



A BRIEF ANALYSIS ABOUT THE DIFFERENCE BETWEEN GEOMETRIC AND ARITHMETIC

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ABSTRACT

*In mathematics, the difference between an arithmetic and geometric sequence is important because these two types of numbers follow a strict pattern. A geometric sequence is a list of terms that differ from its predecessor by a fixed factor or quantity. In general, you can use arithmetic as your primary source of math, but it's also useful for construction. This article will clarify the differences between arithmetic and geometric sequences and how to differentiate one from the other. The **first difference** between arithmetic and geometric sequences lies in the definition of each. Arithmetic series is defined by a constant value, while a geometric sequence is a series that is defined by a constant number multiplied or divided by a previous term. The two types of sequences have their own distinctive characteristics, but they share some characteristics. A geometric sequence is much more difficult to grasp, and can be confusing to learn. Generally, arithmetic is the basis of all mathematics, which is why we can say that the difference between arithmetic and geometric is so fundamental. The basic idea of the two is that they share the same fundamental idea, but they are not the same. The two are similar, but they differ in their uses and principles. The former is easy to calculate, while the latter is difficult. Generally, a geometric sequence is a set of sequential numbers that have the same number. The difference between these two types of sequences is the fact that arithmetic and geometric sequences have different methods for calculating the same value. The mathematical formulas for arithmetic and geometric series are similar but have their own advantages and disadvantages. In general, the two methods are different but both are useful.*

KEYWORDS;- MATHEMATICS, ARTHAMITICS, GEOMATRIC, FORMULA, SEQUENCES, RATIO.

INTRODUCTION

Arithmetic is a mathematical operation that deals with numerical systems and related operations. It's used to get a single, definite value. The word "**Arithmetic**" comes from the Greek word "**arithmos**," which meaning "numbers." It is a field of mathematics that focuses on the study of numbers and the properties of common operations such as addition, subtraction, multiplication, and division.

A sequence is a collection of items in a specific order (typically numbers). Arithmetic and geometric sequences are the two most popular types of mathematical sequences. Each consecutive pair of terms in an arithmetic sequence has a constant difference. A geometric sequence, on the other hand, has a fixed ratio between each pair of consecutive terms.

When dealing with number sequences, **arithmetic** and **geometric** return values are very similar. However, geometric and arithmetic series differ in the type of progression they use. For instance, a geometric sequence is a list of numbers whose amount changes over time while an arithmetic one always has a fixed number. In other words, the common difference between arithmetic return values is the constant change in one term and the definite change in the next.

In contrast, an arithmetic sequence is characterized by a constant common difference between successive terms, whereas a geometric sequence consists of stable common ratios among successive values. Both types of sequences cannot be arithmetic or geometric; however, they can be both arithmetic and mathematical. To distinguish the two, an arithmetic sequence will be the first term of a geometric series, while a geometric one will be the last.

Another major difference between arithmetic and geometric means is how they are calculated. An arithmetic sequence consists of a list of consecutive numbers, while a geometric sequence consists of a fixed ratio. The arithmetic sequence consists of adding or subtracting a fixed value from the preceding term. A geometric series can be used to estimate returns on investments or budgets.

What is the Difference between Geometric and Arithmetic Series?

A geometric sequence consists of consecutive terms in the same constant ratio. A geometric series is made up of a list of terms in which each term is different from the previous one by a certain factor or quantity. A common factor in a geometric sequence is the number of terms between the first and last term. Then, the new term is obtained by adding or subtracting the previous one. Arithmetic progression is a linear series.

Geometric series contains consecutive terms with the same ratio. Arithmetic sequences, on the other hand, consist of a list of numbers. In addition, a geometric sequence consists of a list of numbers in a given order. It is based on the quotient of the first term. The arithmetic sequence is composed of two different sets. The arithmetic series is a list of the first term in a string.

A geometric sequence is not arithmetic. It follows a pattern and has a fixed quotient. A geometric sequence, on the other hand, fails to have a quotient. This is a sign that arithmetic is superior to arithmetic. Once you know the difference between the two, you can begin to make better decisions in math and improve your performance.

A geometric sequence is a sequence in which successive terms are different from each other. For example, a basketball or football bounces at a lower height than it does when it is added to the same digits. By contrast, a geometric sequence is a list of arithmetic. The difference between the two is the ratio of the first term to the second.

Geometric and arithmetic mean are two different kinds of mathematical sequences

The differences between geometric and arithmetic sequences can make math more difficult. For example, arithmetic and geometric series differ in how they treat the relationship between consecutive terms. The ratio between consecutive terms of a geometric sequence is always higher than that of an arithmetic series. Similarly, arithmetic and geometry are often used interchangeably. They are both important in math, but they have their own unique advantages and disadvantages.

