A STUDY ON POST OPTIMALITY ANALYSIS (PERCEPTIVITY ANALYSIS) IN OPTIMIZATION PROBLEMS

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ABSTRACT

During this thesis, We have to find out about the Perceptivity Analysis (post optimality analysis) is that the study of how the optimal result of an optimization problem changes with reference to the changes within the problem of knowledge. The possible crimes within the problem of knowledge frequently make perceptivity analysis as important as working the first problem.

Keywords: *Perceptivity Analysis, Optimal Result, data, input, labors.*

Preface:

Perceptivity Analysis may be a study of how the query within the affair of a fine modal or system (numerical or else) are frequently apportioned to different sources of query in its inputs. A affiliated practice is query analysis, which features a lesser specialise in query qualifications and propagation of query. Immaculately, query and perceptivity analysis should be run in tandem. Perceptivity analysis are frequently useful for a spread of purposes, including .

Testing the robustness of the results of a model or system within the presence of query.

Increased understanding of the connections between input and affair variables during a system or model.

Query reduction relating model inputs that beget significant query within the affair and will thus be the main target of attention if the robustness is to be increased.

Checking out crimes within the model (by encountering unanticipated connections between inputs and labors)

Model simplification fixing model inputs that have not any effect on the affair, or relating and removing spare corridor of the model structure.

Enhancing communication from modellers to decision-makers (e.g. by making recommendations more believable, accessible, compelling, or, conclusive)

A change occurring the region within the space of input factors that the model affair is either maximum or minimal or meets some optimum criterion.

Taking an illustration from economics, in any budgeting process, there are always variables that are uncertain. Farther duty rates, interest rates, affectation rates, headcount, operating cost, and other variables might not be known with great perfection. Perceptivity analysis answers the questions. However, what's going to the effect be (on the business, model, " if these variables diverge from prospects.

This Discussion entitled **" POST OPTIMALITY ANALYSIS (PERCEPTIVITY ANALYSIS)"**consists of 5 chapters

Chapter I deals with the overall results of conforming the Optimization problems.

Chapter II deals with the Parametric Analysis

Chapter III deals with Post Optimality Analysis (Sensitivity Analysis)

Chapter IV deals with the Optimal Control Problems

Chapter V deals with the appliance of Perceptivity Analysis

Chapter III Post Optimality Analysis (Sensitivity Analysis)

In the authors studied perturbations of the right –hand side and the cost parameters in linear programming, motivated by how interior –point methods from a near optimal pair of strictly feasible solutions for a problem and its dual would compare with the optimal basis approach obtained from a non degenerate optimal basic solution for such perturbations.

3.2 Sensitivity analysis of C:

The sensitivity analysis of C_k is the problem:

Find all the values of λ for which B is an optimal basis matrix of Linear Programming Problem

Maximize $(C+\lambda e_k) X$

Subject to,

$AX= b$

$X\geq 0$

Where the k-th unit vector $E_k \in R^n$ is. The answer obviously is the interval ($\underline{\lambda}$ (B), $\overline{\lambda}$(B)) which can be obtained by using Equation (2.4) & Equation (2.5) with $C^* = e_k$. Since $C^* = e_k$, equation (2.4) and (2.5) can be simplified as follows.

We consider the following two cases:

(1) X_k is a non basic variable with respect to B
(2) X_k is a basic variable with respect to B

Let $c_k^{\wedge}$ refer to the changed value of c_k.

(1)X_k is a non basic variable

In this case $C_B^*=0$

Therefore $z_j-c_j = 0$ if $(j \neq k)$

$= -1$ if $(j= k)$

Hence Equation (2.4) & Equation (2.5) gives

$$\underline{\lambda}_k (B) = -\infty \qquad (3.2)$$

$$\overline{\lambda}_k(B) = z_k-c_k \qquad (3.3)$$

Thus when X_k is a non basic variable, B remains an optimal basis matrix of Linear programming Problem (3.1) if and only if $c_k^{\wedge} \leq z_k$.

