A STUDY OF SOME INVENTORY MODELS FOR PRODUCTS WITH TIME-DEPENDENT STORAGE COSTS

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ABSTRACT

In this paper we are presenting a study of some Inventory Models for Declension Products with Time Dependent Storage Cost. There are two major objectives of inventory control, which commonly are in conflict. The operations manager's problem is in striking a balance between the two. These objectives are to maximize the level of customer service, and to minimizing the cost of providing an adequate level of customer service, promoting efficiency in production or purchasing. There are two major objectives of inventory control, which are usually in conflict. The operational manager's problem lies in balancing the two. These objectives are to maximize the level of customer service and reduce the cost of providing an adequate level of customer service, promoting efficiency in production or procurement. Today, the changing scenario of the market has as many advantages as disadvantages. The customer is offered more choice today. This has certainly put him in a winning position. However, a retailer is not better if it has to lose for any reason. This puts the retailer in a tight spot, especially if it has no stock to deliver to the customer. With so many options, a customer can easily place their order with some other retailer who should have the goods ready for delivery. For the retailer this loss of sales may result in a greater loss as the customer will hesitate to approach it the next time.

Keyword: - *Inventory Models, Time, storage Cost, Products etc.*

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1. INTRODUCTION

An important issue in inventory theory is how to deal with unfulfilled demands occurring during shortages or stock outs. In most developed models, researchers assumed that the deficiency is either completely backlogged or lost altogether. The first case, known as a backorder or backlogging case, represents a situation where the incomplete demand is fully ordered back. In the second case, also referred to as the case of lost sales, we believe that incomplete demand is completely lost. In addition, when there is a shortage, some customers are ready to wait for the backorder and others are willing to buy from other vendors. In many cases customers are conditioned for shipping delays and may be willing to wait for a short time to get their first choice. For example, for fashionable items and high-tech products with shorter product life cycles, the customer's willingness to wait for the backlog is decreasing with the length of the waiting time. Thus the length of waiting time for the next replenishment will determine whether the backlog will be accepted or not. In many real-life situations, the longer the time is, the shorter the time, over a shorter period of time. Therefore, the backlog rate for realistic business conditions must be variable and dependent on the waiting time for the next replenishment. Many researchers have modified inventory policies by considering "time proportional partial backlog rate".

Inventory is a repository or repository of an item or resource used by an organization. Good inventory management is important for all firms, whether manufacturing or service. Inventory can be a major commitment of monetary resources. Inventory affects almost every aspect of daily operations. There are two major objectives of inventory control, which are usually in conflict. The operational manager's problem lies in balancing the two. These objectives are to maximize the level of customer service and reduce the cost of providing an adequate level of customer service, promoting efficiency in production or procurement.

A glance at the available literature on inventory shows that many models have been formulated in stable environments, which were stable given demand rates. Such a demand rate had the simplest and only advantage of providing minimalism to the study. It was also a known fact at the time that there could be no commodity that could boast an uncontrolled constant demand rate in its nature over time and other market forces. In realistic business conditions it is observed that the demand rate depends on many factors, such as time, stock, selling price, etc. Such assumptions would be a major improvement on the continuity of demand. They will allow mobility in the nature of demand created by the commodity in the market.

2. THE GENERAL STRUCTURE OF INVENTORY MODELS

There are two situations in which time has been treated as a continuous variable. In one sense, it is assumed that demands occur as a continuous time and that the corresponding amount ordered or produced is a similar function. The second situation is that in which the demand come discontinuously at time points which are not necessary; a firm may find it more profitable not to meet the demand.

- **Static Models:** These do not consider the impact of changes taking place during the planning horizon i.e. they are independent of time. Therefore, in this case only one decision is required for the duration of a given time period.
- **Dynamic Models:** In these models time is considered as one of the important variable and thus, the impact of changes generated by time is considered further a series of interdependent decisions would be required during the planning horizon.

3. INVENTORY MANAGEMENT

Inventory management is a big issue for many large companies and stores, since it can be difficult to track and control large inventories. For retail stores, managing inventories can be extremely challenging, thanks to light fingered patrons. Inventory Management does not make decisions or manage operations; they provide the information to Managers who make more accurate and timely decisions to manage their operations. Inventory systems where customers join waiting line to receive their demanded items are recently considered by many researchers, as these models allow the study of both queue length and size of the inventory.

