Analysis of a Queuing System in a National Museum using Monte Carlo simulation.

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ABSTRACT

The research describes the visitor's flow in a system's model and ways in which the system is constrained by the existing counters, personnel, and equipment. Despite the fact that several predictive algorithms have been developed in an effort to reduce museum crowding. Such predictive models included Model Predictive Control, which was established by Stephen Boyd and Yang Wang aimed at controlling the queuing systems. The model was not effective since it's slow in predicting elements in a queue. But, this study has combined simulating concepts with queuing system to generate an appropriate queuing model, simulating concepts looked into the simulating concepts which handles problems that are difficult to be solved analytically. The study found that NMK is using a single multi-server queuing system having 4 cashiers and the mean arrival rate of visitors is 2 minutes per visitor while the service rate is 3 minutes per visitor. On simulation, holding the rates of arrival constant while varying cashiers' numbers, the study showed that with the presence of 5 cashiers the cumulative visitor waiting to receive the service have reduced such that in the presence of 6 cashiers the cumulative visitor at some durations is zero. The simulation showed that the required number of cashiers to enhance the quality of services NMK offers has to be increased from 4 to 5.

Keywords: *Exponential Distribution; Forecasting; Poisson Distribution; Monte Carlo Simulation.*

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1. INTRODUCTION

1.0. Background Information

There are various factors which are considered to conclude if a museum is bad or good. Cleanliness, level of attraction, layout of the snake park, and its setting are among the key factors. By managing these factors appropriately, any museum is capable of attracting several visitors. However, there are additional factors which have to be considered, especially if a museum has successfully attracted visitors. These factors include visitor's waiting time, service quality, and staffing levels. Improving those processes will have a significant impact on the tourists in the museum.

The transaction resulting from the service organization has to be complete, courteous, and efficient. However, the bitter test results from the time taken for a visitor to get attention this contaminate the general decision about the quality of services offered by the museum. In numerous cases, in a system of waiting lines, a manager may decide the level of services to offer. If the level of service is lower, then it's inexpensive, though the customer rate of dissatisfaction may be high, which might cause loss of future business and a high cost of processing customers' complaints. To enhance the customer's satisfaction, a study of the queuing system has to come in, whereby the queuing theory studies waiting lines or queues. The problem of the services' provision can be reduced by developing a queuing model which reduces the queuing length. In this case, museum administrators can use the predicted model to enhance the quality of service of any museum in the future.

1.1: Statement of the problem.

There are numerous factors which lead to an investigation of this system, queuing is a vital issue which requires resolution in NMK's operation. Some of the problems that motivated the researcher to study this system include:

1. Visitors take a lot of time waiting to receive the anticipated services.
2. Allocation of the resources accordingly to encourage satisfaction of visitors.
3. Due to long queue the rate of visitors dissatisfaction is very high making many visitors shift from NMK to other museums which have a tendency to have short queues and offer similar services.

There are numerous developed predictive models aimed at trying to ease the overcrowding problem in museums. Such models include predictive control model which was developed by Stephen Boyd and Yang Wang aimed at controlling the queuing systems, but the models was very slow in predicting waiting time thus failing to be very effective. Thus this study aims at improving the predictive model for predicting the waiting times using Monte Carlo simulation approach.

1.2: Objectives of the study.

1.2.1: The general objective.

The research's primary is to use Monte Carlo simulation to examine the queueing system now in place at NMK and create a new system that will improve the level of services that NMK provides to its visitors.

1.2.2: The specific objective.

1. Estimating the arrival and service time of the present queuing model used by NMK.
2. To estimate the waiting time of the present queuing model used by NMK.
3. To propose a queuing model that reduces the operational costs and utilization of the resources in the museum

1.3: Research questions

The study intended to improve models that forecast the number of people waiting in line. The investigation was guided by the following research inquiries:

1. Estimating the arrival and service time of the present queuing model used by NMK?
2. Predicting the waiting time of the present queuing model used by NMK?
3. Proposing a queuing model that reduces the operational costs and utilization of the resources in the museum?

1.4: Significance of the study

The researcher's significant includes:

1. The researcher's flexibility is provided since it is safe and easy to change a system's section than changing the whole system.
2. Research provides a suitable but not excessive service facilities. Since providing excessive service facilities, other services facilities will be idle and if service facilities are not changed the rate of visitor's frustration will remain.
3. The research is significant in saving visitor's time and the rate of visitor's satisfaction rises.

1.5: Scope of the study

Waiting lines or queues are commonly used in the construction of queuing models.

a) The study covered queueing models in a dynamic system with probabilistic input.

b) The study employed the First-in, First-out (FIFO) method of queuing.

1.6: Limitations of the study.

The investigation is restricted to the case study of NMK. The simulating of a queuing system in this aspect is limited to an application of a queuing system in service industry

2. LITERATURE REVIEW

2.0: Introduction

This section discusses the background as well as the problems resulting from the overcrowding of visitors that causes inadequacy of visitors' processing.

