A Study on the Numerical Methods of Solving Transportation Problem

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ABSTRACT

Transportation model has been considered as one of the important applications of Linear Programming Problem (LPP). It is mainly concerned in determining the schedule for transportation of goods to a specified place in such a way that minimizes the shipping cost and satisfies all the supply and demand constraints. Transportation model has been considered as a very lucrative method in logistics and supply chain management for reducing cost and improving service. As the transportation problem modeled as linear programming problem, it can be solved by simplex method but this is very time consuming and complex technique. Hence different methods and algorithms have been developed to support the transportation problem and to find the solution in a very easy manner. Some popular methods are simplex method, Vogel Approximation Method (VAM), North West Corner Rule (NWCR), Least Cost Method, Row Minimum Method; Column Minimum method. The transportation problems can be solved by using the simplex methods, which is a time-consuming solution.
The main aim of these thesis transportation problem solution methods is to minimize the cost or the time of transportation. Most of the currently used methods for solving transportation problems are trying to reach the optimal solution, whereby, most of these methods are considered complex and very expansive in term of the execution time. In this study we use the best candidate method (BCM), in which the key idea is to minimize the combinations of the solution by choosing the best candidates to reach the optimal solution.

Keyword: *VAM, NWCR, BCM, Optimal Solution, Key, Minimize*

Introduction

The optimal cost is desirable in the movement of raw materials and goods from the sources to destinations. Mathematical model known as transportation problem tries to provide optimal costs in transportation system. Some well known and long use algorithms to solve transportation problems are Vogel's Approximation Method (VAM), North West Corner (NWC) method, and Matrix Minima method. VAM and matrix minima method always provide IBFS of a transportation problem. Afterwards many researchers provide many methods and algorithms to solve transportation problems. Some of the methods and algorithms that the current research has gone through are: 'Modified Vogel's Approximation Method for Unbalance Transportation Problem' [1] by N. Balakrishnan; Serder Korukoglu and Serkan Balli's 'An Improved Vogel's Approximation Method (IVAM) for the Transportation Problem' [2]; Harvey H. Shore's 'The Transportation Problem and the Vogel's Approximation Method' [3]; 'A modification of Vogel's Approximation Method through the use of Heuristics' [4] by D.G. Shimshak, J.A. Kaslik and T.D. Barelay; A. R. Khan's **'**A Re-solution of the Transportation Problem: An Algorithmic Approach' [5]; 'A new approach for finding an Optimal Solution for Trasportation Problems' by V.J. Sudhakar, N. Arunnsankar, and T. Karpagam [6]. Kasana and Kumar [7] bring in extreme difference method calculating the penalty by taking the differences of the highest cost and lowest cost in each row and each column. The above mentioned algorithms are beneficial to find the IBFS to solve transportation problems. Besides, the current research also presents a useful algorithm which gives a better IBFS in this topic.
Example The per unit transportation cost (in thousand dollar) and the supply and demand (in number) of motor bikes of different factories and showrooms are given in the following transportation table.

Factories	Showrooms				Supply (a_i)
	D_1	D_2	D_3	D_4	
W_1	9	8	5	7	12
W_2	4	6	8	7	14
W_3	5	8	9	5	16
Demand (b_j)	8	18	13	3	42

We want to solve the transportation problem by the current algorithm.

Solution

The row differences and column differences are:

Factories	Showrooms				Supply
	D_1	D_2	D_3	D_4	
W_1	9^{4}_{5}	8^{3}_{2}	5^{0}_{0}	7^{2}_{2}	12
W_2	4^{0}_{0}	6^{2}_{0}	8^{4}_{3}	7^{3}_{2}	14
W_3	5^{0}_{1}	8^{3}_{2}	9^{4}_{4}	5^{0}_{0}	16
Demand	8	18	13	3	42

The allocations with the help of ARP and ACP are:

Factories	Showrooms				Supply				
	D_1	D_2	D_3	D_4		ARP			
W_1	9	8	$^{12}5$	7	12	(2.2)	-	-	-
W_2	$^{8}4$	$^{6}6$	8	7	14	(2.2)	(2.2)	(1)	(1)
W_3	5	$^{12}8$	$^{1}9$	$^{3}5$	16	(1.7)	(1.7)	(2.3)	(0.5)
Demand	8	18	13	3	42				
ACP	(2)	(1.3)	(2.3)	(1.3)					
	(0.5)	(1)	(0.5)	(1)					
	-	(1)	(0.5)	(1)					
	-	(1)	(0.5)	-					

The transportation cost is $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

$$z = 5 \times 12 + 4 \times 8 + 6 \times 6 + 8 \times 12 + 9 \times 1 + 5 \times 3$$
$$= 248 \$$$