Inventory Management and Inventory Control must be designed to meet the dictates of the market place and support the company's strategic plan. The many changes in market demand, new opportunities due to worldwide marketing, global sourcing of materials, and new manufacturing technology, means many companies need to change their Inventory Management approach and change the process for Inventory Control. Despite the many changes that companies go through, the basic principles of Inventory Management and Inventory Control remain the same. Some of the new approaches and techniques are wrapped in new terminology, but the under lying principles for accomplishing good Inventory Management and Inventory activities have not changed. The Inventory Management system and the Inventory Control Process provides information to efficiently manage the flow of materials, effectively utilize people and equipment, coordinate internal activities, and communicate with customers. Inventory Management and the activities of Inventory Control do not make decisions or manage operations; they provide the information to Managers who make more accurate and timely decisions to manage their operations.

4. METHODOLOGY

Although in recent years a large number of inventory models have been made. Yet there is a scope as well as need of more models which are more general and more flexible. It is natural phenomena that the demand for an item declines over time, with passage of time, new replacement for the product are being launched in the market, specially it is the case with fashion goods. Similar is the case with computer and its packages. Our aim is to develop models to face these types of problems.

The models can be developed by using mathematical tools like differential equation, differentiation, integration, difference equations, computational numeric methods, statistical technique, computer application, optimization and approximation. There are two techniques of solving models by cost minimization and profit maximization technique.

5. MODEL WITH LINEAR RATE OF DETERIORATION

In this section, two perishable inventory models have been developed for variable rates of deterioration. In practice it can be observed that constant rate of deterioration occurs rarely. Most of the items deteriorate fast as the time passes. Therefore it is much more realistic to consider deterioration rate as time dependent deterioration rate. Demand for these models is observed to be selling price dependent. Shortages in inventory are allowed and partially backlogged. Backlogging rate is taken as constant.

Let Q (t) be the inventory level at time (0, t, T). The differential equations for the instantaneous state over (0, T) are given by:

$$\frac{dQ(t)}{dt} + (k+\theta t)Q(t) = -(a-p), \quad 0 \le t \le t_1 \qquad \text{...(2.5.1)}$$

$$\frac{dQ(t)}{dt} = -B(a-p), \qquad t^1 \le t \le T \qquad \text{...(2.5.2)}$$

Solution of equation (2.5.1) & (2.5.2) are:

$$Q(t) = (a-p)[(t_1 - t) + k\{t(t-t_1) + \frac{1}{2}(t_1^2 - t^2)$$
$$\theta\left(\frac{t_1^4}{4} - \frac{t^4}{3} - \frac{t^2 t_1}{4} - \frac{t t_1^3}{6} + \frac{t^3 t_1}{2}\right)\}$$
$$+k^2\left\{\frac{t_1^3}{6} - t^3 + \frac{t t_1}{2}(t - t_1)\right\} + \theta\left(\frac{t_1^3}{6} + \frac{t^3}{3} - \frac{t^2 t_1}{2}\right)$$
$$+\theta^2\left(\frac{t_1^5}{40} - \frac{t^5}{15} - \frac{t_1^3 t^2}{12} - \frac{t^4 t_1}{8}\right)] \qquad \text{...(2.5.3)}$$

$$Q(t) = B(a-p)(t_1 - t) \qquad \text{... (2.5.4)}$$

From (2.5.3) and (2.5.4), one gets

$$Q(t) = (a-p)[(t_1 - t) + k\{t(t-t_1) + \frac{1}{2}(t_1^2 - t^2)$$
$$+\theta\left(\frac{t_1^4}{4} - \frac{t^4}{3} - \frac{t^2 t_1}{4} - \frac{t t_1^3}{6} + \frac{t^3 t_1}{2}\right)\}$$
$$+k^2\left\{\frac{t_1^6}{6} - \frac{t^3}{6} + \frac{t t_1}{2}(t - t_1)\right\} + \theta\left(\frac{t_1^3}{6} + \frac{t^3}{3} - \frac{t^2 t_1}{2}\right)$$
$$+\theta^2\left(\frac{t_1^5}{40} - \frac{t^5}{15} - \frac{t_1^3 t^2}{12} - \frac{t^4 t_1}{8}\right)] = (a-p)(t_1 - t) \qquad \text{... (2.5.5)}$$