The queuing models provide tools which are powerful in evaluation and designing the queuing systems' performance (Eze & Odunukwe, 2015). Every time visitors arrive at the NMK's service facility, some of them have to wait before receiving the anticipated service. This shows that a visitor has to wait his/her chance, maybe in line to be served. As shown in the figure below.

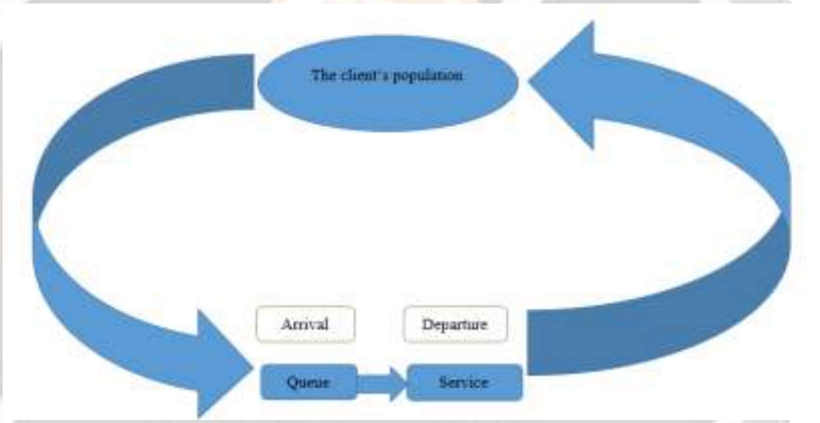

Figure 1: The simplest structure of a queuing system.

Yearly, visitation grows as a result of seasonal variations in western countries. With more tourists visiting the country as a result of the cold season in their home countries, various tourist-attracting sites are flooded with tourists.

Kenya has various tourist attraction sites; Nairobi National Museum is among them. Overcrowding is a vital problem that is faced during the cold season in western countries and during the closure of schools both within and outside the country. The demands of visitors are frequently reprioritized by employees when museums are packed. The personnel will speak with those who scheduled their trip before D-day.

For example, the tour guides will take their time in educating the visitors about the services they offer and will guide them throughout their tour and even answer any questions they might have, when the museum is not fully staffed. This ensures that the visitors are well informed about their touring activities. However, when museums are crowded, the provision of information might be difficult. From the past, queuing theory has effectively been used in the service industry for modeling policy making, staff scheduling, visitors' waiting in the queue, and analysis of counter requirements.

It is practiced commonly in a service industry to estimate the number of required counters as an average staying length in hours times average number of daily visits divided by average counter's rate of occupancy (that is the average number of busy counters during a day) (Ding et.al., 2022), (Zhang et.al., 2019), and (Niu et.al., 2019).

$$Counter's\ requirement = \frac{\text{average staying length in hours}}{\text{average counter's rate of occupancy}} * \text{ average daily visits}$$

However, Dwars et.al. (2013), argues that any model that is based on the average number has no capability of describing the dynamics and complexity of visitor flow in a system.

Recently, the use of queuing models has provided an effective means of estimating the required number of counters based on the measures of sound performance. Using M/G/∞ queue in modelling the number of counters in a museum. They showed that in the presence of the steady state, the average counter's rate of occupancy follows a Poisson distribution, having λW as the mean, where W is the average length of stay and λ is the rate of daily visits. The researcher can use this model to calculate how many grantee counters are needed to ensure that the service is provided at the right time for any given target percentage of arrivals.

In addition, the M/G/∞ system is used to simulate the line of visitors who need alternative special assistance, such as those who pre-book the services and those who are waiting to enter the snake park. In the presence of many visitors in the snake park, those who booked first are prioritized. Keeping visitors in case the viewing area is full reduces the museum's utilization. The model that was developed can help the museum's planning department predict the effects of certain policy changes on practical ways to gain access to the facilities. Skorokhod, (1956) argues that instead of using the M/G/∞ queue (an infinite capacity queue), the M/G/c queue can be used due to its state-dependent rate of arrival in addressing the problem of long queues in museums.

2.1: Present model of predicting the visitors in the museums and their related limitations.

2.2: DES (Discrete Event Simulation) and MCQT (Markov Chain contained in Queuing Theory).

HM (hidden Markov) models have been used in a variety of domains, including bioinformatics and storage workloads. (Harrison, et.al, 2012). Leonard E. Baum used HM models for the first time in a statistical work for the interpretation of Markov chains in the late 1960s (Baum & Petrie, 1966) as well as for estimating the probability functions of Markov chains statistically (Baum & Eagon, 1967). Speech recognition which are still applied today were fields of training HM models in 1970s and 1980s (Shwartz, 1991) and (Rabiner, 1989).

A probabilistic bivariate Markov chain called the HM model is used to hold information on time series development. The model consists of an HM chain $\{Ct\}$ (so that t is an integer) with an observable discrete stochastic time $\{0t; t \geq 0\}$ and states that are not directly observable. The $\{Ct, Ot\}; t \geq 0$ is a bivariate Markov chain results from the combining of the two.

