# A STUDY OF MALARIA TRANSMISSION DYNAMICS BY MATHEMATICAL MODELLING

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## ABSTRACT

*In this paper we are presenting a study of Malaria Transmission Dynamics by Mathematical Modelling. The model is an autonomous system, constructed by considering two models: a model of vector population and a model of virus transmission. Therefore, given that partially immune individuals remain slightly infectious to mosquitoes, a similar work has been done in this paper for malaria global transmission dynamics following the SIRS-SI pattern. The original reproduction ratio has been calculated using the next generation matrix method. These incidence rates produce antibodies in response to the presence of malaria caused by parasites in both human and mosquito populations. The existence of an area where model epidemiology is possible is established. Stability analysis of the disease-free equilibrium is performed through a threshold parameter (reproduction number R0) obtained using the next generation matrix technique.*

**Keyword: -** *Malaria, Transmission, Time, Dynamics, Modelling etc.*

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## 1. INTRODUCTION

Mathematical and theoretical biology are the interdisciplinary scientific research fields with a range of their applications. The branch is sometimes called "Mathematical Biology" or "Biomathematics[1]" to stress the mathematical side. It may also called as "Theoretical Biology" to stress the biological side. The "Theoretical Biology" use to focus more on the development of theoretical principles for biology. "Mathematical Biology" focuses on the use of mathematical tools to study biological systems.

*"The two terms Mathematical Biology and*
*Theoretical Biology are sometimes interchanged,*
*the concept remains one and the same".*

The disease, malaria, which is one of the most prevalent and fatal human infections worldwide, is caused by infection with a single-celled (protozoan) parasite of the genus Plasmodium and causes chills, fever, headaches, pain, and vomiting is characterized by paroxysms. Transformation Parasites reach humans by the bite of the female anophilius mosquito (vectors). [1]Chasnov the five parasitic species (Plasmodium falciparum, Plasmodium vivix, Plasmodium ovale, Plasmodium malariae and Plasmodium nolaceae) that cause malaria in humans, [1] Plasmodium falciparum is the most deadly form and prevails in Africa [2] The largest number of deaths in the parasitic tropics And is responsible for clinical cases. Its infection can cause serious diseases affecting the brain, lungs, spine, and other organs [4].

Mathematical biology argued that population growth would be "geometric" while resources (the carrying capacity of the environment) could only increase arithmetic (Mallet James, 2001). A founding text by D'Arcy Thompson (Ian Stewart, 1998) is considered "On Growth and Form" (1917), and other early pioneers include Ronald Fisher, Hans Leo Prizibram, Nicholas Shevsky and Vito Voltra (Evelyn Fox Keller) Are included. 2002).

1960 is the year that shows the rapid development of "interest in bio-mathematics". Following are some possible reasons for the rapid development of "interest in bio-mathematics":

• Genomics, the newly launched field, created a rich collection of data in the form of "information".

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[1]Chasnov, R Jeffrey,(2016), "Mathematical Biology",Hong Kong

## 2. MALARIA

Malaria is a widely spread infectious disease, which is also one of the most prevalent disease in tropical and subtropical areas.in addition to being widespread, malaria is also known as a massive killer because annually 300 million to 500 million infections are reported, in which most of the deaths are either children under five are pregnant children.

Malaria is a mosquito-borne infectious disease that affects humans and other animals. Malaria causes symptoms that typically include fever, tiredness, vomiting, and headaches. In severe cases it can cause yellow skin, seizures, coma, or death.

**Spread**

The plasmodium parasite is spread by female Anopheles mosquitoes, which are known as “night biting” mosquitoes because they mostly commonly bite between desk and dawn.

If a mosquito bites a person already infected with malaria, it can also become infected and spread the parasite on to other people. How-ever malaria cannot be spread directly from person to person.

Once we are bitten, the parasite enters the blood stream and travels to the liver the infection develops in the liver before re-entering the blood stream and invading the red blood cells.

The parasite grow and multiply in the red blood cells. At regular intervals, the infected blood cells burst, releasing more parasite into the blood. Infected blood cells usually burst every 48-72 hours. Each time they burst, you will have a bout of fever ,chills and sweating.