Stock due to deterioration during $(0, t_1)$ is:

$$D = \int_0^{t_1} (k+\theta t)Q(t) \quad dt$$
$$= (a-p)\left(\frac{k t_1^2}{2} + \frac{1}{6}(k^2\theta)t_1^3 + \frac{k\theta t_1^4}{8} + \frac{\theta t^3}{6} + \frac{\theta^2 t_1^5}{40}\right) \qquad \text{... (2.5.6)}$$

$$q = D + \int_0^t (a-p)dt$$
$$= (a-p)\left(\frac{k t_1^2}{2} + \frac{1}{6}(k^2\theta)t_1^3 + \frac{k\theta t_1^4}{8} + \frac{\theta^2 t_1^5}{40} + T\right) \qquad \text{... (2.5.7)}$$

Holding cost occurs during $(0, t_1)$ is:

$$H = \int_0^{t_1} (h+\alpha t)Q(t)dt$$
$$= (a-p)[\{h(\frac{t_1^2}{2} + \frac{k^2 t_1^4}{24} + \frac{k t_1^3}{6} + \frac{\theta t_1^4}{12} + \frac{\theta^2 t_1^6}{90} + \frac{k\theta t_1^5}{24})$$
$$+\alpha(\frac{t_1^3}{6} + \frac{k t_1^4}{24} + \frac{1}{120}k^2 t_1^5 + \frac{1}{40}\theta t_1^5 + \frac{7}{720}k\theta t_1^6 + \frac{1}{336}\theta^2 t_1^7)\}] \qquad \text{... (2.5.8)}$$

Shortage cost occurs during (t_1, T) is:

$$S = \int_{t_1}^{T} (-Q(t)dt$$

$$= \frac{1}{2}(a-p)(T-t_1)^2 \qquad \text{... (2.5.9)}$$

Lost cost occurs during (t_1, T) is:

$$L = \int_{t_1}^{T} (1-B)(a-p)dt$$

$$= (1-B)(a-p)(T-t_1) \qquad \text{... (2.5.10)}$$

Total profit per unit time is given by:

$$P(T,p) = p(a-p) - \frac{1}{T}(A + Cq + H + C_1S + C_2L)$$

$$= P(a-p) - \frac{1}{T}[A + C\{(a-p)\left(T + \frac{kt_1^2}{2} + \frac{1}{6}(k^2+\theta)t_1^3 + \frac{k\theta t_1^4}{8} + \frac{\theta^2 t_1^5}{40}\right)\}$$

$$+(a-p)\{h(\frac{t_1^2}{2} + \frac{k^2t_1^4}{24} + \frac{kt_1^3}{6} + \frac{\theta t_1^4}{12} + \frac{1}{90}\theta^2 t_1^6)$$

$$+\alpha\left(\frac{t_1^3}{6} + \frac{kt_1^4}{24} + \frac{1}{120}k^2t_1^5 + \frac{1}{40}\theta t_1^5 + \frac{13}{5040}k\theta t_1^6\right)$$

$$+\frac{1}{336}\theta^2 t_1^7 + C_1\left\{\frac{1}{2}(a-p)(T-t_1)^2\right\} + C_2(1-B)(a-p)(T-t_1)]$$

… (2.5.11)

Let $t_1 = \beta T \quad , \quad 0 \leq \beta \leq 1$

$$P(T,p) = P(a-p) - \frac{1}{T}[A + C\{(a-p)(T + k\beta^2T^2$$

$$+\frac{1}{6}(k^2+\theta)\beta^3T^3 + \frac{k\theta}{8}\beta^4T^4 + \frac{\theta^2\beta^5T^5}{40})\}$$

$$+(a-p)\{h(\frac{\beta^2T^2}{2} + \frac{k^2\beta^4T^4}{24} + \frac{k\beta^3T^3}{6} + \frac{\theta\beta^4T^4}{12} + \frac{1}{90}\theta^2\beta^6T^6)$$

$$+\alpha(\frac{\beta^3T^3}{6} + \frac{k\beta^4T^4}{24} + \frac{1}{120}k^2\beta^5T^5 + \frac{1}{40}\theta\beta^5T^5 + \frac{13}{5040}k\theta\beta^6T^6)$$

$$+\frac{1}{336}\theta^2\beta^7T^7 + \frac{9T^2}{2}(1-\beta)^2 + C_2(1-B)(a-p)(T-\beta T)] \quad \text{... (2.5.12)}$$

