A STUDY ON SOME FUZZY HYPERSOFT SET SETS

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Abstract

Soft Set theory was first introduced by Molodtsov in 1999 as a general mathematical tool for dealing with fuzzy object. In 2019 Samarandache developed hypersoft set theory as an extension of soft set theory. In this work, Fuzzy Hypersoft sets operations such as like subset, union, Intersection, equal set, complement, AND and OR are introduced..

Keywords:- *Soft set, fuzzy soft set, hypersoft set, Complement, aggregation operations* .

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1. Introduction:

Numerous complicated problems include unclear data in social sciences, economics, medical scinces, engineering and other fields. These problems, with which one is faced in life, cannot be solved classical mathematical tools. In classical mathematics, a model is designed and the exact solution of this model is calculated. However, if the given situation contains uncertainty, solving the model with classical mathematical method is very complex. Fuzzy set theory[1], rough set theory [2] and other theories have been described to solve situations involving uncertainty. The fuzzy set theory introduced by Zadeh has become very popular for uncertainty problems and has been a suitable construct for representing uncertain concepts as it allows for the partial membership function. Mathematicians and computer scientists have worked on fuzzy sets ad over the years many useful applications of fuzzy set structure have emerged such as fuzzy control systems, automata, fuzzy logic, fuzzy topology .The soft set concept was developed by [3] as completely new math tool for solving difficulty in dealing with uncertainly. Molodtsov[3] defined a soft set that is sub-set as a parameterized fail of the set of the universe where each element is considered a set approximate elements of the soft set. In the past few years, the fundamentals of soft set theory have been studied by different researchers. Maji et al. [4] by combining the fuzzy set an soft set structure, it has created the fuzzy soft claster structure, which is a hybrid structure .Smarandache [8]introduced new technique handling uncertainty. By transforming the functionality into a multi-decision function, he generalized the soft set to hypersoft set. Although Hypersoft set theory is more recent, it has attracted great attention from researchers [9-14].
In this paper, we have discussed the fundamentals of fuzzy hypersoft set such as fuzzy hypersoft subset, complement, union, Intersection, AND and OR operators n a numerical example are also illustrated.

2. Preliminaries:-

2.1. Soft Set:-

Let U be an universe of discourse and P(U) is the power set with respect to U . Let E be the set of the parameters or attributes and $A \subseteq E$. Then the pair (F_A, E), where $f_A: E \to P(U)$ such that $f_A(x) = 0 \; if \; x \notin A$
is called **Soft Set** over U .
Thus a Soft Set over U can be represented by the set of order pairs

$$F_A = \{(x, f_A(x)); x \in E, f_A(x) \in \mathrm{P(U)}\} .$$

Example 1:-

Let $\mathrm{U} = \{u_1, u_2, u_3, u_4, u_5\}$ be an universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a subset of parameters.

If $A = \{x_1, x_2, x_4\} \subseteq E$,$f_A(x_1) = \{u_2, u_4\}$, $f_A(x_2) = U$ and $f_A(x_4) = \{u_1, u_3, u_5\}$
then the soft set F_A is written by

$$F_A = \{\ (x_1, \{u_2, u_4\}),\ \ (x_2\ , U),\ \ (x_4, \{u_1, u_3, u_5\})\ \ \ \}\ \ .$$

2.2.Fuzzy Soft Set:-

Let U be an universe of discourse and F^{U} is the collection of all fuzzy subsets of U . Let E be the set of parameters or attributes and $A \subseteq E$ .Then the pair (F_A, E) is called **fuzzy Soft Set** over U such that

$$f_A : E \rightarrow \mathrm{F}^{\mathrm{U}}\ .$$

Thus a Fuzzy Soft Set over U can be represented by the set of order pairs

$$F_A = \{(x, f_A(x)); x \in E, f_A(x) \in F(U)\}$$

where $f_A(x)$ is called the membership function of $x \in E$.

Example 2:-
Let $\mathrm{U} = \{u_1, u_2, u_3, u_4, u_5\}$ be an universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a subset of parameters.