**Causes:**Malaria is caused by Plasmodium parasites. The parasites are spread to people through the bites of infected female Anopheles mosquitoes, called "malaria vectors." There are 5 parasite species that cause malaria in humans, and 2 of these species – P. falciparum and P. vivax – pose the greatest threat.

There are four types of human malaria: Plasmodium vivax, P. malaria, P. ovale and P. falciparum. P. vivax and P. falciparum are the most common forms. Falciparum malaria—the most deadly type—is most common in sub-Saharan Africa, where it causes more than 400 000 deaths a year.

**Symptoms**

Malaria is an acute febrile illness. In a non-immune individual, symptoms usually appear 10–15 days after the infective mosquito bite. The first symptoms – fever, headache, and chills – may be mild and difficult to recognize as malaria. If not treated within 24 hours, P. falciparum malaria can progress to severe illness, often leading to death.

Children with severe malaria frequently develop one or more of the following symptoms: severe anaemia, respiratory distress in relation to metabolic acidosis, or cerebral malaria. In adults, multi-organ failure is also frequent. In malaria endemic areas, people may develop partial immunity, allowing asymptomatic infections to occur.

**Transmission**

In most cases, malaria is transmitted through the bites of female Anopheles mosquitoes. There are more than 400 different species of Anopheles mosquito; around 30 are malaria vectors of major importance. All of the important vector species bite between dusk and dawn. The intensity of transmission depends on factors related to the parasite, the vector, the human host, and the environment.

Anopheles mosquitoes lay their eggs in water, which hatch into larvae, eventually emerging as adult mosquitoes. The female mosquitoes seek a blood meal to nurture their eggs. Each species of Anopheles mosquito has its own preferred aquatic habitat; for example, some prefer small, shallow collections of fresh water, such as puddles and hoof prints, which are abundant during the rainy season in tropical countries.

Transmission is more intense in places where the mosquito lifespan is longer (so that the parasite has time to complete its development inside the mosquito) and where it prefers to bite humans rather than other animals. The long lifespan and strong human-biting habit of the African vector species is the main reason why approximately 90% of the world's malaria cases are in Africa.

Transmission also depends on climatic conditions that may affect the number and survival of mosquitoes, such as rainfall patterns, temperature and humidity. In many places, transmission is seasonal, with the peak during and just after the rainy season. Malaria epidemics can occur when climate and other conditions suddenly favour transmission in areas where people have little or no immunity to malaria. They can also occur when people with low immunity move into areas with intense malaria transmission, for instance to find work, or as refugees.

Human immunity is another important factor, especially among adults in areas of moderate or intense transmission conditions. Partial immunity is developed over years of exposure, and while it never provides complete protection, it does reduce the risk that malaria infection will cause severe disease. For this reason, most malaria deaths in Africa occur in young children, whereas in areas with less transmission and low immunity, all age groups are at risk.

## 3. MATHEMATICAL MODEL

Mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modelling. Mathematical models are used in the natural sciences (such as physics, biology, earth Science, chemistry) and engineering disciplines(suchas computer science, electrical sciences and engineering disciplines) as well as in the social sciences (such as economics, physiology, sociology, political science).

## 4. MATHEMATICAL MODELLING

Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application.

Models exactly represent the real problem situations. Models help managers to take decisions faster and more accurately. They typically offer convenience and cost advantages over other means of obtaining the required information on reality. Large and complex problems can be solved with ease.

**Theorem 4.1** Assume that if Ro>1 there exists η>0 such that for any initial function Φ=$(\phi_1, \phi_2, \phi_3, \phi_4)\epsilon X_+^0$, the corresponding solution $(s_1(t), I_1(t), s_2(t), I_2(t))$satisfies

(i) $\frac{d_1}{ae_{1m+d_1}} \leq s_{2\infty}\, and\, \frac{2}{ae_{2m+d_2}} \leq s_{2\infty};$

(ii) $\eta \leq I_{1\infty}$ and $\eta \leq I_{2\infty}$.