For maximize of the function P(T,p)

$$\frac{\partial P(T,p)}{\partial T} = 0 \ , \quad \frac{\partial P(T,p)}{\partial p} = 0$$

$$\frac{\partial P(T,p)}{\partial T} = [-\frac{A}{T^2} + C\{(a-p)(\frac{k\beta^2}{2} + \frac{1}{6}(k^2+\theta)\beta^3.2T$$

$$+\frac{k\theta}{8}\beta^4.3T + \frac{\theta^2\beta^5}{40}.4T^3)\}$$

$$-(a-p)\{h(\frac{\beta^2}{2} + \frac{k^2\beta^4}{24}.3T + \frac{k\beta^3}{6}.2T + \frac{\theta\beta^4}{12}3T + \frac{1}{90}\theta^2\beta^6.5T)$$

$$+\alpha(\frac{\beta^3}{6}.2T+\frac{k\beta^4}{24}.3T^2+\frac{1}{120}k^2\beta^5.4T^3$$

$$\frac{1}{40}\theta\beta^5.4T^3+\frac{13}{5040}k\theta\beta^6.5T^4+\frac{1}{336}\theta^2\beta^7.6T^5)\}$$

$$-\frac{9}{2}(1-\beta^2)+(1-B)(a-p)(1-\beta)=0$$

$$\frac{\partial P(T,p)}{\partial p}=a-2p+\frac{1}{T}[\{Tc+\frac{k\beta^2T^2}{2}c+\frac{1}{6}(k^2+\theta)\beta^3T^3c+\frac{k\theta}{8}\beta^4T^4c$$

$$\frac{+\theta^2\beta^5T^5c}{40}\}+\frac{9}{2}T^2(1-\beta)^2$$

$$+\frac{h}{T}(\frac{\beta^2T^2}{2}+\frac{k^2\beta^4T^4}{24}+\frac{k\beta^3T^3}{6}+\frac{\theta\beta^4T^4}{12}+\frac{1}{90}\theta^2\beta^6T^6$$

$$+\frac{\alpha}{T}(\frac{\beta^3T^3}{6}+\frac{k\beta^4T^4}{24}+\frac{1}{120}k^2\beta^5T^5+\frac{1}{40}\theta\beta^5T^5$$

$$+\frac{13}{5040}k\theta\beta^6T^6+\frac{1}{336}\theta^2\beta^7T^7)-T(1-B)(1-\beta)]=0$$

$$\Rightarrow A-\frac{h\beta^2T^2}{2}(a-p)-(\frac{k\beta^2T^2}{2}(a-p)-\frac{1}{3}(k^2+kh+\alpha)(a-p)\beta^3T^3$$

$$-\frac{1}{8}(k^2h+\alpha)\beta^4T^4(a-p)$$

$$-\frac{k^2}{30}\alpha\beta^5T^5-\frac{C_1}{2}T^2(1-p)^2+\theta\{-\frac{\beta^3T^3}{3}c(a-p)$$

$$-\frac{3k}{8}\beta^4T^4c(a-p)-\frac{\beta^4T^4}{4}(a-p)h$$

$$-\frac{\beta^5T^5}{10}\alpha(a-p)+\frac{13}{1008}k\beta^6T^6\alpha(a-p)\}$$

$$+\theta^2\{-\frac{\beta^5T^5}{10}c(a-p)-\frac{1}{18}h(a-p)\beta^6T^6$$

$$-\frac{\alpha}{58}(a-p)\beta^7T^7\}-T(1-B)(1-\beta)=0 \qquad \text{... (2.5.13)}$$