The first MCQT model considered the most frequently occurring scenarios. Discrete event simulation is commonly used to validate other models and has gained widespread acceptance in institutions including banks, healthcare facilities, and museums. The model offers tools which are valuable in researching the trade-off between the variability sources, visitor flow times, and the capacity structure (Collins, et.al, 2015).

2.2.1: Weakness of DES (Discrete Event Simulation) and MCQT (Markov Chain contained in Queuing Theory).

The plots generated from DES have numerous fluctuations between every simulation plot. The unstable feature, which is improved through the increment of the ensemble size, the large simulation of the ensemble size causes higher costs of computation. Such that if several visitors are allowed to enter the museum simultaneously, the model will count for only one visitor (Sparacino, 2002).

2.3: Queueing theory and DES (Discrete-Event Simulation)

The system's operation in DES is represented in a sequential manner of events. Whereby every event happens in an instant of time and consequently changes the marks in the system's state. The model result is stochastic and dynamic. The DES comprises event lists, clocks, and even generators of random numbers (Xia et. al., 2009).

For example, if a visitor is waiting for a service, the system's state is the queue length or visitors' number in the queue. The events of the system are the arrival of visitors and their departure. As a result, in this system, the model must be described stochastically depending on the arrival and departure of visitors. It is important to first build a number of random entities depending on the distribution in order to simulate such a system.

Consider (n, t) where n represents the number of visitors coming into museum at time t, such that the visitors coming at time ($t_1,\ t_2 = dt + t_1, t_3 = dt + t_2, t_4 = dt + t_3, \ldots\ , t_k$) can be expressed as $\{(\ n_1, t_1), (n_2, t_2), (\ n_3, t_3) \ldots, (n_k, t_k)$ whereby dt is constant and $n_1, n_2, n_3, \ldots. n_k$ are random numbers. The simulator generator will generate each visitor's service rate $v_1, v_2, v_3, \ldots. v_k$ which are also random numbers. . These random numbers all follow the same distribution. That is, their stay in the system ends when they depart. All the data is saved by the simulator. Thus, the visitor count can only be determined by reviewing the recorded data.

For example, in computing the number of visitors during t_1 the simulator will check the visitors at time t_1 being between the visitors' departure and arrival time. Building such models has various implications, like modeling the detailed behaviors of the system and also modeling the dependency and performance of the system (Hardy & Aryal, 2020). Compared to other models, such models have a long time of execution and the interpretation of the results is very difficult (Wiler et.al. 2011).

The two most commonly used approaches in improving and modeling visitor flow are queueing theory and DES (Hoseini et al., 2018). Compared to the rich literature about visitor flow models in museums.

2.4: Poisson distribution and Stochastic Simulation.

Consider, Torres-Ortega, et.al. (2018) they conducted a study and found that the booking of visitations resulted in an overflow of visitors who are few with similar places of visitation in comparison with the daily un-booked visitations. The model they used proved that variable departure was more significant compared to the entrance rate's variable in contributing to the visitor overflow in museums. Several researchers have used the regression model to analyze the needed capacity of museums. The statistical assumption that is common in modelling the data that is counted is that it has to follow a Poisson distribution, thus assuming that the number of visitors in a museum is Poisson distributed (Farmer, 1990).

2.4.1: Weakness of Poisson distribution and Stochastic Simulation.

From Torres-Ortega, et.al. (2018), the conclusion of the goodness of fit test lacked sufficient support. The researchers tried to use the goodness of fit from the chi square in justification of the Poisson process. The obtained p values ranged from 0.136 to 0.802 for varied weekdays. The obtained p value was higher than the threshold for significance (0.05), hence the Poisson distribution null hypothesis was not rejected (Farmer, 1990). From the Poisson property's fundamental rule, the variance and mean are equal. However, the study indicated that variance resulting from the 4 days was not even close to the calculated mean, hence the Poisson distribution was not even justified conveniently per visitors' arrival. Additionally, the chi square test's p values were lower (not high enough) to support the null hypothesis.

2.5: Model of the Decisions tree.

The plots generated from DES have numerous fluctuations between every simulation plot. The unstable feature, which is improved through the increment of the ensemble size, the large simulation of the ensemble size causes higher costs of computation. Such that if several visitors are allowed to enter the museum simultaneously, the model will count for only one visitor (Sparacino, 2002). Therefore, in the event that the process grows complex and the variable control restrictions are successful, it is crucial to create a more significant mathematical model, such as one that takes into account constraints and integrated variables. The decision tree model lacks the MPCT of practice that should optimize and minimize the computation of matrices, which is time-consuming and uses a lot of resources.

2.5.1: Weakness of the Decisions tree model.

The model does not consider the factors that are time-related to be partially correlated with the outcomes. Thus, it is difficult to remember the exact time an event collapsed in a situation of visitation.

2.6: ARIMA (Autoregressive Inductive Moving Average).

