X and Y that the factory will produce. Then the mathematical model of the problem has the form: $f(x_1, x_2) = a_1x_1 + a_2x_2 \to \max$

with material constraints: $b_1x_1 + b_2x_2 \le b$; $c_1x_1 + c_2x_2 \le c$; $d_1x_1 + d_2x_2 \le d$

and the sign constraints of the variable: $x_1 \ge 0; x_2 \ge 0$.

This is a linear programming problem. Solve this optimization problem, we can determine the optimal solution is (x_1, x_2), which in turn is the optimal solution for the number of products X and Y that the factory needs to produce in order to maximize the profit.

5. CONCLUSIONS

The paper presents a method application of the optimization problem for production cost problems in business. This further elucidates the two-way intimate relationship between mathematics and practice and the important role of mathematics in practice. In the future, the author's follow-up studies will carry out research on optimization for the processes occurring in the engineering process.

6. ACKNOWLEDGEMENT

This work is supported by Thai Nguyen University of Technology, Vietnam

7. REFERENCES

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