$$a-2p+c+\frac{k\beta^2Tc}{2}+\frac{1}{6}k^2\beta^3T^2c+\frac{c1}{2}T(1-\beta)^2+\frac{h\beta^2T}{2}$$

$$+\frac{hk^2\beta^4T^3}{24}+\frac{hk\beta^3T^2}{6}+\frac{\alpha\beta^3T^2}{6}+\frac{\alpha k\beta^4T^3}{24}+\frac{\alpha k^2\beta^5T^4}{120}+\frac{\theta\beta^3T^2}{6}$$

$$+\frac{k}{8}\beta^4T^3c+\frac{\beta^4hT^3}{12}+\frac{1}{40}\alpha\beta^5T^4+\frac{13\alpha k\beta^6T^5}{5040})$$

$$+\theta^2\left(\frac{\beta^5T^4c}{40}+\frac{h\beta^6T^5}{90}+\frac{\alpha}{336}\beta^7T^6\right)-T(1-B)(1-\beta)=0 \qquad \text{... (2.5.14)}$$

And satisfy the sufficient condition for minimizing P(T,p)are:

$$\frac{\partial^2P(T,p)}{\partial T^2}\le 0, \quad \frac{\partial^2P(T,p)}{\partial p^2}\le 0, \qquad \text{... (2.5.15)}$$

$$and \quad \frac{\partial^2 P(T,p)}{\partial T^2}, \quad \frac{\partial^2 P(T,p)}{\partial p^2} - \frac{\partial^2 P(T,p)}{\partial T \partial P} \geq 0$$

at p = p* and T = T*.

Example 1: A =200, a =100, C=z_o, h (t) =0.4, C1 = 1.2, $\beta =$ 0.95
k =0, $\alpha =$ 0.1, $\theta =$ 0.01
In appropriate units based on these input data. The complete outputs are as follows, profit =1471.0240,T*=3.3501,P* =61.35627
Example 2 : A =200, a=100, C =20, h(t)=.4, C1 =1.4
$\beta =$ 1, $\alpha =$ .1, $\theta =$ .1, in appropriate units based on these input data the complete Outputs are follows, profit =1419.0463, T*=1.1862, P*=60.36611

6. Effects of Backlogging Parameter (α):

The backlogging parameter (α) has initially been taken as 0.1. Now, we vary backlogging parameter from 0.08 to 0.12 and observe the effects it has over the solution.

Table 3.1

Parameter Value	% Change	T_1	S	K
0.08	-20	0.797928	44.689239	229.687594
0.09	-10	0.798138	44.695144	229.707430
0.10	0	0.798349	44.701076	229.727292
0.11	+10	0.798560	44.707007	229.747148
0.12	+20	0.798771	44.712938	229.766997

The study of above table (3.1) reveals the following interesting facts with the increment in backlogging parameter:
We notice an increase in the inventory period.
An increase in the initial inventory level is observed.
The value of total average cost of the system also keeps increasing.

2. Effects of Deterioration Parameter (θ):

Initially, the deterioration parameter (θ) has been taken as 0.02. We observe the following effects with the variation in deterioration parameter from 0.016 to 0.024.

Table 3.2

Parameter Value	% Change	T_1	S	K
0.016	-20	0.798871	44.710567	229.683879
0.018	-10	0.798724	44.703975	229.709333
0.020	0	0.798349	44.701076	229.727292
0.022	+10	0.797975	44.693138	229.745263
0.024	+20	0.797601	44.685186	229.763212

From the above table (3.2) we observe some interesting facts, which are quite obvious when considered in the light of reality. If we increase the value of deterioration parameter then we notice that:
Period of inventory decreases.
The initial inventory decreases.
Total average cost of the system increases.

3. Effects of Life Time Parameter (μ):
Initially, the life time parameter (μ) has been taken as 0.40. We observe the following effects with the variation in this parameter from 0.32 to 0.48.

Table 3.3

Parameter Value	% Change	T_1	S	K
0.32	-20	0.797926	44.697616	229.806831
0.36	-10	0.798125	44.698991	229.766853
0.40	0	0.798349	44.701076	229.727292
0.44	+10	0.798599	44.703976	229.689058
0.48	+20	0.798876	44.707791	229.653044

The following observations have been made on the basis of above table (3.3). If we increase the value of this parameter then:
We notice an increment in the period in which inventory holds.
We notice an increment in the initial inventory level.
The total average cost of the system goes on decreasing progressively.

