A Study of Number Sense: Pre-Service Teachers Learning

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Abstract

The purpose of this study was to identify the use of number sense strategies by pre-service teachers studying at the department of elementary education. Compared to the previous one; new mathematics curriculum places more emphasis on various strategies such as estimation strategies, computational estimation strategies, rounding and mental computation, which are among the fundamental components of the number sense. The study was undertaken within the framework of the question "What strategies do the pre-service teachers, who will implement this curriculum, use while solving problems requiring number sense?". 133 pre-service teachers from the Elementary Education Department in a State University in the province of Istanbul participated in this study. "Number Sense Test" about five different number sense components was used as a testing instrument. Findings were analyzed with qualitative and quantitative methods. At the end of the research, pre-service teachers' number sense was found to be very low. When solution methods were analyzed, it was found that pre-service teachers preferred using "rule based methods" instead of "number sense" in each of the components. This finding was consistent with prior studies that show pre-service teachers' preference of written methods to number sense methods. With regard to research findings, suggestions were offered for future studies by emphasizing the necessity for measures to increase pre-service teachers' knowledge on number sense as well as its use.

Keywords: *Number Sense-Based Strategies, Partial Number Sense-Based Strategies, Estimation, Pre-service Elementary Teachers.*

1. INTRODUCTION

The importance of number sense in school mathematics has been highlighted by many national reports. However, research has shown that many students in the middle grades are poor in number sense. Many factors, such as lack of attention to number sense in mathematics textbooks together with heavy emphasis on rules associated with written computation may account for this situation. The number sense knowledge that teachers have and the value they place on its importance are other factors. Thus children's lack of number sense may be partly due to their teachers' lack of number sense as well as not knowing how to help students develop number sense.

Reys (1994) advocates that "teachers play an important role in building number sense in the type of classroom environment they create, in the teaching practices they employ, and in the activities they select.".She goes on to argue that number sense becomes meaningful and valuable to students when teachers believe that developing number sense is more important than mastering the rules associated with written computation. Research has demonstrated that teachers play a key role in helping children develop number sense through creating a good learning environment that encourages children to explore numbers, operations, and their relationships freely and meaningfully.

When a student is asked to do the multiplication of 4.5 x 1.2, s/he gives the answer "54.0" by multiplying decimal numbers and by paying attention to the decimal digit (Reys et al., 1991, p. 3). When a thirteen-year-old student was asked to estimate the addition of 12 3 and 7 8 in America, among the choices of 1, 2, 19, 21 and "I don't know", 50% of the students chose the alternatives of 19 or 21 (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). When 8th-grade students were asked the question "How many fractions are there between 2 5 and 3 5 ?" 46% of these students answered as "There is no fraction" (McIntosh, Reys, & Reys, 1992). When the question "If 750 is divided by 0.98, is the result larger, smaller or equal to 750?" was posed to the students in Turkey, 74% of the 4th-grade students and 70% of the 5th-grade students could not give the correct answer. Most of the students who provided the

correct answer did so only after performing mathematical operations. Generally, the students thought that the answer should be smaller than 750 by making a generalization about the fact that division makes numbers smaller (Şengül & Gürel, 2003). There aremany examples to the lack of number sense such as the ones stated above. Answers provided to those kinds of questions present student levels to estimate and understand the effects of mathematical operations concerning numbers and number sense.

2. WHAT IS NUMBER SENSE?

According to Reys et al. (1999) number sense refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. As Schneider and Thompson (2000) state a student who has a good number sense is successful in flexible thinking about numbers, understanding their meanings and the relationships among them. Development of number sense is important in mathematics education. The National Council of Teachers of Mathematics (NCTM), in their Principles and Standards for School Mathematics, notes that number sense is one of the foundational ideas in mathematics in that students (1) Understand numbers, ways of representing numbers, relationships among numbers, and number system; (2) Understand meanings of operations and how they related to one another; (3) Compute fluently and make reasonable estimates (NCTM, 2000, p. 32).

3. NUMBER SENSE COMPONENTS

Number sense is a complex process that includes many different components of numbers, operations and their relationships and it has generated much research and discussions among mathematics educators, cognitive psychologists, researchers, teachers and mathematics curricula developers. As a result, different psychological perspectives have been provided; theoretical frameworks of number sense have been proposed; characteristics of number sense have been described and essential components of number sense have been enumerated. Based on a review of the number sense literature, this study focused on number sense to include:

• Understanding of the meaning and size of numbers: This skill is associated with the ability to recognize the relative size of numbers. For example, when a student is asked to compare 2 5 with, 1 2 knowledge of how to do this is the indicator of this skill.