Research by Farmer (1990) on "the models for forecasting the number of beds required in a hospital in the acute sector" discovered that ARIMA modeling can estimate how many beds a hospital will need. The model has been used by various researchers to predict the number of attractive sections in several museums across the world. The ACF (auto covariance functions) of a wide range of stationary processes, whose values trend to zero as the log tends to infinity, are approximated or described by the ARMA processes.

2.6.1: Weakness of ARIMA (Autoregressive Inductive Moving Average).

The model found that the number of attractive sections in several museums across the world is as a result of attraction facilities available or the required attraction section. Also, it was found that high volatility periods, which were shown by the presence of the GARCH errors, resulted in rising waiting times in the sections.

Thus, the model seems to have limitations, especially the inherent variability of the flow of events, especially visitors or patients in a facility. There are numerous developed methodologies for forecasting the census and arrival counts in several attractive sections of museums evaluated by time series regression, seasonal ARIMA (autoregressive integrated moving average), artificial neural networks, and exponential smoothing in forecasting the number of daily visitors in snake park sections and gallery sections of various museums. Mostly, the use of the time series method provides an improved absolute estimation error, which is relative to the MLR approach, which is considered the benchmark model in forecasting the number of visitors to the museum daily.

2.7: Stochastic Simulation Model (SSM)

A stochastic simulation model (SSM), which is used to describe systems or events that are unpredictable due to the effect of random variables, includes the Monte Carlo simulation model. The SSM model presupposes that the elements of the random input are at least similar. Inventory and queuing systems are typically modeled stochastically. The fundamental drawback of SSMs is that their outputs are themselves random and must thus be interpreted as estimations of the underlying model properties.

A set of variables that are random, comparable in order, and specified in a common sample space are referred to as a stochastic process. The state space is the collection of all possible values that these random variables could have. For instance, the stochastic process is continuous if the collection is $\{X(t), t \geq 0\}$ whereas it is discrete if the collection is X_1, X_2, X_3 .... (Ucar et.al, 2017). When t is a parameter that spans an appropriate index set T, a stochastic process is one of the families of random variables X. When the index value T is one of 0 or 1, 2, 3, 4, 5… then the discrete time unit is displayed by the index. In this case, the random variable X can represent repeated responses from subjects of a learning experiment, outcomes from successively tossing a fair coin, or sequential observations resulting from an experiment of a certain population. Thus, the distinction of the stochastic processes is based on their range of values, which are possible or state space for the random variable X through their index T.

2.7.1: Simulation and Stochastic Model

Stochastic models are used accurately in representing world processes and phenomena that are real, especially in monitoring visitors and the tourism sector in general. The model used the visitor flow where their admission were modelled as a Poisson process having a parameter whose estimation is inter-arrival times. The in-visitor was represented by an exponential distribution with parameter μ.

2.8: The model in use.

The researcher used Monte Carlo simulation to create a queuing model from an existing model; as a result, Monte Carlo simulation was crucial in determining how many people would eventually join the queue. By sampling the probability distribution for each system variable, the Monte Carlo simulation method can generate tens, hundreds, thousands, or even tens of thousands of alternative results.

2.8.1: Monte Carlo Simulation.

Monte Carlo simulation falls under the category of stochastic models, which describes systems or events that are unpredictable as a result of the influence of various random variables. The literature shows that the present models are not accurate and effective. They also fail in producing the optimal results due to the variables used in computing them.

The stochastic model category, which describes systems or occurrences that are uncertain due to the influence of numerous random factors, includes Monte Carlo simulation. The literature demonstrates that the current models are insufficiently 20 precise and efficient. The variables used in their computation cause them to generate less than ideal results as well. By calculating the distribution of conceivable total completion dates or costs, Monte Carlo simulation is a technique for computing or iterating the project costs and schedules several times using input numbers or values that are randomly chosen from a distribution of probabilities of possible duration or costs (Avlijaš, 2019). The method samples probability distributions as per the system's variable in producing a thousand or hundred possible outcomes. The model has been applied to performance processors to forecast index performance costs of in-order architecture and test its effectiveness versus Itanium-2.In order to simplify how the model (Monte Carlo Simulation) functions, this research established a model that estimates the visitors' number in the museum using the arrival time, service time, and waiting time using exponential and Poisson distributions

To facilitate the model, the study considered M/M/c/∞ the simple multi-server queuing model. In order to execute a Monte Carlo simulation, the study employed R programming and made the assumption that the service time follows an exponential distribution and the time of arrival follows a Poisson distribution.

Application of Monte Carlo Simulation

The method existed long ago and it is still used in various fields.

I) In individual financial planning, Monte Carlo Simulation is applied in most cases to predict the total amount of money an individual requires for retirement and how much they will spend annually once the retirement starts.

II) Monte Carlo Simulation is applied by various organizations to determine if or not future researches are worth the effort and costs through modeling potential outcomes of the study.
Currently, simulation is being used in public health in estimating the direct costs associated to prevent diabetes (type 1 diabetes) via the application of nasal insulin, given that it is part of a routine healthcare organization.

3. Research methodology

3.0: Introduction

This section illustrates the method that was adapted in the research. It discusses the following: data collection methods, queuing length measurement, and models of queuing systems that are based on death and birth processes.