6. CONCLUSION

In this paper, we discussed the model of the list of constrained items with a constant and variable rate. The demand rate taken in this paper is called power pattern demand (PPD) and if it does, the stockier can use a policy other than the traditional policy based on the liner pattern. If the bulk of demand occurs at the beginning of the period, we use n> 1 and if it occurs at the end of the period, we use 0 <n 1. The constant demand pass corresponds to n = 1 and n = onds corresponds to the instantaneous demand. Shortage is allowed and the backlog rate depends on the duration of the waiting time and varies inversely. The cost minimization technique is used to derive the expressions for the total cost and other parameters. From the numerical depiction of the model, it is observed that over which period the inventory holds and the prior inventory level increases with an increase in backlog and life time parameters, while these investments decrease with a declining increment. The overall average cost of the system increases with an increase in backlog and constraining parameters while life is declining with an increase in time frame.

REFERENCES

1. **Manna, S.K. and Chaudhuri, K.S. (2006)**, "An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages", E.J.O.R., 171, 557-566.
2. **Roy, T. and Chaudhuri, K.S. (2006)**, "Deterministic inventory model for deteriorating items with stock-level dependent demand and shortage", Nonlinear Phenomena in Complex Systems, 9, 1, 43-52.
3. **Song, X. and Cai, X. (2006)**, "On optimal payment time for a retailer under permitted delay of payment by the wholesaler", International Journal of Production Economics, 103, 1, 246-251.
4. **Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006),** "An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging", I.J.P.E., 101, 369-384.
5. **Zhou, Y.W. and Yang, S.L. (2005),** "A two-warehouse inventory model for items with stock-level-dependent demand rate. I.J.P.E., 95 (2), 215-228.
6. **Wu, M.Y. and Wee, H.M. (2005),** "An integrated production inventory model with shortage for deteriorating items in a supply chain", Journal of Information and Optimization Sciences. 26(1), 233-246
7. **Teng, J.T., Chang, C.T. and Goyal, S.K. (2005)**, "Optimal pricing and ordering policy under permissible delay in payments", I.J.P.E., 97, 121-129.
8. **Sana, S., Goyal, S.K. and Chaudhuri, K.S. (2004)**, "A production-inventory model for a deteriorating item with trended demand and shortages", E.J.O.R., 157, 357-371
9. **Balkhi, Z.T. and Benkherouf, L. (2004)**, "On an inventory model for deteriorating items with stock dependent and time varying demand rates", Computers & Operations Research, 31, 223- 240.
10. **Chang, C.T. (2004),** "An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity", I.J.P.E., 88, 307-316.
11. **Chung, K.J. and Huang, Y.F. (2003)**, "The optimal cycle time for EPQ inventory model under permissible delay in payments", International Journal of Production Economics, 84, 3, 307-318
12. **Giri, B.C., Jalan, A.K. and Chaudhuri, K.S. (2003),** "Economic order quantity model with Weibull distribution deterioration, shortage and ramp-type demand", I.J.S.S., 34(4), 237-243.
13. **Goyal, S.K. and Giri, B.C. (2003)**, "The production-inventory problem of a product with time varying demand, production and deterioration rates", E.J.O.R., 147, 549-557.

14. **Rau, H., Wu, M.Y. and Wee, H.M. (2003), "** Integrated inventory model for deteriorating items under a multi - echelon supply chain environment", I.J.P.E., 86(2), 155-168.
15. **Yang, P.C. and Wee, H.M. (2002)**, "A single-vendor and multiple-buyers production inventory policy for deteriorating item", E.J.O.R., 143, 570-581
16. **Teng, J.T., Chang, H.J., Dye, C.Y. and Hung, C.H. (2002),** "An optimal replenishment policy for deteriorating items with time varying demand and partial backlogging", O.R.L., 30, 387-393.
17. **Papachristos, S. and Skouri, K. (2000),** "An optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential-type backlogging", O.R.L., 27, 175-184