Vogel's Approximation Method (VAM) is one of the conventional methods that gives better Initial Basic Feasible Solution (IBFS) of a Transportation Problem (TP). This method considers the row penalty and column penalty of a Transportation Table (TT) which are the differences between the lowest and next lowest cost of each row and each column of the TT respectively. In a little bit different way, the current method consider the Average Row Penalty (ARP) and Average Column Penalty (ACP) which are the averages of the differences of cell values of each row and each column respectively from the lowest cell value of the corresponding row and column of the TT. Allocations of costs are started in the cell along the row or column which has the highest ARP or ACP. These cells are called basic cells. The details of the developed algorithm with some numerical illustrations are discussed in this article to show that it gives better solution than VAM and some other familiar methods in some cases.

The optimal cost is desirable in the movement of raw materials and goods from the sources to destinations. Mathematical model known as transportation problem tries to provide optimal costs in transportation system. Some well known and long use algorithms to solve transportation problems are Vogel's Approximation Method (VAM), North West Corner (NWC) method, and Matrix Minima method. VAM and matrix minima method always provide IBFS of a transportation problem. Afterwards many researchers provide many methods and algorithms to solve transportation problems. Some of the methods and algorithms that the current research has gone through are: 'Modified Vogel's Approximation Method for Unbalance Transportation Problem' [1] by N. Balakrishnan; Serder Korukoglu and Serkan Balli's 'An Improved Vogel's Approximation Method (IVAM) for the Transportation Problem' [2]; Harvey H. Shore's 'The Transportation Problem and the Vogel's Approximation Method' [3]; 'A modification of Vogel's Approximation Method through the use of Heuristics' [4] by D.G. Shimshak, J.A. Kaslik and T.D. Barelay; A. R. Khan's 'A Re-solution of the Transportation Problem: An Algorithmic Approach' [5]; 'A new approach for finding an Optimal Solution for Trasportation Problems' by V.J. Sudhakar, N. Arunnsankar, and T. Karpagam [6]. Kasana and Kumar [7] bring in extreme difference method calculating the penalty by taking the differences of the highest cost and lowest cost in each row and each column. The above mentioned algorithms are beneficial to find the IBFS to solve transportation problems. Besides, the current research also presents a useful algorithm which gives a better IBFS in this topic.

Ttransportation problem is concerned with the optimal pattern of the product units' distribution from several original points to several destinations. Suppose there are m points of original $A_1, \ldots, A_i, \ldots, A_m$ and n destinations $B_1, \ldots, B_j, \ldots, B_n$. The point $A_i (i = 1, \ldots, m)$ can supply ai units, and the destination $B_j (j = 1, \ldots, n)$ requires bj units (see Equation 1).

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_i \tag{1}$$

Whereby, the cost of shipping a unit from A_i to Bj, is computed as c_{ij}. As well as, the problem in determining the optimal distribution pattern consists the pattern for which shipping costs are at a

minimum. Moreover, the requirements of the destinations Bj, j = 1, . . ., n, must be satisfied by the supply of available units at the points of origin Aj, i = 1, . . .,m.

As shown by Equation 2, if *xij* is the number of units that are shipped from *Ai* to *Bj*, then the problem in determining the values of the variables *xij*, *i* = 1, . . ., *m* and *j* = 1, . . ., *n*, should minimize the total of the shipping costs.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \tag{2}$$

While,

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i \,;\; i = 1, \ldots, m \\ &\sum_{i=1}^{m} x_{ij} = b_j \,;\; j = 1, \ldots, n \\ &x_{ij} > 0 \,;\, i = 1, \ldots, m \,;\; and\; j = 1, \ldots, n \end{aligned}$$

Mathematically, the transportation problem can be represented as a linear programming model. Since the objective function in this problem is to minimize the total transportation cost as given by Equation 3.

$$Z = c_{11}\, x_{11} + c_{12}\, x_{12} + \cdots + c_{mn}\, x_{mn} \tag{3}$$

Equation 3 is a mathematical formulation of a transportation problem that can adopt the linear programming (LP) technique with equality constraints. LP technique can be used in different product areas such as oil plum industry [1]. However, The LP technique can be generally used by

genetic algorithm such as Sudha at el. article [2]. The transportation solution problem can be found with a good success in the improving the service quality of the public transport systems [3]. Also it is found in Zuhaimy Ismail at el. article [4]. As well as, the transportation solution problem is used in the electronic commerce where the area of globalization the degree of competition in the market article [5], and it can be used in a scientific fields such as the simulated data for biochemical and chemical Oxygen demands transport [6], and many other fields.