1. Using M/M/s as a model of a queuing system while measuring the performance of the system in NMK
2. The discipline of the priority formulation-Using queuing model in measuring system performance in NMK.
3. Dependent on the State formulation-Using service time model in measuring the performance of the system in NMK.

3.1: Data collection method.

The research used secondary data, which consisted of recorded information about the service time and the arrival times of the visitors. The data collection instruments were the museum's recorded sheets.

The data was gathered over the course of two days each week, on Monday and Friday to 3 p.m from 8.00a.m. Due to the waits they experience—they are quite long on Mondays and relatively short on Fridays—the researcher favored the day.

Among the museums in Kenya the researcher preferred NNM due to the number of visitors it handles and the queue it experiences.

3.2: Queuing length's measure.

The measures was:

1) The number of people in the line was counted every 30 minutes to allow the researcher to create the model. The arrival records were used to gather the data.
2) Time taken for a visitor to be served which was considered constant for all visitors (3 minutes per visitor).

3.3: Queuing models basing on birth-death processes.

The queues and waiting lines result from the service and arrival variabilities. They occur as a result of the random clustering of arrivals and high patterns of variable service; this causes the system to be temporarily overloaded. In various cases, the variabilities of service time are described by a negative exponential distribution, while those of arrival time are described by a theoretical Poisson distribution. In the queuing theory's context, the term "death" is the departure of the served visitor, while "birth" is the arrival of a new visitor in the queuing system.

3.4: The M/M/c queuing system.

If λ is the arrival rate, N total arrivals, and T total time then;

$$\lambda = \frac{N}{T}$$

Consider μ to be the service rate (number of visitors served per time unit), N total arrivals, and S be the total services time, therefore,

$$= \frac{N}{T}$$

In addition, 1/μ is spent on average in the system during service time.

- Since λ is the mean arrival rate and p_o is the long-run time proportion of the system being in state 0, the rate at which a process leaves state 0 and enters state 1 is equal to λp_o.
- Given that μ is the rate at which a process departs from state 1 following completion or departure, and that p_1 is the long-run time fraction of the system being in state 1, the rate at which a process moves from state 1 to 0 is μp_1 .
- In light of the rate of equality concept, we arrive at: The rate at which a process transitions from state 0 to state 1 is =. The rate at which a process moves from state 1 to state 0

$$\lambda p_0 = \mu p_1 \quad \dots (1)$$

In the state 0 < n < c, there are two possible methods for a process to leave state n: by departure or via arrival. Taking into account that p_n is the system's long-run time percentage while it is in state n and that the overall rate of a process departing or arriving from state n is $\lambda p_n + n\mu p_n$. since n servers are busy (this is the additive property of Poisson process).

Additionally, there are two methods for a process to enter state n: either by leaving state 1 + n or by arriving from state n − 1. Therefore, the entering rate in state n by a process $\mu(n+1)p_{n+1} + \lambda p_{n-1}$.

Thus, using rate of equality principle, we get,

$$+np_n\mu = \mu(n+1)p_{n+1} + \lambda p_{n-1} \quad \dots (2)$$

Correspondingly, if c ≤ n we find;

$$c\mu p_n + p_n\lambda = c\mu p_{n+1}\lambda p_{n-1} \quad \dots (3)$$

Repeatedly applying equation (2) alongside equation (3) we get the following at last step:

$$\lambda p_n - (n+1)\mu p_{n+1} = \lambda p_{n-1} - n\mu p_n$$

$$= \lambda p_{n-2} - (n-1)\mu p_{n-1}$$

$$\vdots$$

$$\lambda p_0 = \mu p_1$$

$$= 0$$

For, $0 < n \leq c$

$$\mu n p_n = \lambda p_{n-1}$$

In iterating and rearranging terms, we get for c ≥ n > 0

$$p_n = p_{n-1}\frac{\lambda/\mu}{n} = p_{n-2}\frac{(\lambda/\mu)^2}{n(n-1)} = \ldots = p_0\frac{(\lambda/\mu)^n}{n!} \quad \dots (4)$$

In similar manner we can get for n > c as follows

$$p_n = \frac{(\lambda/\mu)^n}{c!c^{n-c}} p_0 \; ... (5)$$

Given $\sum_{n=0}^{\infty} p_n = 1$ as that the normalization condition then the queuing length expected L can be calculated as for n customers and $\lambda/c\mu < 1$

$$p_0 = [\sum_{n=0}^{c-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!} \frac{c\mu}{(c\mu-\lambda)}]^{-1} \; ... (6)$$

$$L = \sum_{n=0}^{\infty}(-c+n)\, p_n = \frac{p_c\, \lambda}{\mu(-p+1)^2} \; ... \; (7)$$

Given that $p = \lambda/c\mu$ is the rate of utilizing the server. Then, we determine the expected queue's waiting time as;

$$W = \frac{L}{\lambda} = \frac{p_c}{\mu(1-p)^2} \; ... \; (8)$$

Now that we are aware of the probability distribution, we can compute the probability-generating function of the number of persons in the lines or queue.