If $A = \{x_1, x_2, x_4\} \subseteq E$,$f_A(x_1) = \{0.9/u_2, 0.5/u_4\}$, $f_A(x_2) = U$ and $f_A(x_4) = \{0.2/u_1, 0.4/u_3, 0.8/u_5\}$
then the soft set F_A is written by

$$F_A = \{\ (x_1, \{0.9/u_2, 0.5/u_4\}),\ \ (x_2\ , U),\ \ (x_4, \{0.2/u_1, 0.4/u_3, 0.8/u_5\})\ \}\ \ .$$

2.3. Hypersoft Set :

Let $a_1, a_2, a_3, \ldots \ldots a_n$ be the distinct attributes whose attributes values belongs to the sets $A_1, A_2, A_3, \ldots A_n$ respectively, where $A_i \cap A_j = \emptyset$ for $\neq j$. A pair $(\phi, A_1 \times A_2 \times A_3 \times A_4 \times A_5)$ is called a **hypersoft set** over the universal set U, where ϕ is the mapping given by

$$\phi : A_1 \times A_2 \times A_3 \times A_4 \times A_5 \rightarrow P(U)\ .$$

Example 3:-

Suppose that a Mr. X wants to buy a computer from a market . There are three kind of computers which from the set of discourse $\mathrm{U} = \{u_1, u_2, u_3\}$. Also consider the set of attributes given as

$$E_1 = CPU\ Type = \{Ryzen(\alpha_1), Intel(\alpha_2)\ \}$$

$$E_2 = Case\ Size = \{Mid\ Toweer(\beta_1)\ , Full\ Tower(\beta_2)\ , Compact\ Case(\beta_3)\}$$

$$E_3 = Hard\ Drive = \{1TB(\gamma_1)\ , 512GB\ (\gamma_2)\ , 256GB(\gamma_3)\ \ \}$$

Suppose that

$$A_1 = \{\alpha_2\}\ ,\quad A_2 = \{\beta_2\ , \beta_3\}\ \ ,\quad A_3 = \{\gamma_1, \gamma_2\}$$

$$B_1 = \{\alpha_1, \alpha_2\},\qquad B_2 = \{\beta_1, \beta_2\}\ ,\quad B_3 = \{\gamma_1\}$$

are subset of E_i ,for each $= 1,2,3$. Then the Hypersoft set $(\phi_1, A_1 \times A_2 \times A_3)$ and $(\phi_2, B_1 \times B_2 \times B_3)$ defined as follows-

$$(\phi_1, A_1 \times A_2 \times A_3) = \quad \left\{ \begin{matrix} < (\alpha_2, \beta_2, \gamma_1), \{u_1\ , u_2\} >\ , < (\alpha_2, \beta_2, \gamma_2), \{u_1, u_2, u_3\} > \\ , < (\alpha_2, \beta_3, \gamma_1), \{u_1, u_3\} >, < (\alpha_2, \beta_3, \gamma_2), \{u_2, u_3\} > \end{matrix} \right\}$$

$$(\phi_2, B_1 \times B_2 \times B_3) = \quad \left\{ \begin{matrix} < (\alpha_1, \beta_1, \gamma_1), \{u_1, u_3\} >\ , < (\alpha_1, \beta_2, \gamma_1), \{u_2, u_3\} > \\ , < (\alpha_2, \beta_2, \gamma_1), \{u_1, u_3\} >, < (\alpha_2, \beta_2, \gamma_1), \{u_2, u_3\} > \end{matrix} \right\}$$

3. Fuzzy Hypersoft Set:

Let U be an initial universal set and $E_1 \times E_2 \times E_3 \times \dots \times E_n$ be a set of parameters . Let $A_1 \times A_2 \times A_3 \times \dots \times A_n$ be the non-empty subset of $E_1 \times E_2 \times E_3 \times \dots \times E_n$. Then the pair $(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ is called a **Fuzzy Hypersoft set** over U , where F is the mapping given by

$$\phi: A_1 \times A_2 \times A_3 \times \dots \times A_n \to \mathrm{F}^{\mathrm{U}} \ .$$

where F^{U} denotes the collection of all fuzzy subsets of U.