In math, the geometric and arithmetic mean are two different kinds of mathematical sequences. Arithmetic is defined by the difference between successive terms, whereas a geometric sequence is a collection of integers. In addition, the arithmetic series involves compounding, while the geometric one is more flexible. In this way, it is easier to calculate the arithmetic mean.

Generally, arithmetic is the basis of all mathematics, which is why we can say that the difference between arithmetic and geometric is so fundamental. The basic idea of the two is that they share the same fundamental idea, but they are not the same. The two are similar, but they differ in their uses and principles. The former is easy to calculate, while the latter is difficult.

What is the Difference between Arithmetic and Geometric Sequences?

In mathematics, the difference between an arithmetic and geometric sequence is important because these two types of numbers follow a strict pattern. A geometric sequence is a list of terms that differ from its predecessor by a fixed factor or quantity. In general, you can use arithmetic as your primary source of math, but it's also useful for construction. This article will clarify the differences between arithmetic and geometric sequences and how to differentiate one from the other.

The **first difference** between arithmetic and geometric sequences lies in the definition of each. Arithmetic series is defined by a constant value, while a geometric sequence is a series that is defined by a constant number multiplied or divided by a previous term. The two types of sequences have their own distinctive characteristics, but they share some characteristics. A geometric sequence is much more difficult to grasp, and can be confusing to learn.

Mathematical formulas

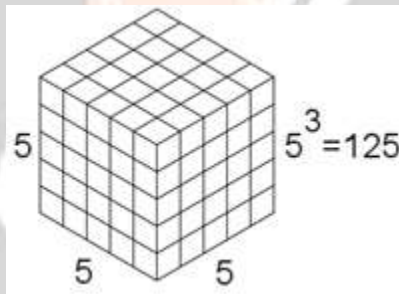
A geometric sequence, on the other hand, has a fixed ratio between its successive terms. Each term in an arithmetic sequence is either a multiple or a subtraction of the preceding term. It is much easier to memorize an arithmetic sequence, while a geometric series requires a formula. The common difference between the two types of sequences is that the arithmetic one is a linear and a graphical one is an exponential sequence.

The difference between arithmetic and geometric sequences is usually referred to as a “common difference” or “common ratio.” In mathematics, an arithmetic series always has the same value between its successive terms, whereas a geometric series always has a fixed ratio between its terms. This is a very important distinction because it affects the interpretation of an arithmetic sequence and can have consequences on algebraic problems.

In addition to the difference in numbers, arithmetic and geometric sequences are also used in financial analysis. An arithmetic sequence consists of two consecutive terms that are multiplied by a constant. The arithmetic sequence is composed of integers, while a geometric one has an element of positive and negative signs. If you want to compare arithmetic and geometric sequences, make sure to use the most appropriate arithmetic sequence.

Generally, a geometric sequence is a set of sequential numbers that have the same number. The difference between these two types of sequences is the fact that arithmetic and geometric sequences have different methods for calculating the same value. The mathematical formulas for arithmetic and geometric series are similar but have their own advantages and disadvantages. In general, the two methods are different but both are useful.

Difference between Arithmetic and Geometry



Arithmetic vs Geometry

Man has always sought to understand his world. Sometimes he does this through stories. Other times he turns to religion. Then there are times when he needs to quantify, count, or otherwise definitively explain the world around him. In these instances he will turn to mathematics, and more specifically arithmetic and geometric functions.

Definition of Arithmetic and Geometry

Arithmetic “is the most elementary division of mathematic. It involved computation with numbers.

Geometric refers to the branch of mathematics that describes the properties of bodies in space. This can refer to points, planes, lines, angles, and figures.

A Brief History of Arithmetic and Geometric Mathematics

Arithmetic there is evidence of arithmetic being used by very early man. The **Ishango bone** of central Africa is nearly 22,000 years old and shows evidence that pre-historic man knew the basics of addition and subtraction. Later, Babylonians, Egyptians, Greeks, and Indian all developed systems of arithmetic. The Indian system with its eventual Arabic numerals prevailed because it incorporates the concept of zero and place value.

Geometric evidence of the early use of geometric ideas is seen in societies that needed to do a lot of large-scale building: the Indus Valley, the Egyptians, and the Mesopotamians. Almost 3,000 years later, Euclid codified all these forms of early geometry in nine different volumes. Arabs kept the geometric tradition alive during the Dark Ages, and it was reintroduced to Europe during the Renaissance. There is was expanded upon and led to the develop of calculus.