$$\mathrm{P}(x) = \sum_{n=0}^{c-1} p_n + \sum_{n=c}^{\infty} p_n x^{n-c} = 1 - \frac{p_c}{1-p} + \frac{p_c}{1-p^2} \; ... (9)$$

This allows us to determine the waiting time distribution as;

$$W^*(s) = P\left(1 - \frac{s}{\lambda}\right) = 1 - \frac{p_c}{1-p} + \frac{p_c}{1-p+s/c\mu} \; ... \; (10)$$

The model m/m/c has service and inter-arrival time as parameters, which are distributed as exponential and Poisson, respectively. This place used a first-come, first-serve (FCFS) queueing policy. The size of waiting queue's space was infinite.

Consequently, the model was just a birth-death process' special case, having a constant mean service rate for each busy server. After thorough simulation and evaluation, the model was checked for significance in helping the museum managers decide on an appropriate facility level. Also, to attain a balance among the delaying cost in service offering and the cost of service offered.

3.5: Priority formulation - Queuing model's discipline

The queue's discipline in this priority formulation is based on the system priority. As a result, the priority rankings are used to determine the order in which visitors are chosen for the service. Several queuing systems frequently fit these discipline models more accurately than any other model on the market.

In this way, urgent work takes precedence over other tasks, and important visitors or booked visitation visitors are given higher priority than others. Thus, using a priority discipline model usually offers a refinement that is very welcoming over other normal queuing models. The priority discipline model uses FCFS and always incorporates every assumption of the multiple server model.

3.6: A model where the arrival rate, service rate, or both are state-dependent

All previous models have assumed that the mean rate of service will remain constant regardless of the number of users accessing the system. Unfortunately, in real queueing systems where the servers are actual people, the average rate of service is not constant. The servers frequently operate more swiftly when there is a backlog of work that is significant or when there is a long line than when there is little or no backlog.

The long queue's strain is the only reason for the rise in service rate; servers are just exerting more effort. Nevertheless, it sometimes results partially due to the compromised quality of services or assistance attained in certain phases of the service.

3.7: The description of the visitors.

The visitor's arrival at the Nairobi National Museum is at random and the queuing discipline is FCFS. The museum under this research operates from 8:00 am to 5:00 pm. All visitors go to the reception for an initial screening before starting their tour of the museum. Therefore, Monte Carlo simulation method is successful in describing the dynamics and complexity of the visitor's flow.

3.8: The software modelling used.

This research used R programming to produce well-made, publication-worthy plots, which included the required mathematical symbols and formulas. Also, the study used descriptive research methodology.

4. RESULT AND DISCUSSION

Monte Carlo simulation is based on the experimenting the change (or probability) on elements via random sampling. The following are five steps that were used.

a) Setting up of the significant variance's probability distribution.
b) Generating a cumulative distribution of the line's customers
c) Create an interval of every variable's random numbers
d) Producing the needed random numbers. –
e) Last but not least, simulating several trials

The distribution of arrival and cumulative visitors is shown below, with service time for each visitor being 3 minutes, arrival time being 2 minutes, and anticipated wait times

	Number of cashiers	Expected waiting time
[1,]	1	30.00
[2,]	2	25.57
[3,]	3	4.74
[4,]	4	3.90
[5,]	5	2.17
[6,]	6	0.03
[7,]	7	0.01
[8,]	8	0.00
[9,]	9	0.00
[10,]	10	0.00

It can be seen that in the presence of 5 cashiers the que exists and in the presence of 6 cashiers the que seems not to exist. Thus developing an M/M/5 model from the present M/M/4 model.

4.1. Arrival time and wait time

The simulation is computed using arrival times (daily arrivals) and servers (cashiers) which kept on changing depending on the number of cashiers that are operating in NMK.

The number of cashiers is changed while keeping the arrival time constant during several iterations of the Monte Carlo simulation. Following up to 1000 trials with two slots, three slots, four slots, five slots, and six slots, the resulting simulated data is as follows: 56, 51, 15, 15, 48, 17, 69, 20, 69, 56, 6, 15, 17, 37, 69; 23, 6, 56, 69, 51, 29, 48, 48, 69, 48, 19, 20, 19, 15, 51; 37, 132, 17, 20, 132, 20, 15, 20, 17, 37, 69, 56, 51, 37, 23; 23, 17, 29, 132, 132, 37, 37, 15, 20, 19, 17, 15, 48, 51, 48; 1, 18, 12, 28, 16, 5, 16, 10, 12, 19, 11, 8, 6, 9, 6; 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 3

4.2: Performance of varied waiting times.

As collected data from the recorded sheets is plotted against the projected value from the Monte Carlo simulation, various NMK outcome's graphs are shown below.