Example 4:-

We consider that attributes in example 3 .Then the fuzzy sets $(\phi_1, A_1 \times A_2 \times A_3)$ and $(\phi_2, B_1 \times B_2 \times B_3)$ defined as follows -

$$(\phi_1, A_1 \times A_2 \times A_3) = \begin{pmatrix} < (\alpha_2, \beta_2, \gamma_1), \left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}\right\} >, < (\alpha_2, \beta_2, \gamma_2), \left\{\frac{u_1}{0.2}, \frac{u_2}{0.5}, \frac{u_3}{0.1}\right\} > \\ , < (\alpha_2, \beta_3, \gamma_1), \left\{\frac{u_1}{0.6}, \frac{u_3}{0.7}\right\} >, < (\alpha_2, \beta_3, \gamma_2), \{\frac{u_2}{0.4}, \frac{u_3}{0.5}\} > \end{pmatrix}$$

$$(\phi_2, B_1 \times B_2 \times B_3) = \begin{Bmatrix} < (\alpha_1, \beta_1, \gamma_1), \left\{\frac{u_1}{0.4}, \frac{u_3}{0.7}\right\} >, < (\alpha_1, \beta_2, \gamma_1), \left\{\frac{u_2}{0.3}, \frac{u_3}{0.6}\right\} > \\ , < (\alpha_2, \beta_2, \gamma_1), \left\{\frac{u_1}{0.7}, \frac{u_3}{0.8}\right\} >, < (\alpha_2, \beta_2, \gamma_1), \{\frac{u_2}{0.5}, \frac{u_3}{0.7}\} > \end{Bmatrix}$$

3.2. Fuzzy Hypersoft Subset:

Assume that $(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ and $(\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ be the two fuzzy hypersoft sets over the same universal sets U .

(a) $(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ is the **Fuzzy Hypersoft subset** of $(\psi, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ denoted
$(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n) \subseteq (\psi, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ if
(i) $(A_1 \times A_2 \times A_3 \times \dots \times A_n) \subseteq (B_1 \times B_2 \times B_3 \times \dots \times B_n)$
(ii) $\forall e \in A_1 \times A_2 \times A_3 \times \dots \times A_n$ $\phi_1(e)$ and $\phi_2(e)$ are identical approximations .

(b) $(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ is **Fuzzy Hypersoft equal set** to $(\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ and is denoted by
$(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n) = (\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ if

(i) $(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n) \subseteq (\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ and

(ii) $(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n) \supseteq (\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$

3.3. Complement of Fuzzy Hypersoft Set:

If $(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ be the fuzzy hypersoft set and complement is denoted by $(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n)^c$ and it is defined in such a way that

$$(\phi, A_1 \times A_2 \times A_3 \times \dots \times A_n)^c = (\phi^c, \neg A_1 \times A_2 \times A_3 \times \dots \times A_n)$$

Where

$$\phi: A_1 \times A_2 \times A_3 \times \dots \times A_n \to \mathrm{F}^{\mathrm{U}} \text{ be a mapping as follows}$$

$$\phi^c(\alpha) = U - \phi(\neg\alpha) \ , \forall \alpha \in \neg A_1 \times A_2 \times A_3 \times \dots \times A_n$$

3.4.Union of Fuzzy Hypersoft Sets:

Assume that $(\phi_1, A_1 \times A_2 \times A_3 \times \dots \times A_n)$ and $(\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ be the two hypersoft sets over the same universal sets U, then union between them is denoted by
$(\phi_1 , A_1 \times A_2 \times A_3 \times \dots \times A_n) \cup (\phi_2, B_1 \times B_2 \times B_3 \times \dots \times B_n)$ is hypersoft set (ϕ, C)

where $C = A_1 \times A_2 \times A_3 \times \ldots \times A_n) \cup (B_1 \times B_2 \times B_3 \times \ldots \times B_n)$ and $\forall e \in C$, such that

$$F(e) = \begin{cases} \phi_1(e) & if\ e \in (A_1 \times A_2 \times A_3 \times \ldots \times A_n) - (B_1 \times B_2 \times B_3 \times \ldots \times B_n) \\ \phi_2(e) & if\ e \in (B_1 \times B_2 \times B_3 \times \ldots \times B_n) - (A_1 \times A_2 \times A_3 \times \ldots \times A_n) \\ \phi_1(e) \cup \phi_2(e) & if\ e \in (A_1 \times A_2 \times A_3 \times \ldots \times A_n) \cap (B_1 \times B_2 \times B_3 \times \ldots \times B_n) \end{cases}$$

