Analysis of plasma behaving as a high pass filter for EM waves using the FDTD method

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ABSTRACT

In electromagnetics, a dispersive material is a material with electromagnetic parameters dependent on frequency. In this paper, the implementation of simulations of media made up of plasma is studied. More particularly, the studies will focus on the behavior of the plasma as being a high pass filter for an EM wave. The FDTD 1D method, using the DB-FDTD formulation will be used for numerical implementations. Wave propagations at different frequencies in aluminum plasma will be analyzed. In order to corroborate the results of the numerical implementations, calculations of phase velocities as a function of various frequencies will be carried out. These calculations are based on the dispersion relation defined by the Klein-Gordon equation.

Keyword: *Plasma, 1D FDTD, DB-FDTD, EM wave, Numerical implementation, Klein-Gordon dispersion*

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1. INTRODUCTION

The FDTD method (Finite Difference Time Domain) can not only be used for the modeling of isotropic and anisotropic medium, but can also be used for the modeling of dispersive materials. Plasma is one of these dispersive materials. The plasma behaves like a high pass filter for EM (Electromagnetic) waves. A wave with a relatively high frequency, that's frequency is greater than the frequency of the plasma, passes through the plasma as if the plasma was a transparent medium. However, this is true for a frequency much higher than the frequency of the plasma. For frequencies with orders of magnitude relative to the frequency of the plasma, the medium is not completely transparent to the EM wave [1].

The aim of this paper is to model the behavior of the plasma for EM waves passing through the medium formed by the plasma. To do this the FDTD method, using the DB-FDTD formulation that involved electric and magnetic flux density, is used to model the propagation of the wave in 1D space.

2. FDTD 1D FORMULATION FOR PLASMA

2.1. 1D update equations

In order to model a dispersive medium with the FDTD method, the DB-FDTD formulation is used [2]. The electric and magnetic flux density update equations are given in Eq.1, for propagation along the x axis [3].

$$D_z^{n+1}(i) = C_{dd}(i)D_z^n(i) + C_{dh}(i)\left(\frac{H_y^{n+\frac{1}{2}}(i) - H_y^{n+\frac{1}{2}}(i-1)}{\Delta x}\right) \quad (1.a)$$

$$C_{dd}(i) = \frac{1-\frac{\sigma_{ep}(i,)\Delta t}{2\varepsilon(i)}}{1+\frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)}} \quad ; \quad C_{dh}(i) = \frac{\Delta t}{1+\frac{\sigma_{ep}(i,)\Delta t}{2\varepsilon(i)}} \quad (1.b)$$

$$B_y^{n+\frac{1}{2}}(i) = C_{bb}(i)B_y^{n-\frac{1}{2}}(i) + C_{be}(i)\left(\frac{E_z^n(i+1) - E_z^n(i)}{\Delta x}\right) \quad (1.c)$$

$$C_{bb}(i) = \frac{1-\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}}{1+\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}} \quad ; \quad C_{be}(i) = \frac{\Delta t}{1+\frac{\sigma_{mp}(i)\Delta t}{2\mu(i)}} \quad (1.d)$$

Using the Z transform and Drude model for a dispersive material, the update equations for the electric field are given in Eq.2, and those for the magnetic field in Eq.3 [1][3].

$$E^n = \frac{1}{\varepsilon_0 \varepsilon_\infty}(D^n - I_e^{n-1}) \quad (2.a)$$

$$I_e^n = (1 + e^{-\nu_e \Delta t})I_e^{n-1} - e^{-\nu_e \Delta t} I_e^{n-2} + \frac{\varepsilon_0 \omega_{pe}^2 \Delta t}{\nu_e}(1 - e^{-\nu_e \Delta t})E^n \quad (2.b)$$

$$H^n = \frac{1}{\mu_0 \mu_\infty}(B^n - I_m^{n-1}) \quad (3.a)$$

$$I_m^n = (1 + e^{-\nu_m \Delta t})I_m^{n-1} - e^{-\nu_m \Delta t} I_m^{n-2} + \frac{\mu_0 \omega_{pm}^2 \Delta t}{\nu_m}(1 - e^{-\nu_m \Delta t})H^n \quad (3.b)$$

Where ω_p is the plasma frequency in $rad.s^{-1}$. ν is the electron collision frequency in the plasma.

2.2. Grid termination with absorbent layers

In order for the grid to behave like an infinite space, the boundaries of the grid will be made up of absorbent layers whose absorption factors are gradually increased as the layers progress. Eq.4 represent the calculation of loss factors and assignment of loss layers in a 1D FDTD grid [3][4].

$$pe(i) = \frac{\sigma_{ep}(i)\Delta t}{2\varepsilon(i)} = pm(i) = \frac{\sigma_{mp}(i)\Delta t}{2\mu(i)} = 0.333\left(\frac{i}{taille_{perte}}\right)^3 ; \quad (4.a)$$

$$i = \left[1, taille_{perte}\right],\ et\ i = [taille_{grille} - taille_{perte}, taille_{grille}] \quad (4.b)$$

2.3. TFSF formulation

The injection of the source is done by TFSF formulation by applying the corrections to the flux densities. This makes it possible to simulate a wave propagating only in the direction of the x positives. The TFSF formulation for this scenario is described in Eq. 5 [3][4].