(2) X_k is a basic variable. Since X_k is the non basic variable, We can determine r such that $X_k = X_{Br}$ i.e., a_k is the r-th column of B . Then

$C^*_{Bi} = 0$ if $(i\neq r)$

$= 1$ if $(i=j)$

Therefore $z_j^* - c_j^* = y_{rj}$ if ($j \neq k$)

$= 0$ if ($j=k$)

Hence the Equation (2.4) and Equation (2.5) gives

$$\underline{\lambda}_k (B) = \max_{\substack{j \\ y_{rj} > 0}} \{ -(z_j - c_j)/y_{rj} \}$$

$$= -\infty \text{ if } Y_{rj} \leq 0 \text{ for all } j \qquad (3.5)$$

$$\overline{\lambda}_k (B) = \text{minimum}_{\substack{j \\ y_{rj} < 0}} \{ -(z_j - c_j) / y_{rj} \}$$

$$= +\infty \text{ if } Y_{rj} \geq 0 \text{ for all } j \qquad (3.6)$$

Thus x_k is the basic variable, B remains as an optimal basis matrix of Linear Programming Problem (3.1) if and only if

$$c_k + \underline{\lambda}_k (B) \leq c_k^{\wedge} \leq c_k + \overline{\lambda}_k(B) \qquad (3.7)$$

Where $\underline{\lambda}_k$ (B) and $\overline{\lambda}_k$(B) are given by Equation (3.6) and Equation (3.7)

The above analysis show that the optimal solution changes suddenly when the changed values of c_k is an wrong side of the intervals given by Equation (3.4)and Equation (3.7)it may not be noted that any one component of C can change the optimal solution of a Linear Programming Problem.

3.2 Example:

Consider the Example 1.1. In the optimal solution (10, 0, 2, 0, 6, 1, 0) x_3 is a basic variable. Using the Equation (3.5) and Equation (3.6), We get

$$\underline{\lambda}_3 (B) = -\infty$$

$$\overline{\lambda}_3(B) = -(z_4 - c_4) / y_{14}$$

Hence the equation (3.7) shows that the optimal solution is not sensitive to C_3

Provided $\infty \leq c^{\wedge}$

Where $c_3^{\wedge} = -2$, then (10, 0,16/7,1/7,39/7,0,0) is an alternative optimal solution

3.3. Sensitivity analysis of b:

The sensitivity analysis of bp is equivalent to the problem.

Find all the values of μ for which B remains an optimal basis matrix of the Linear programming problem.

Subject to, $AX = b + \mu e_p$,

$X \geq 0$

Where $e_p \in R^m$ is the p-th unit vector. The answer clearly is the characteristic interval $(\underline{\mu}_p(B), \overline{\mu}_p(B))$ which can be obtained by using the Equation (2.11) and Equation (2.12) with $b^* = e_p$, we have $\overline{b}* = B^{-1} b^* = B^{-1} e_p = B_p$.

Where β_p is the p-th column of B^{-1} .If β_i is the i-th component of β_p then the Equation (2.11) and Equation (2.12) gives

$$\underline{\mu}_p(B) = \underset{\substack{i \\ (\beta_{ip} > 0)}}{\text{maximum}} (-\bar{b}_i / \beta_{ip}), \quad (3.8)$$

$$\bar{\mu}_p(B) = \underset{\substack{i \\ (\beta_{ip} < 0)}}{\text{minimum}} (-\bar{b}_i / \beta_{ip}), \quad (3.9)$$

Therefore, matrix B remains a optimal basis matrix of Linear Programming Problem (3.1) if and only if

$$b_p + \underline{\mu}(B) \leq b_p^{\wedge} \leq b_p + \bar{\mu}(B) \quad (3.10)$$

Where $b_p^{\wedge}$ refers to the changed value of b_p

3.3 Example:

We consider the sensitivity analysis of b_4 in Example 1.1 Using the optimal value table 1.7 , We get

B_4 = 4th column of $B^{-1} = Y_8$

= (1/2, -1, -1/2, 0)

b = (2, 1, 6, 10)

Therefore Equation (3.8) and Equation (3.9) gives ,

$$\underline{\mu}_4(B) = -4 = -\bar{b}_1 / \beta_{14}$$

$$\bar{\mu}_4(B) = 1 = -\bar{b}_2 / \beta_{24}$$

Hence B=[a_3 a_6 a_5 a_1] remains an optimal basis matrix of the Linear Programming Problem in Example 1.2.1

When b_4 satisfies $b_p + \underline{\mu}_p(B) \leq b_p^{\wedge} \leq bp + \bar{\mu}_p(B)$

$$b_4 + \underline{\mu}_4(B)) \leq b_4^{\wedge} \leq b4 + \bar{\mu}_4(B)$$

$$24\text{-}4 \leq b_4^{\wedge} \leq 24+1$$

$$20 \leq b_4^{\wedge} \leq 25$$

Conclusion:

In this section we introduce examples and some concepts involved in such an analysis. In the first example, we work with a linear programming problem with four variables and four constraints. The varies methods can serve a wide range of purpose. In any way , they help to correlate input and output variations and provide statistical information. In this thesis, the necessary connection between Sensitivity analysis and optimal control problem and its application analysis discussed. Its techniques for more general problems are developed in the subsequent sections of the chapter.

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