Objective:

1. To find the transportation problem solution methods is to minimize the cost or the time of transportation.
2. The best candidate method (BCM), in which the key idea is to minimize the combinations of the solution by choosing the best candidates to reach the optimal solution.

3. To compare the best method of solving the transportation problem with the help of example

Literature Review

Ilija NIKOLIĆ (2007) Total transportation time problem regarding the time of the active transportation routes. If the multiple optimal solutions exist, it is possible to include other criteria as second level of criteria and find the corresponding solutions. Furthermore, if there is a multiple solution, again, the third objective can be optimized in lexicographic order. The methods of generation of the optimal solution in selected cases are developed. The numerical example is included.

F. Cornillier, F. F. Boctor, G. Laporte, and J. Renaud (2008), Intermodal Transportation Problems. The Intermodal Transportation Problems using more than one transportation mode are as follows. The main variants of the Intermodal Transportation Problems are intermodal multicommodity routing problem with scheduled services, tour planning problem (TPP) [50], tourist trip design problems, railroad blocking problem (RBP), and intertemporal demand for international tourist air travel.

F. Cornillier, G. Laporte, F. F. Boctor, and J. Renaud (2009), Transportation Mode. Buses; trucks; motorcycles; bicycles; cars and pickups; box trucks and dock highs; cargo and sprinter vans; less than truck Load (LTL); full truck load (FTL); flatbeds; reefers (refrigerated units); longer combination vehicles (LCV) with double semitrailer, recovery vehicle, scooters, and pedestrians; main battle tank; infantry fighting vehicles; armored personnel carriers; light armored vehicles; self-propelled artillery and anti-Air mine protected vehicles; combat engineering vehicles; prime movers and trucks; unmanned combat vehicles; military robot; joint light tactical vehicle (JLTV), utility vehicle, refrigerator truck, landfill compaction vehicle, garbage truck, waste collection vehicle; armored cash transport car; and security van.

P. Pandian and G.Natarajan (2010), Transportation cost has significant impact on the cost and the pricing of raw materials and goods. Supplier and producer try to control the cost of transportation. The way how the desirable transportation cost can be obtained is the subject matter of transportation problems in linear programming. Some conventional methods to find the minimum transportation cost are North West Corner (NWC) method, Matrix Minima method/ Least Cost method, Row Minima method, Column Minima method, and Vogel's Approximation Method (VAM).

Aminur Rahman Khan (2011), Matrix Minima method and VAM are considered to provide the better IBFS. Besides the covenantal methods many researchers has provide many methods to find a better IBFS of a TP. Some of the important related works the current research has deal with are: 'A New Approach for Solving Transportation Problems with Mixed Constraints'.

V.J. Sudhakar, N. Arunnsankar, T. Karpagam, (2012) calculate the Difference Indicators by taking the difference of the largest and the next largest cell value of each row and each column of the TOCT for the allocation of units of the TT. The cited algorithms in this article are beneficial to find the IBFS to solve transportation problems. Also, the current research presents a useful algorithm which gives a better IBFS in this connection.

C. I. Hsua, H. C. Lib, and L. H. Yanga (2013), Rocket-powered aircraft, nuclear powered aircraft, spacecraft, space shuttle, space, space planes, rockets, missiles, and advanced Hall electric propulsion, crew exploration vehicle (CEV), automated transfer vehicle (ATV), evolved expendable launch (EELV), National Aerospace Plane (NASP), transatmospheric vehicle (TAV), orbital space plane, next generation launch technology, winged shuttle (WSLEO), expendable interorbital ferry vehicle (EIOFV), reusable interorbital ferry vehicle (RIOFV), and spaceship.

Prof. Reena. G. Patel, Dr. P.H. Bhathawala. (2014), The current method is an attractive method which is very simple, easy to understand and gives result exactly or even lesser to VAM method. The solution obtained by the current method is near modi method.

Ocotlán Díaz-Parra,1 Jorge A. Ruiz-Vanoye (2014), The aims at being a guide to understand the different types of transportation problems by presenting a survey of mathematical models and algorithms used to solve different types of transportation modes (ship, plane, train, bus, truck, Motorcycle, Cars, and others) by air, water, space, cables, tubes, and road. Some problems are as follows: bus scheduling problem, delivery problem, combining truck trip problem, open vehicle routing problem, helicopter routing problem, truck loading problem, truck dispatching problem, truck routing problem, truck transportation problem, vehicle routing problem and variants, convoy routing problem, railroad blocking problem (RBP), inventory routing problem (IRP), air traffic flow management problem (TFMP), cash transportation vehicle routing problem, and so forth.