On Monday

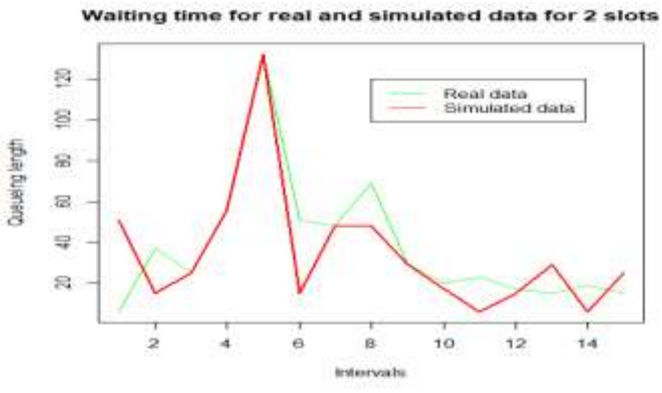

Figure 2: Two slots of waiting for real and simulated data.

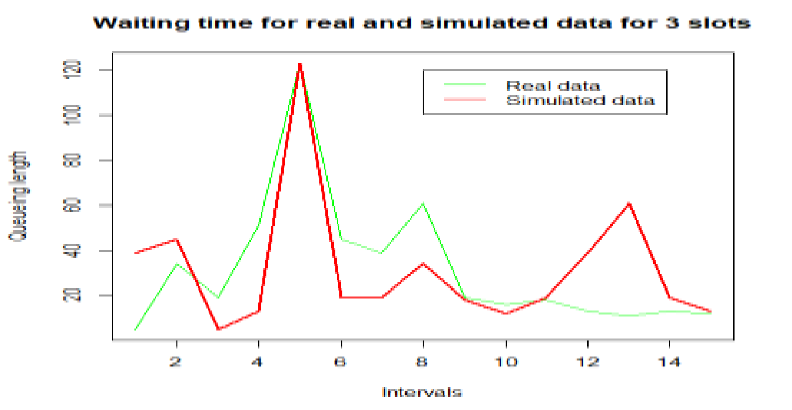

Figure 3: Three slots of waiting for real and simulated data.

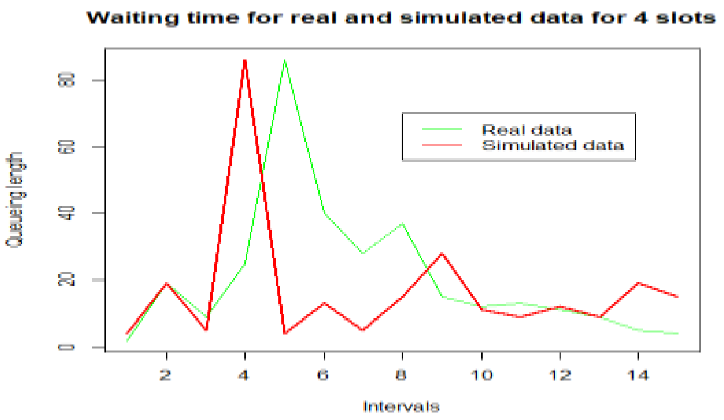

Figure 4: Four slots of waiting for real and simulated data.

On Friday

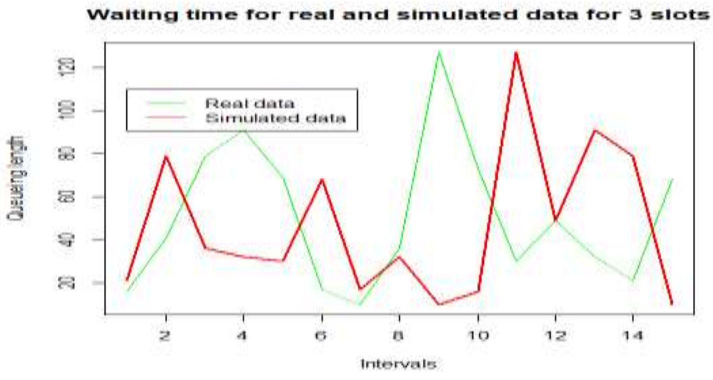

Figure 5: Three slots of waiting for real and simulated data.