3.5. Intersection of Fuzzy Hypersoft Sets

Assume that $(\phi_1, A_1 \times A_2 \times A_3 \times \ldots \times A_n)$ and $(\phi_2, B_1 \times B_2 \times B_3 \times \ldots \times B_n)$ be the two hypersoft sets over the same universal sets U, then intersection between them is denoted by $(\phi_1, A_1 \times A_2 \times A_3 \times \ldots \times A_n) \cap (\phi_2, B_1 \times B_2 \times B_3 \times \ldots \times B_n)$ is hypersoft set (ϕ, C) where $C = (A_1 \times A_2 \times A_3 \times \ldots \times A_n) \cap (B_1 \times B_2 \times B_3 \times \ldots \times B_n)$ and $\forall e \in C$, such that

$$F(e) = \phi_1(e) \cap \phi_2(e)$$

3.6. AND Operation

Let U be an initial universe set and (ϕ_1, A) and (ϕ_2, B) be two fuzzy hypersoft sets over the universe U. The 'AND' operation on them is denoted by $(\phi_1, A) \wedge (\phi_2, B) = (\psi, A \times B)$ is given by

$$(\psi, A \times B) = \{< u, \psi_{(\epsilon_1, \epsilon_2)}(u) >; u \in U, (\epsilon_1, \epsilon_2) \in A \times B\}$$

Where $\psi_{(\epsilon_1, \epsilon_2)}(u) = \{< u, \min\{\phi_{1_{\epsilon_1}}(u), \phi_{2_{\epsilon_2}}(u)\} >\}$

3.7. OR Operation

Let U be an initial universe set and (ϕ_1, A) and (ϕ_2, B) be two fuzzy hypersoft sets over the universe U. The 'OR' operation on them is denoted by $(\phi_1, A) \vee (\phi_2, B) = (\psi, A \times B)$ is given by

$$(\psi, A \times B) = \{< u, \psi_{(\epsilon_1, \epsilon_2)}(u) >; u \in U, (\epsilon_1, \epsilon_2) \in A \times B\}$$

Where $\psi_{(\epsilon_1, \epsilon_2)}(u) = \{< u, \max\{\phi_{1_{\epsilon_1}}(u), \phi_{2_{\epsilon_2}}(u)\} >\}$.

Example 5:-

We consider that attributes in example 3 .Then the fuzzy sets $(\phi_1, A_1 \times A_2 \times A_3)$ and $(\phi_2, B_1 \times B_2 \times B_3)$ defined as follows-

$$(\phi_1, A_1 \times A_2 \times A_3) = \left\{ \begin{array}{l} < (\alpha_2, \beta_2, \gamma_1), \left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}\right\} >, < (\alpha_2, \beta_2, \gamma_2), \left\{\frac{u_1}{0.2}, \frac{u_2}{0.5}, \frac{u_3}{0.1}\right\} > \\ , < (\alpha_2, \beta_3, \gamma_1), \left\{\frac{u_1}{0.6}, \frac{u_3}{0.7}\right\} >, < (\alpha_2, \beta_3, \gamma_2), \left\{\frac{u_2}{0.4}, \frac{u_3}{0.5}\right\} > \end{array} \right\}$$

$$(\phi_2, B_1 \times B_2 \times B_3) = \left\{ \begin{array}{l} < (\alpha_1, \beta_1, \gamma_1), \left\{\frac{u_1}{0.4}, \frac{u_3}{0.7}\right\} >, < (\alpha_1, \beta_2, \gamma_1), \left\{\frac{u_2}{0.3}, \frac{u_3}{0.6}\right\} > \\ , < (\alpha_2, \beta_2, \gamma_1), \left\{\frac{u_1}{0.7}, \frac{u_3}{0.8}\right\} >, < (\alpha_2, \beta_2, \gamma_1), \left\{\frac{u_2}{0.5}, \frac{u_3}{0.7}\right\} > \end{array} \right\}$$

