# Advancement of Patterns of Remarkable forms of Polygonal Numbers with Amicable Conditions

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**Abstract:**

Technique to construct the sequences of Diophantine 3-tuple with the property are illustrated in this paper.

**Keywords:** Diophantine 3-tuple, m-gonal numbers, Icosioctgonal numbers.

**1. Introduction:** In this article, a pattern of numbers known as a Diophantine 3-tuple is demonstrated. A set of three distinct integers is called a Diophantine 3-tuple with property D(n) if the product of any two members of the set with the addition of n (a non-zero integer or a polynomial with integer coefficients) is a perfect square. [1-13] provides an extensive review of various problems on Diophantine triples with suitable properties. This paper describes the process of obtaining sequences of Diophantine 3-tuple with property $D\,(-431)$ and $D\,(-456)$.

**2. Development of Patterns of significant forms of Polygonal numbers.**

**2.1: Patterns of 3-tuples with $(3,2)$ – Icosihexagonal numbers.**

The general term of sequence of 26-side $m$-gonal numbers named as Icosihexagonal is defined by

$$M(26,n) = 12n^2 - 11n$$

The $n^{th}$ member of sequence of (3,2) – Icosihexagonal number is provided the relation

$$I_{(3,2)}(n) = 2M(26,n) + 3M(26,n+1)$$

Generate

$$\omega_1 = I_{(3,2)}(n) = 60n^2 + 17n + 3$$

$$\omega_2 = I_{(3,2)}(n+2) = 60n^2 + 257n + 277$$

where $\omega_1\omega_2 - 431 = (60n^2 + 137n + 20)^2$

Assume that

$$\omega_1\omega_2 - 431 = \alpha^2 \quad (1)$$

To stretch this preferred 2-tuples into 3-tuples, let $\omega_3$ be the third member combined with the conditions that

$$\omega_1\omega_3 - 431 = \beta^2 \quad (2)$$

$$\omega_2\omega_3 - 431 = \gamma^2 \quad (3)$$

Resolving (2) and (3) by utilizing simple arithmetic operations, it is evaluated by

$$431(\omega_1 - \omega_2) = \omega_2\,\beta^2 - \omega_1\gamma^2 \quad (4)$$

Let $\beta = A_1 + \omega_1$ and $\gamma = A_1 + \omega_2(n)$ (5)

Substitute (5) in (4), the value of $\nabla$ and hence $\beta$ is calculated by

$$A_1 = \alpha \text{ and } \beta = \omega_1 + \alpha \quad (6)$$

Employing (6) in (2) and comparing the resultant equation with (1), it is noted that

$$\omega_3 = \omega_1 + \omega_2 + 2\alpha = 240n^2 + 548n + 320 \quad (7)$$

Consequently, $\{60n^2 + 17n + 3, 60n^2 + 257n + 277,\ 240n^2 + 548n + 320\}$ is a 3-tuples consisting (3,2)-Icosihexagonal numbers with the provision that the deduction of 431 with the product of two members remains a square number.

In a similar way if $(\omega_2, \omega_3)$, $(\omega_3, \omega_4)$ etc are pairs concerning (3,2)-Icosihexagonal numbers, then following the above procedure it is possible to stretch into patterns of triples $(\omega_2, \omega_3, \omega_4)$, $(\omega_3, \omega_4, \omega_5)$ etc where

$\omega_4 = 540n^2 + 1593n + 1191$ and $\omega_5 = 1500n^2 + 4025n + 2745$.

Hence, it is concluded that $(\omega_1, \omega_2, \omega_3)$, $(\omega_2, \omega_3, \omega_4)$, $(\omega_3, \omega_4, \omega_5)$ etc are special forms of triples in which the product of two numbers subtracted by 431 is a square.

**Numerical illustrations for limited cases of $n$ are given in table below:**

| $n$ | $(\omega_1, \omega_2, \omega_3)$ | $(\omega_2, \omega_3, \omega_4)$ | $(\omega_3, \omega_4, \omega_5)$ |
|---|---|---|---|
| 1 | (80, 594, 1108) | (594, 1108, 3324) | (1108, 3324, 8270) |
| 2 | (277, 1031, 2376) | (1031, 2376, 6537) | (2376, 6537, 16795) |
| 3 | (594, 1588, 4124) | (1588, 4124,10830) | (4124, 10830, 28320) |
| 4 | (1031, 2265, 6352) | (2265, 6352, 16203) | (6532, 16203, 42845) |
| 5 | (1588, 3062, 9060) | (3062, 9060, 22656) | (9060, 22656, 60370) |