Khan, A.R., Vilcu, A., Sultana, N. and Ahmed, S.S. (2015), In today's highly competitive market, various organizations want to deliver products to the customers in a cost effective way, so that the market becomes competitive. To meet this challenge, transportation model provides a powerful framework to determine the best ways to deliver goods to the customer. In this article, a new approach named allocation table method (ATM) for finding an initial basic feasible solution of transportation problems is proposed. Efficiency of allocation table method has also been tested by solving several number of cost minimizing transportation problems and it is found that the allocation table method yields comparatively a better result. Finally it can be claimed that the allocation table method may provides a remarkable Initial Basic Feasible Solution by ensuring minimum transportation cost. This will help to achieve the goal to those who want to maximize their profit by minimizing the transportation cost.

A. Amaravathy, K. Thiagarajan and S. Vimala (2016), A transportation problem has been widely studied in computer science and Operation Research. It is one of the fundamental problems of network flow problem which is usually use to minimize the transportation cost for industries with number of destination while satisfying the supply limit and earliest and most important applications of linear programming problem.

S.M. Abul Kalam Azad , Md. Bellel Hossain , Md. Mizanur Rahman (2017), The current new algorithmic approach to solve the Transportation Problem (TP) is based upon the Total Opportunity Cost (TOC) of a Transportation Table (TT). Opportunity cost in each cell along each row of a TT is the difference of the corresponding cell value from the lowest cell value of the corresponding row. Similarly, opportunity cost in each cell along each column of a TT is the difference of the corresponding cell value from the lowest cell value of the corresponding column. Total Opportunity Cost Table (TOCT) is formed by adding the opportunity cost in each cell along each row and the opportunity cost in each cell along each column and putting the summation value in the corresponding cell. The current algorithm considers the average of total opportunity costs of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of total opportunity costs of cells along each column identified as Column Average Total Opportunity Cost (CATOC). Allocations of costs are started in the cell along the row or column which has the highest RATOCs or CATOCs. The Initial Basic Feasible Solution (IBFS) obtained by the current method is better than some other familiar methods which are discussed in this article.

Sushma Duraphe , Sarla Raigar (2017), we are trying to find the optimum solution of a transportation problem and is to minimize the cost. The current new algorithmic approach to solve the transportation problem is based upon the Total Opportunity Cost (TOC) of a transportation table (TT) and maximum minimum penalty approach. The most attractive feature of this method is that it requires very simple arithmetical and logical calculation, which compared to the existing method an optimal solution and illustrated with numerical example.

Niluka Rodrigo, Lashika Rjapaksha (2018), The problem of locating distribution centers is one of the most important issues in design of supply chain. The design of the distribution system is an important issue for almost every company. Wide range of problems arising in practical applications can be formulated as Mixed-integer nonlinear Models. Multi-commodity distribution system design is a generalization of a facility location problem where we have several commodities, and shipment from a plant to customer occurs through a distribution center. This report presents a real life distribution problem. The problem is to determine which distribution centers to use so that all customer demands are satisfied, production capacities are not exceeded, and the total distribution cost that is the fixed cost of operating the distribution center and the transportation cost is minimized. A computer program (Software R) is developed to obtain the optimal solution.

Materials and Method

Transportation model: In a transportation problem, we are focusing on the original points. These points may represent factories to produce items, and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to

illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory i ($i = 1, 2, \ldots, m$) produces a_i units, and the warehouse j ($j = 1,2, \ldots, n$) requires b_j units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij}. The decision variables x_{ij} is being the transported amount from the factory i to

the warehouse j. Typically, our objective is to find the transportation pattern that will minimize the total of the transportation cost (see Table 1).

Table 1: The model of a transportation problem

Origins (Factories)	Destinations (Warehouses)		Available	
	1 2 …….. n			
1	c_{11}	c_{12}	c_{1n}	a_1
2	c_{21}	c_{22}	c_{2n}	a_2
…	…	…	…	…
m	c_{m1}	c_{m2}	c_{mn}	a_m
Required	b_1	b_2	b_n	

Algorithms for solving: ther are several algorithm for solving ttransportation problems which are based on different of special linear programming methods, among these are:
1. Northwest Corner method
2. Minimum cost method
1. Genetic algorithm
3. Vogel's approximation method
4. Row Minimum Method
5. Column Minimum Method

Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields smaller objective value. In this study, we used the Vogel's approximation method, since it generally produces better starting solutions than other solving methods; as well we have used the BCM solution steps [8].

Proposed Method

In this study, we proposed a new solving method for transportation problems by using BCM. The proposed method must operate the as following:

Step1: We must check the matrix balance, if the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Step2: Appling BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.

Step3: Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit\ cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an element with lowest cost comparing to the determined element in the chosen combination, then we elect it.

Step4: Elect the next least cost from the chosen combination and repeat Step 3 until all columns and rows are exhausted.

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