Let us assume that, $(\alpha_2, \beta_2, \gamma_1) = J_1$, $(\alpha_2, \beta_2, \gamma_2) = J_2$, $(\alpha_2, \beta_3, \gamma_1) = J_3$, $(\alpha_2, \beta_3, \gamma_2) = J_4$ in $(\phi_1, A_1 \times A_2 \times A_3)$ and $(\alpha_1, \beta_1,) = K_1$, $(\alpha_1, \beta_2, \gamma_1) = K_2$, $(\alpha_2, \beta_2, \gamma_1) = K_3$,$(\alpha_2, \beta_2, \gamma_1) = K_4$ in $(\phi_2, B_1 \times B_2 \times B_3)$.

The tabular form of these sets are as follows.

$(\phi_1, A_1 \times A_2 \times A_3)$	u_1	u_2	u_3
J_1	0.3	0.4	0
J_2	0.2	0.5	0.1
J_3	0.6	0	0.7

J_4	0	0.4	0.5

$(\phi_2, B_1 \times B_2 \times B_3)$	u_1	u_2	u_3
K_1	0.4	0	0.7
K_2	0	0.3	0.6
K_3	0.7	0	0.8
K_4	0	0.5	0.7

Then the AND and OR operations of these sets are given as below.

$(\phi_1, A_1 \times A_2 \times A_3) \wedge (\phi_2, B_1 \times B_2 \times B_3)$	u_1	u_2	u_3
$J_1 \times K_1$	0.3	0	0
$J_1 \times K_2$	0	0.3	0
$J_1 \times K_3$	0.3	0	0
$J_1 \times K_4$	0	0.4	0
$J_2 \times K_1$	0.2	0	0.1
$J_2 \times K_2$	0	0.3	0.1
$J_2 \times K_3$	0.2	0	0.1
$J_2 \times K_4$	0	0.5	0.1
$J_3 \times K_1$	0.4	0	0.7
$J_3 \times K_2$	0	0	0.6
$J_3 \times K_3$	0.6	0	0.7
$J_3 \times K_4$	0	0	0.7
$J_4 \times K_1$	0	0	0.5
$J_4 \times K_2$	0	0.3	0.5
$J_4 \times K_3$	0	0	0.5
$J_4 \times K_4$	0	0.4	0.5

$(\phi_1, A_1 \times A_2 \times A_3) \vee (\phi_2, B_1 \times B_2 \times B_3)$	u_1	u_2	u_3
$J_1 \times K_1$	0.4	0.4	0.7
$J_1 \times K_2$	0.3	0.4	0.6
$J_1 \times K_3$	0.7	0.4	0.8
$J_1 \times K_4$	0.3	0.5	0.7
$J_2 \times K_1$	0.4	0.5	0.7
$J_2 \times K_2$	0.2	0.5	0.6
$J_2 \times K_3$	0.7	0.5	0.8

$J_2 \times K_4$	0.2	0.5	0.7
$J_3 \times K_1$	0.6	0	0.7
$J_3 \times K_2$	0.6	0.3	0.7
$J_3 \times K_3$	0.7	0	0.8
$J_3 \times K_4$	0.6	0.5	0.7
$J_4 \times K_1$	0.4	0.4	0.7
$J_4 \times K_2$	0	0.4	0.6
$J_4 \times K_3$	0.7	0.4	0.8
$J_4 \times K_4$	0	0.5	0.7

3. Conclusions:

In this paper, we have discussed the fundamentals of fuzzy hypersoft set such as fuzzy hypersoft subset, complement, union, Intersection, AND and OR operators. This results will be very helpful for future experts to enhance the work for Intuitionistic fuzzy hypersoft set, Neutrosophic fuzzy hypersoft set, Plithogenic hypersoft set and hypersoft multi sets among others .

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