**2.2. Patterns of 3-tuples with $(3, 2)$ – Icosioctagonal numbers.**

The n[th] term of the sequence of 28-sides $m$-gonal numbers is symbolized by

$$M(28, n) = 13n^2 - 12n$$

Define the n[th] term of (3,2) Icosioctagonal numbers by the relation

$$J_{(3,2)}(n) = 2M(28, n) + 3M(28, n+1)$$

Then the sequence of the above defined numbers is given by

$$86, 299, 642, 1115, 1718, 2451, 3314, 4307, 5430, 6683, \ldots.$$

Let

$$\mu_1 = J_{(3,2)}(n) = 65n^2 + 18n + 3$$

$$\mu_2 = J_{(3,2)}(n+2) = 65n^2 + 278n + 299$$

Then $(\mu_1, \mu_2)$ is a pair such that the multiplication of two numbers diminished by 456 is a square. As in the previous section, this pair is extended to the triple $(\mu_1, \mu_2, \mu_3)$ where $\mu_3 = 260n^2 + 592n + 344$

Correspondingly if $(\mu_2, \mu_3), (\mu_3, \mu_4)$ etc is a sequence of pairs of elements, then the repetition of the procedures as in section 2.1 all the pairs can be prolonged into triples $(\mu_2, \mu_3, \mu_4)$,$(\mu_3, \mu_4, \mu_5)$ etc.

Here

$$\mu_4 = 585n^2 + 1722n + 1283$$

$$\mu_5 = 1625n^2 + 4350n + 2955$$

**Examples for the derived triples with the desired condition for few samples of $n$ are given in table below.**

| $n$ | $(\mu_1, \mu_2, \mu_3)$ | $(\mu_2, \mu_3, \mu_4)$ | $(\mu_3, \mu_4, \mu_5)$ |
|---|---|---|---|
| 1 | (86, 642, 1196) | (642, 1196, 3590) | (1196, 3590, 8930) |
| 2 | (299, 1115, 2568) | (1115, 2568, 7067) | (2568, 7067, 18155) |
| 3 | (642, 1718, 4460) | (1718, 4460, 11714) | (4460, 11714, 30630) |

| | | | |
|---|---|---|---|
| 4 | (1115, 2451, 6872) | (2451, 6872, 17531) | (6872, 17531, 46355) |
| 5 | (1718, 3314, 9804) | (3314, 9804, 24518) | (9804, 24518, 65330) |

**3. Python Program for verification of the needed triples with numerical values satisfying two exclusive conditions in Section 2.1and 2.2 is displayed below.**

```
import math
Section = int(input('ENTER THE VALUE OF SECTION'))
if Section == 1:
  n = int(input('ENTER THE VALUE OF n = '))
  ω1 = 60 * n * n + 17 * n + 3
  ω2 = 60 * n * n + 257 * n + 277
  ω3 = 240 * n * n + 548 * n + 320
  ω4 = 540 * n * n + 1593 * n + 1191
  ω5 = 1500 * n * n + 4025 * n + 2745
  print(' ω1 = ', ω1, 'ω2 = ', ω2, 'ω3 = ', ω3, 'ω4 = ', ω4, 'ω5 = ', ω5)
  X = ω1 * ω2 − 431
  Y = ω2 * ω3 − 431
  Z = ω3 * ω1 − 431
  M = ω2 * ω4 − 431
  N = ω3 * ω4 − 431
  P = ω3 * ω5 − 431
  Q = ω4 * ω5 − 431
  root1 = math.sqrt(X)
  root2 = math.sqrt(Y)
  root3 = math.sqrt(Z)
  root4 = math.sqrt(M)
  root5 = math.sqrt(N)
  root6 = math.sqrt(P)
  root7 = math.sqrt(Q)
  if (int(root1 + 0.5) ** 2 == X) and (int(root2 + 0.5) ** 2 == Y) and (int(root3 + 0.5) ** 2 == Z):
    print('(ω1, ω2, ω3) = ', ( ω1, ω2, ω3), "is a Diophantine triple with D(−431)")
  else:
    print('(ω1, ω2, ω3) = ', ( ω1, ω2, ω3), "is not a Diophantine triple with D(−431)")
  if (int(root2 + 0.5) ** 2 == Y) and (int(root4 + 0.5) ** 2 == M) and (int(root5 + 0.5) ** 2 == N):
    print('(ω2, ω3, ω4) = ', ( ω2, ω3, ω4), "is a Diophantine triple with D(−431)")
  else:
```

```
    print('(ω2, ω3, ω4) = ', ( ω2, ω3, ω4), "is not a Diophantine triple with D(−431)")
  if (int(root5 + 0.5) ** 2 == N) and (int(root6 + 0.5) ** 2 == P) and (int(root7 + 0.5) ** 2 == Q):
    print('(ω3, ω4, ω5) = ', ( ω3, ω4, ω5), "is a Diophantine triple with D(−431)")
  else:
    print('(ω3, ω4, ω5) = ', ( ω3, ω4, ω5), "is not a Diophantine triple with D(−431)")
elif Section == 2:
  n = int(input('ENTER THE VALUE OF n = '))
  μ1 = 65 * n * n + 18 * n + 3
  μ2 = 65 * n * n + 278 * n + 299
  μ3 = 260 * n * n + 592 * n + 344
  μ4 = 585 * n * n + 1722 * n + 1283
  μ5 = 1625 * n * n + 4350 * n + 2955
  print('μ1 = ', μ1, 'μ2 = ', μ2, 'μ3 = ', μ3, 'μ4 = ', μ4, 'μ5 = ', μ5)
  A = μ1 * μ2 − 456
  B = μ1 * μ3 − 456
  C = μ2 * μ3 − 456
  D = μ2 * μ4 − 456
  E = μ3 * μ4 − 456
  F = μ3 * μ5 − 456
  G = μ4 * μ5 − 456
  root1 = math.sqrt(A)
  root2 = math.sqrt(B)
  root3 = math.sqrt(C)
  root4 = math.sqrt(D)
  root5 = math.sqrt(E)
  root6 = math.sqrt(F)
  root7 = math.sqrt(G)
  if (int(root1 + 0.5) ** 2 == A) and (int(root2 + 0.5) ** 2 == B) and (int(root3 + 0.5) ** 2 == C):
      print('(μ1, μ2, μ3)) = ', ( μ1, μ2, μ3), "is a Diophantine triple with D(−456)")
  else:
    print('(μ1, μ2, μ3)) = ', ( μ1, μ2, μ3),     "is not a Diophantine triple with D(−456)")
  if (int(root3 + 0.5) ** 2 == C) and (int(root4 + 0.5) ** 2 == D) and (int(root5 + 0.5) ** 2 == E):
    print('(μ2, μ3, μ4)) = ', ( μ2, μ3, μ4), "is a Diophantine triple with D(−456)")
  else:
   print('(μ2, μ3, μ4)) = ', ( μ2, μ3, μ4), "is not a Diophantine triple with D(−456)")
```