$\boldsymbol{proof}$: since $0 \leq I_1(t) \leq 1 for\ t \geq \tau_2\ and\ 0 \leq I_2(t) \leq 1 for\ all\ t \geq -\tau_1, the\ s_1'$ and $s_2'(t)\ equations\ in\ system\ ((2.3.27) lead\ to$

$$s_1'(t) \geq d_1 - ae_1ms_1(t) - d_1s_1(t) = d_1 - (ae_1m + d_1)s_1(t),$$
$$s_2'(t) \geq d_2 - ae_2s_2(t) - d_2s_2(t) = d_2 - (ae_2 + d_2)s_2(t).$$

By the standard comparison theorem it follows that

$s_1(t) \geq w_1(t), and\ s_2(t) \geq w_2(t), where\ (w_1(t), w_2(t) is\ the\ solution\ of$

$$w_1'(t) = d_1 - ae_1mw_1(t) - d_1w_1(t),$$
$$w_2'(t) = d_2 - ae_2mw_2(t) - d_2w_2(t).$$

$with\ w_1(0) \leq \phi_1(0), w_2(0) \leq \phi_3(0). thus,$

$$s_{1\infty} \geq w_{1\infty} = \frac{d_1}{ae_1m + d_1}, and\ s_{2\infty} \geq w_{2\infty} = \frac{d_2}{ae_1ms_{1\infty}}. \quad (2.3.35)$$

$next\ applying\ the\ functional\ Lemma\ to\ the\ s_1'(t)\ and\ s_2'(t)\ equations\ in\ system\ (2.3.27) gives$

$$I_1^\infty \geq \frac{d_2 - d_2s_{2\infty}}{ae_2s_{2\infty}}, I_2^\infty \geq \frac{d_2 - d_2s_{1\infty}}{ae_2s_{1\infty}}$$

$by\ the\ le2.3.2$ and the inequalities in(2.3.36), we know that

$\partial x_+^1 = {X_+^o}/{X_+^0}\ is\ a\ uniform\ weak\ repeller\ for\ X_+^0. applying\ thoerm\ 1.4\ to\ the\ solution\ of$

Semi flow $\psi$(t,$\Phi) = (s_1(t), I_1(t), s_2(t)) of\ system$ (2.3.37) for $t \geq \max(\tau_1, \tau_2)\ with\ \Phi\epsilon X_+^0$,we conclude that $\partial X_+^t$ is also a uniform strong repellerfor $X_+^0$,implying that the disease is uniformly strongly persistent .This means that there exists an $\eta \geq 0$ such that $I_{1\infty} \geq \eta$,where $\eta$ is independent of the initial function $\Phi\epsilon X_+^0$.This completes the proof of theorem.

The following theorem, parallels to theorem 2.3.2 for (2.3.18),and conforms the global asymptotical stability of the endemic equilibrium $E*$ for the system (2.3.27) under the assumption $\gamma_1 = 0.$

**Lemma 4.1.1** If ($S_1(0)$; $I_1(0)$; $S_2(0)$; $I_2(0)$) ∈Ω satisfies $S_1(0) + I_1(0) = 1$ and $S_2(0)+I_2(0) = 1$, then system (2.2.4) has a unique solution ($S_1(t)$; $I_1(t)$; $S_2(t)$; $I_2(t)$) satisfying the initial data$\{s_1(0), I_1(0), s_2(0), I_2(0)$
To which remains in $\Omega$ for all
t$\geq 0, more\ over, if I_1(0) + I_1(0) > 0, then\ I_1(0) and I_2(0) > 0\ for\ t > 0.$
**Proof:** The proof of the lemma is as under
Let pi'=$\lim_{n\to\infty} \int_o^t e^{-d_i u} P_i(u) du \qquad i = 1,2,3 \ldots\ldots$
Clearly,$P_i'(respectively\ p_2')is\ the\ average\ time\ that\ an\ infected\ human\ beings$
Respectively female mosquito remains in the talent class before becoming infectious or dying by

$$0 < p_i' < \lim_{n\to\infty} \int_o^t e^{-d_i u}\, du = \frac{1}{d_i}, \quad i = 1,,2,3 \ldots..$$