• Understanding the meaning and effect of operations: This component is related to the ability to recognize how the result will change when operations or numbers are changed in calculations.

• Understanding and use of equivalent expressions: It is the ability to know the equivalent numbers and using them when necessary. For example, being able to answer the question, "Which product of m number gives the same result when the m number is divided by 0.25?"

• Flexible computing and counting strategies for mental computation: Individual problem solving without resorting to written calculations and estimations in order to investigate the appropriateness of the result emphasizes the ability to do mental calculations.

• Measurement benchmarks: This skill is comprised of the ability to determine and use reference points that can vary according to situations.

In the last 20 years, studies on the improvement of number sense have been carried out with increasing interest. With respect to the increased importance of number sense, it is a significant responsibility to improve students' number sense in mathematics education. Both international studies and the studies carried out in Turkey in the field demonstrate that the primary school students' number senses are low.

4. THE IMPORTANCE OF MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE

Several studies report the importance of mathematics teachers' subject matter knowledge and assert that the mathematical knowledge teachers possess has a profound impact on what and how they teach argues that if teachers are to promote and teach students to recognize reasonable answers, the teachers must then respect and value the

importance of reasonable answers. Schifter (1999) says mathematics teachers need to "understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise." Furthermore, the Principles and Standards for School Mathematics (NCTM, 2000) emphasizes that "effective teaching requires knowing and understanding mathematics.". These statements significantly underscore the importance of mathematics teacher's subject matter knowledge. Teachers play a key role in teaching number sense, and lead children to learn and value the importance of number sense. Research in Kuwait examined one aspect of number sense, namely reasonableness of answers. This research reported low performance of eighth graders on questions related to determining the reasonableness of answers to a range of mathematics problems. It also reported that teachers valued exact answers and did not consider determining whether answers were reasonable an important instructional goal. Severalstudies have demonstrated that teachers need to have a profound understanding on number sense if they are to capitalize on classroom events and help students develop number sense. Teachers need to create a learning environment that provides opportunities for students to explore numbers, ask questions, and handle unexpected student responses that arise as operations and their relationships with numbers are encountered. If teachers don't understand the mathematics and have a solid knowledge of number sense, it is unlikely they will be able to promote number sense in their students. Investigating pre-service teachers' understanding of number sense will provide baseline data that may be useful in reshaping future experiences in mathematics education courses for pre-service elementary teachers.

5. CONCLUSION

This study suggests that teacher education should put more emphasis on helping pre-service teachers break away from the shackles of rule-based written computation and broaden their ability to include important features of number sense as an integral part of number development. If we want to improve students' number sense, then action should be taken to improve the quality of their teachers' knowledge on number sense.Reys et al. (1999) stated in their study regarding the number sense of the students that although performance levels of the students concerning number sense differentiated on the basis of the countries, students showed consistently low performance. In Turkey, similar findings were stated in the studies of Harç (2010) and Kayhan Altay (2010). According to Ekenstam (1977) number sense covers the improvement of various relationships among mathematical concepts, knowledge and skills; therefore, it provides access to many concepts at the same time when necessary. Students who do not comprehend these relationships have to remember and learn various rules in order to cope with practical problems in everyday life. Besides, it is stated that over-emphasized standard written algorithms prevent students from both using number sense and from improving important thinking skills such as reasoning, estimating and interpreting. On the other hand, by instructing pre-service teachers on the current situation of the students' number sense, it will be possible to provide them with experiences related to how students' number sense abilities can be improved, how lesson plans can be prepared and what kind of activities should be used. Additionally, pre-service teachers can understand the necessity and importance of number sense and this will help them to improve students' number sense as well by providing pre-service teachers with proper training programs in which mental calculations and estimation skills can be improved. For future studies, it will be useful to analyze the relationships among pre-service teachers' meta-cognitive levels and their abilities to use number sense; mathematical self-efficacy and the level of using number sense components, classroom teachers' level of mathematical attitudes and concerns and their abilities to use number sense components.

There is an important mathematics curriculum reform effort underway in Taiwan. However, number sense has not been visible in the mathematics textbooks or integrated into teacher education programs. If significant improvements in mathematics education are to be made, the topic of number sense must become a more central focus in teacher education programs. If we want to help children develop number sense, we must first enhance teachers' number sense. Teachers empowered with knowledge and appreciation for number sense will be more likely to attend to number sense when working with students. While the focus of this research has been on Taiwan, children and teachers in other countries, such as Australia, Kuwait, Japan, and Sweden, have similar struggles. Breaking the shackles of rotely applying algorithms and promoting greater development and application of number sense components in mathematical problem solving is an international challenge. We hope our findings will encourage other institutions to examine the role number sense plays in their teacher education programs.

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