$if\ (int(root5 + 0.5) ** 2 == E)\ and\ (int(root6 + 0.5) ** 2 == F)\ and\ (int(root7 + 0.5) ** 2 == G)$:

$print('(\mu3, \mu4, \mu5)) = ', (\mu3, \mu4, \mu5),$ "$is\ a\ Diophantine\ triple\ with\ D(-456)$")

$else$:

$print('(\mu3, \mu4, \mu5)) = ', (\mu3, \mu4, \mu5),$ "$is\ not\ a\ Diophantine\ triple\ with\ D(-456)$")

**Output of Some values of n:**

$ENTER\ THE\ VALUE\ OF\ SECTION\ 1$

$ENTER\ THE\ VALUE\ OF\ n = 1$

$\omega1 = 80\ \omega2 = 594\ \omega3 = 1108\ \omega4 = 3324\ \omega5 = 8270$

$(\omega1, \omega2, \omega3) = (80, 594, 1108)$ *is a Diophantine triple with* $D(-431)$

$(\omega2, \omega3, \omega4) = (594, 1108, 3324)$ *is a Diophantine triple with* $D(-431)$

$(\omega3, \omega4, \omega5) = (1108,3324,8270)$*is a DiophantinetriplewithD*$(-431)$

ENTER THE VALUE OF SECTION 1

ENTER THE VALUE OF n = 2

ω1 = 277 ω2 = 1031 ω3 = 2376 ω4 = 6537 ω5 = 16795

(ω1, ω2, ω3) = (277, 1031, 2376) is a Diophantine triple with D(−431)

(ω2, ω3, ω4) = (1031, 2376, 6537) is a Diophantine triple with D(−431)

(ω3, ω4, ω5) = (2376,6537,16795)is a DiophantinetriplewithD(−431)

ENTER THE VALUE OF SECTION 2

ENTER THE VALUE OF n = 2

μ1 = 299 μ2 = 1115 μ3 = 2568 μ4 = 7067 μ5 = 18155

(μ1, μ2, μ3)) = (299, 1115, 2568) is a Diophantine triple with D(−456)

(μ2, μ3, μ4)) = (1115, 2568, 7067) is a Diophantine triple with D(−456)

(μ3, μ4, μ5)) = (2568, 7067, 18155) is a Diophantine triple with D(−456)

ENTER THE VALUE OF SECTION 2

ENTER THE VALUE OF n = 2

μ1 = 299 μ2 = 1115 μ3 = 2568 μ4 = 7067 μ5 = 18155

(μ1, μ2, μ3)) = (299, 1115, 2568) is a Diophantine triple with D(−456)

(μ2, μ3, μ4)) = (1115, 2568, 7067) is a Diophantine triple with D(−456)

(μ3, μ4, μ5)) = (2568, 7067, 18155) is a Diophantine triple with D(−456)

## V. Conclusion:

In this paper, we presented a few examples of constructing Diophantine triples for icosihexagonal and icosioctgonal numbers. To summarise, one can take a glance for Diophantine triples for other numbers which have the corresponding suitable properties.

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