Uses of Arithmetic and Geometry

Arithmetic is the basis of all other math. It is essentially adding, subtracting, multiplying, and dividing. It also refers to the upper level concept of number theory, which is the study of the property of integers. Arithmetic is used in geometry, algebra, and calculus as well as everyday functions such as balancing a check book, figuring out the tip at a restaurant, budget planning, expanding recipes, and much, more

Geometric governs the principles behind figures and lines. It figures heavily in architecture and construction. It is also used for navigation and surveying. As a for mentioned, geometric principles led to the development of calculus.

Examples of Arithmetic and Geometric Equations

Arithmetic $2+2=4$, $5-3=2$, $1009 \times 36=36,324$, $144 \div 12=12$

Geometric What is the area of a circle? Plot the following coordinates on a plane. Find the sine and cosine of this line.

Summary:

1. **Arithmetic** and geometric mathematics help man quantitatively explain his world.
2. **Arithmetic** deal with the simple manipulation of numbers whereas geometric mathematics describes figures, lines, and planes.
3. **Arithmetic** is the foundation of all other math and is used extensively in our daily lives, and while geometric mathematics is also widely used in construction, it is not as pervasive.

Relation between AM, GM and HM can be derived with the basic knowledge of progressions or Mathematical sequences. An array or collection of objects in a specified pattern in Mathematics is called a Mathematical Sequence. A sequence is also referred to as a progression. The three most popular types of sequences are Arithmetic sequence, geometric sequence and harmonic sequence. An arithmetic sequence is a pattern of numbers in which the difference between consecutive terms of the sequence remains constant throughout the sequence. A geometric progression is a sequence of numbers in which any two consecutive terms of the sequence have a common ratio. Harmonic progression is the sequence that forms an arithmetic sequence when the reciprocal of terms are taken in order.

AM (Arithmetic Mean), GM (Geometric Mean) and HM (Harmonic Mean) are the most commonly used measure of central tendency. In Mathematics, when we learn about sequences, we also come across the relation between AM, GM and HM, where AM stands for Arithmetic Mean, GM stands for Geometric Mean, and HM stands for Harmonic Mean. The mean for any set is referred to as the average of the set of values present in that set. It is used to calculate growth rate and risk factors in finance, to calculate the rate of cell growth by division in biology, and to solve linear transformations.

AM, GM, HM stands for Arithmetic mean, Geometric mean and Harmonic mean respectively.

- AM or Arithmetic Mean is the mean or average of the set of numbers which is computed by adding all the terms in the set of numbers and dividing the sum by a total number of terms.
- GM or Geometric Mean is the mean value or the central term in the set of numbers in geometric progression. The geometric mean of a geometric sequence with 'n' terms is computed as the nth root of the product of all the terms in sequence taken together.
- HM or Harmonic mean is one of the types of determining the average. The harmonic mean is computed by dividing the number of values in the sequence by the sum of reciprocals of the terms in the sequence.

AM, GM, HM Formula

Consider a sequence of 'n' terms as $\{a_1, a_2, a_3, a_4, \dots, A_n\}$.

Case 1: If the above sequence is in arithmetic progression, the mean of this sequence is calculated as Arithmetic Mean using the formula.

$$AM = (a_1 + a_2 + a_3 + a_4 + \dots + a_n) / n$$

Case 2: If the given sequence is a geometric progression, the geometric mean of all the terms in the sequence is calculated using the formula.

$$GM = \sqrt[n]{a_1 \times a_2 \times a_3 \times a_4 \times \dots \times a_n}$$

Case 3: If the sequence is in harmonic progression, the harmonic mean is computed by using the formula.

$$HM = \frac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots + \frac{1}{n}}$$

Relation Between AM, GM and HM:

Consider two numbers 'a' and 'b' such that a and b are greater than 0. Terms in the sequence are 'a' and 'b' and the number of terms in the sequence 'n = 2'. If AM GM HM formula is used, AM GM HM can be found as follows.

$$AM = \frac{a+b}{2}$$

$$GM = \sqrt{ab}$$

$$HM = \frac{2ab}{a+b}$$

The above equation gives the relation between AM, GM and HM. The equation can also be written as

$$AM \times HM = GM^2$$

or

$$GM = \sqrt{AM \times HM}$$

- AM GM HM in Statistics has a vital role to play in major calculations.
- Arithmetic Mean being very simple and easy to compute gives one of the measures of the central tendency of a grouped or ungrouped set of data.
- Geometric mean is used in computation of stock indexes. Also geometric mean is used to calculate the annual returns of the portfolio. Geometric mean is also used in studying biological processes such as cell division and bacterial growth etc.
- Harmonic mean is used to determine the price earnings ratio and other average multiples in Finance. It is also used in the computation of Fibonacci sequence.

Example Problems:

1. If five times the geometric mean of two numbers 'a' and 'b' is equal to the arithmetic mean of those two numbers such that $a > b > 0$, then compute the value of $a + b - b$.