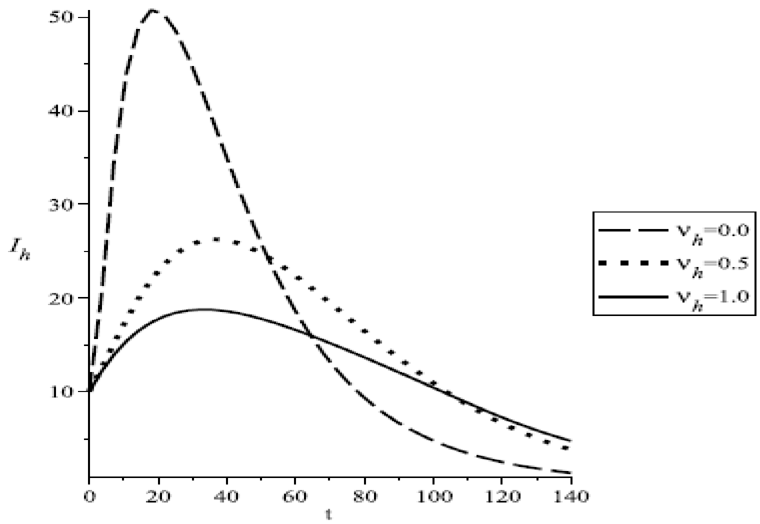
$here\ pi'di'\ (respectively\ P_2'd_2) is\ the\ probability\ \ that\ an\ infected\ host$
$(respectively\ mosquito) will\ die\ during\ the\ latent\ period$. Hence Q1 (respectively Q2) represents the proportion of the exposed hosts (respective vectors). That could survive during the latent period, where

$$Q_{i=-} \lim_{n\to\infty} \int_0^t e^{-d_i(t-\xi)} D_t P_i(t-\xi) d\xi$$
$$= 1 - d_i p_i' \varepsilon(0,1), \quad i = 1,2,3 \ldots\ldots$$

Using Qi=1,2,3, the basic representation number for model(2.2.4) can be defined as

$$Ro = m\frac{ae_1}{v_1 + d_1}.Q_{1.}\frac{ae_2}{d_2}.Q_2 \qquad (2.3.1)$$

accounting for the average number of secondary infections that a single infectious
human being (or female mosquito), once introduced into a fully susceptible population of humans and mosquitoes, is expected to cause with respect to humans (female mosquitoes) during the infection period. Here, due to the transmission nature of this vector-host disease, $R_0$ consists of two parts:$m\frac{ae_1}{v_1+d_1}.Q_{1.}$accounts for how many new infectious mosquitoes results from an infectious human being can result in during his infection period; and $\frac{ae_2}{d_2}.Q_2$explains how many new infectious human beings are infected by an infectious mosquito during its infection period.
System (2.2.4) has a disease-free equilibrium E0, giving by $Eo = (1,0,1,0)$ in terms of the biological meaning of the basic reproduction number, Ro=1 should be a threshold value for the stability /instability of E0 for the model (2.2.4) conformed in the following theorem.




In view of the above, humans need to boost their antibody production to be able to reduce the invasion of parasites into the blood. The immune status of the person, that is, the general health and nutritional status of the infected person, is a factor to prevent or aid the occurrence of malaria. Thus, leading a healthy lifestyle and eating the right foods can help boost antibody levels in humans. It is also important to note that reducing human-mosquito contact rates plays a large role in preventing the spread of malaria. Therefore, we can achieve a malaria-free state by reducing the mosquito bite rate; Use of regular indoor residual spraying, with pesticides

Insecticide-treated bednets, closing doors and windows against mosquitoes, cleaning stagnant water and drainage, and the use of mosquito repellent lotion, which are all considered vector control measures.

**CONCLUSION**

We have presented a mathematical model of malaria transmission in this paper. The model is an autonomous system, constructed by considering two models: a model of vector population and a model of virus transmission. The threshold dynamics of each model are determined and a relationship between them is established.

Transmission also depends on climatic conditions that can affect the number and survival of mosquitoes, such as rainfall patterns, temperature, and humidity. In many places, broadcasting is seasonal, with peak during and after the rainy season. Malaria epidemics can occur when climatic and other conditions suddenly favor transmission in areas where people have little or no immunity to malaria. They can also occur when people with low immunity visit areas with acute malaria transmission, for example to find work, or as refugees.

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