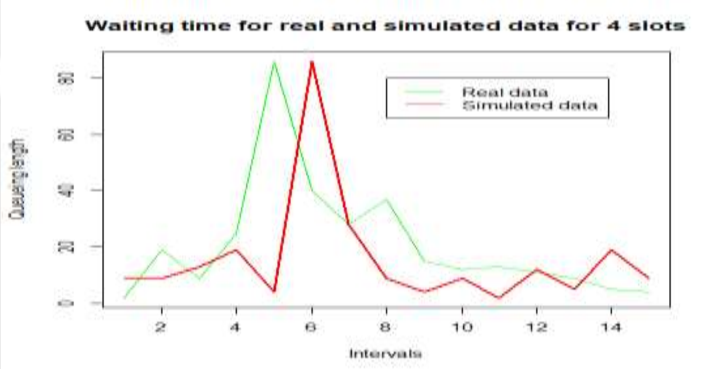

Figure 6: *Four slots of waiting for simulated and real data.*

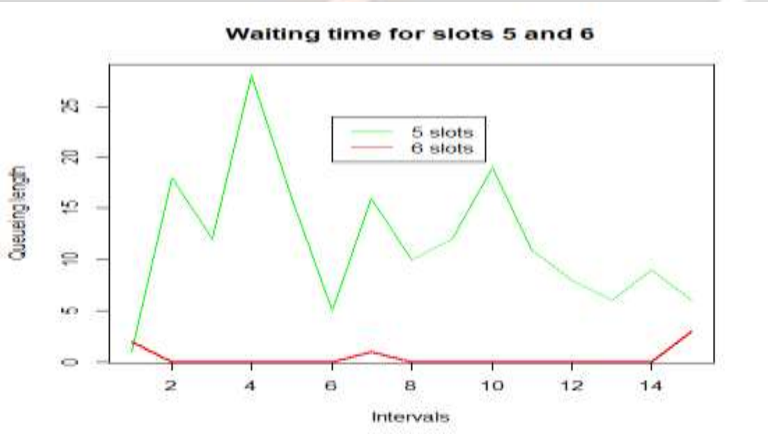

Figure 7: Waiting time for data that is simulated of 5 and 6 slots.

4.3: Summary of figures 2 to 7

From the above figures, the labeling indicates real data that was collected from NMK and data that was collected from the Monte Carlo simulation. Also, the intervals on the x-axis are the time in hours, and queuing length is represented by the y-axis and measured in terms of cumulated visitors.

Figure 2 shows the results from the real collected data from NMK in operation of two cashiers in a day. The output shows that at around 10:00 am the queue is very long with above 100 visitors. This is from the assumptions that 2 cashiers can serve 20 visitors within 30 minutes. The service and arrival times are not constant for each visitor. The results shows that within 30 minutes there are those visitors who takes utmost one minute to be served while others can take more than three minutes to be served depending on the place they need to visit.

Figure 3 show three cashiers' operation during a day. The result shows that at around 10:00 am the queue is very long with about 120 visitors. Since the service times are not constant for each visitor. Figure 4 shows the results from the real collected data from NMK in operation of four tellers in a day. The output shows that at around 10:00 am the queue was very long with about 80 visitors. Since the service times are not constant for each visitor. The queue has begun to diminish since the tellers are increasing. Figure 5 shows the results in operation of 3 cashiers in a day. The output shows that at around 11am the queue was very long with about 120 visitors and diminishes from 1:00pm. The figure 6 shows the results in operation of four cashiers in a day. The output shows that at around 11am the queue is very long with about 80 visitors.

Figure 7 shows the results from Monte Carlo simulation in operation of five and 6 cashiers in a day. The output shows that at around 9am the queue is very long with over 20 visitors. Since the service times are constant for each visitor. The queue is diminishing, and when six tellers are present, there is almost no queue.

The study came to the conclusion that having 5 cashiers working is ideal because having 6 cashiers working results in some cashiers being inactive.

5. Discussion, conclusion, and future work.

The key study findings are discussed and outlined in this chapter. It also makes important deductions and suggestions.

5.1: Discussion.

From the above obtained results from the plotted graphs, it has been seen that as the number of cashiers increases the queuing length reduces. Thus as the cashiers increases the queue diminishes to almost zero, thus in the presence of six cashiers the queue is almost zero, it means other cashiers are inactive, which is an added cost of paying them and they have no work at NMK. Thus the study concluded that the optimal number of cashiers to serve the coming visitors in NNM is five. Since Six cashiers would be a waste of money because most of the time one or more of them would be unavailable; with just four cashiers working, many people would choose to visit other museums.

Furthermore, the research has found that its possible to combine Monte Carlo simulation with queuing system to generate an appropriate queuing model which is very accurate, thus Monte Carlo simulation is very accurate as it is run for several times generating more waiting times. When the generated waiting times are plotted against collected or real data the results produced is exact. That is the discrepancy between two lines is very small.

5.2: Conclusion.

Given that NMK has a desire to aggregate its services in order to draw in more customers and get international recognition, analysts will need to use the key queuing models to optimize the operations that will boost its performance. The study revealed that although numerous models have been developed to gauge waiting times and resource usage in this modern age, they have not yet been fully implemented in our nation. However, the analysts will need to quickly think about how the gallery and snake park lineups interact with one another within the museum.

This study employs simulation models to investigate the applicability and use of queuing models in the service industry. Experimental results have shown that combining Monte Carlo simulation with queuing model generate an appropriate queuing model which is very accurate in generating waiting times, thus Monte Carlo simulation provides highest accuracy in predicting the queues. This is due to the fact that Monte Carlo simulation when run for numerous times it generate waiting times randomly. The study uses those randomly generated waiting times from the simulation and the actual collected data from museum to plot graphs.

5.3: Future work.

1. The study recommends an additional study in the same area but now comparing between Monte Carlo Simulation with either univariate SARIMA model or the multivariate SARIMA Model.
2. It is advised to carry out a comparable experiment using a finite queue.
3. It is also advised to compute the waiting times for each visitor using a Monte Carlo simulation.

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