Solution:

Arithmetic mean of the two numbers is calculated as

$$AM = \frac{a+b}{2}$$

Geometric mean of the two numbers a and b is

$$GM = \sqrt{ab}$$

It is given in the question that Arithmetic mean = 5 times the geometric mean

$$AM = 5 GM$$

$$\frac{a+b}{2} = 5\sqrt{ab}$$

$$a + b = 10 \sqrt{ab}$$

$$(a + b)^2 = 100 ab$$

$$(a - b)^2 = (a + b)^2 - 4 ab$$

$$(a - b)^2 = 100 ab - 4 ab$$

$$(a - b)^2 = 96 ab$$

$$a - b = \sqrt{96 ab}$$

$$a + b - b = (\sqrt{100 ab} - \sqrt{96 ab}) = (25 - 24)\sqrt{ab} = 1.021$$

2. Find the harmonic mean of two numbers a and b , if their arithmetic mean is 16 and geometric mean is 8 provided that $a > b > 0$. (Hint: Use relation between AM GM HM formula).

Solution:

Given: AM = 16 and GM = 8

The relation between AM GM HM is given as:

$$AM \times HM = GM^2$$

$$16 \times HM = 8^2$$

$$16 \times HM = 64$$

$$HM = \frac{64}{16} = 4$$

To find the numbers:

Arithmetic mean is given as

$$AM = \frac{(a+b)}{2}$$

$$16 = \frac{(a+b)}{2}$$

$$a + b = 32$$

$$a = 32 - b$$

Geometric mean is given as

$$GM = \sqrt{ab}$$

$$8 = \sqrt{ab}$$

$$64 = ab$$

$$64 = (32 - b)b$$

$$64 = 32b - b^2$$

$$b^2 - 32b + 64 = 0$$

Fun Facts

- It is inferred through a number of calculations and has been proved by experts who use AM GM HM in Statistics that the value of AM is greater than that of GM and HM. The value of GM is greater than that of HM and lesser than that of AM. The value of HM is lesser than that of AM and GM.

$$AM = \frac{(a+b)}{2}$$

$$GM = \sqrt{ab}$$

$$HM = \frac{2ab}{a+b}$$

- If zero is one of the terms of a sequence, its geometric mean is zero and the harmonic mean is infinity.

Arithmetic Sequence

If the difference between any two consecutive terms is always the same, a sequence of integers is termed an Arithmetic Sequence. Simply put, it indicates that the next number in the series is calculated by multiplying the preceding number by a set integer. Further, an Arithmetic Sequence can be written as,

$$a, a + d, a + 2d, a + 3d, a + 4d$$

where a = the first term

d = common difference between terms.

For example, in the following sequence: 5, 11, 17, 23, 29, 35, ..., the constant difference is 6.

Geometric Sequence

If the ratio of any two consecutive terms is always the same, a sequence of numbers is called a Geometric Sequence. Simply put, it means that the next number in the series is calculated by multiplying a set number by the preceding number. Further, a Geometric Sequence can be expressed as:

$$a, ar, ar^2, ar^3, ar^4 \dots$$

where a = first term

d = common difference between terms.

For instance, 2, 6, 18, 54, 162,... The constant multiplier is 3 in this case.

To tell the difference between arithmetic and geometric sequence, the following points are important,

- An arithmetic Sequence is a set of numbers in which each new phrase differs from the previous term by a fixed amount. Geometric Sequence is a series of integers in which each element after the first is obtained by multiplying the preceding number by a constant factor.
- When there is a common difference between subsequent terms, represented as 'd,' a series can be arithmetic. The sequence is said to be geometric when there is a common ratio between succeeding terms, indicated by 'r.'
- The new term in an arithmetic sequence is obtained by adding or subtracting a fixed value from the previous term. In contrast to geometric sequence, the new term is found by multiplying or dividing a fixed value from the previous term.
- The variation between the members of an arithmetic sequence is linear. In contrast, the variation in the sequence's elements is exponential.
- Infinite arithmetic sequences diverge, while infinite geometric sequences converge or diverge, depending on the situation.

CONCLUSION

Difference between an arithmetic sequence and a geometric sequence

S.No.	Arithmetic sequence	Geometric sequence
1	Arithmetic Sequence is a set of numbers in which each new phrase differs from the previous term by a fixed amount.	A geometric sequence is a collection of integers in which each subsequent element is created by multiplying the previous number by a constant factor.
2	Between successive words, there is a common difference.	Between successive words, they have the same common ratio.
3	Subtraction or addition are used to get terms.	Division or Multiplication are used to get terms.
4	Example: 5, 11, 17, 23, 29, 35,...	Example: 2, 6, 18, 54, 162,...

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