$$D_z^{n+1}(i_{src}) = D_z^{n+1}(i_{src}) + \frac{C_{dh}}{\eta} E_{zinc}\left(-\frac{1}{2}, n + \frac{1}{2}\right) \quad (5.a)$$

$$B_y^{n+\frac{1}{2}}\left(i_{src} - \frac{1}{2}\right) = B_y^{n+\frac{1}{2}}\left(i_{src} - \frac{1}{2}\right) - C_{be}E_{zinc}(0, n) \quad (5.b)$$

3. MODELING OF ALUMINUM PLASMA

3.1. Simulation parameters

For an aluminum plasma, the angular frequency is $\omega_p = 22941\, rad.s^{-1}$ and the collision frequency of the electrons is $\nu_e = 146.29\, rad.s^{-1}$ [5]. The plasma medium is defined from node 120 to node 240 of the grid. In order to simulate a non-magnetized plasma the collision frequency of electrons for magnetic susceptibility is $\nu_m = 0.0001\, rad.s^{-1}$. The same is taken for defining the vacuum in the grid $\nu_e = 0.0001\, Rad.s^{-1}$.

The source is a modulated Gaussian introduced at nodes 30 of the grid of size of 400 $nodes$. The spatial resolution is defined by $N_\lambda = 20$. The source frequencies for the various simulations are respectively $f_{src} = 7\,302.3\, THz$, $f_{src} = 14\,605\, THz$, and $f_{src} = 520\,120\, THz$. Tab.1 give the grid parameters for the different angular frequencies for the simulations. Fig.1 show the different permittivity and conductivity of copper plasma at various simulation angular frequencies.

Tab.1: FDTD Grid Parameters at various simulation angular frequencies

$\boldsymbol{\omega_{src}\ (rad.s^{-1})}$	$\mathbf{4.5882 \times 10^{15}}$	$\mathbf{9.1764 \times 10^{16}}$	$\mathbf{3.268 \times 10^{18}}$
$\boldsymbol{\Delta t\ (fs)}$	6.77×10^{-3}	3.38×10^{-3}	9.51×10^{-5}
$\boldsymbol{\Delta x\ (nm)}$	2.05	1.02	2.88×10^{-2}

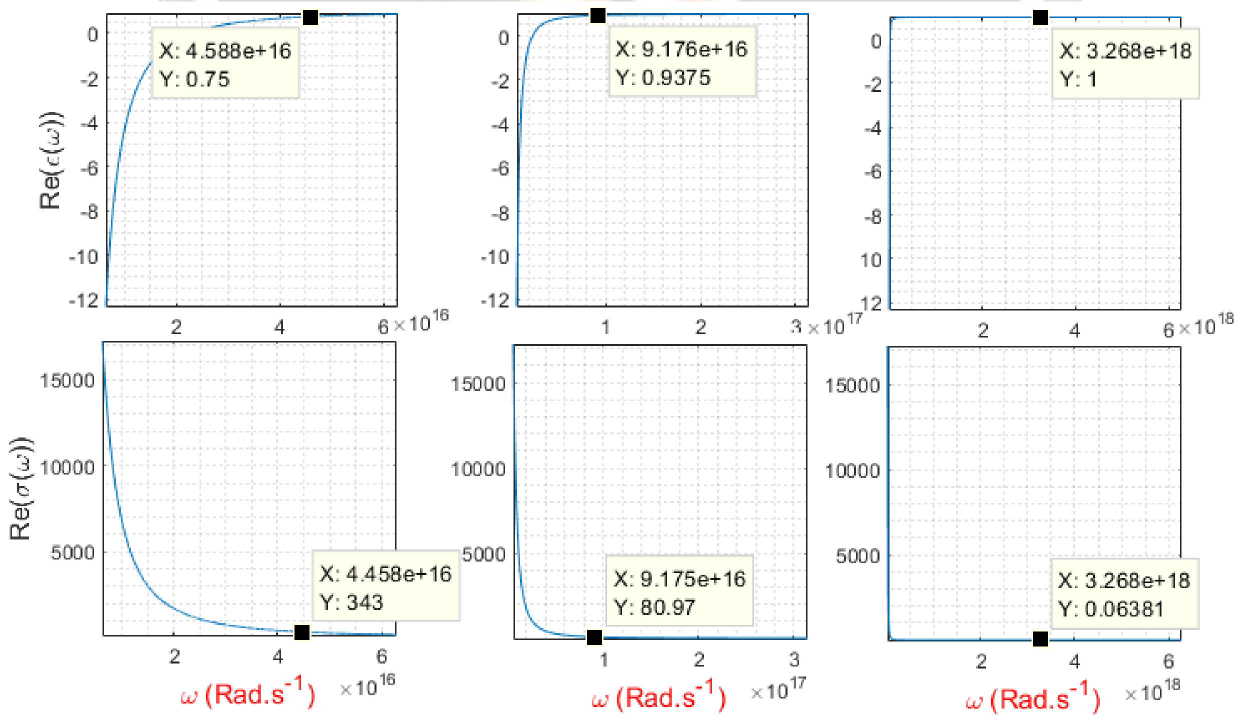

Fig.1: Permittivity and conductivity of copper plasma at various simulation frequencies

3.2. Numerical results

Fig.2, Fig.3 and Fig.4 give the superposition of the snapshots of a modulated Gaussian propagating in a grid made up with vacuum and plasma (curve in blue) and propagating in a grid made up only with vacuum (curve in red). The angular frequency in the simulation is $4.5882 \times 10^{15}\, rad.s^{-1}$ for Fig. 1. For Fig.2 the wave frequency is $9.1764 \times 10^{16}\, rad.s^{-1}$ and $3.268 \times 10^{18}\, rad.s^{-1}$ for Fig.3.

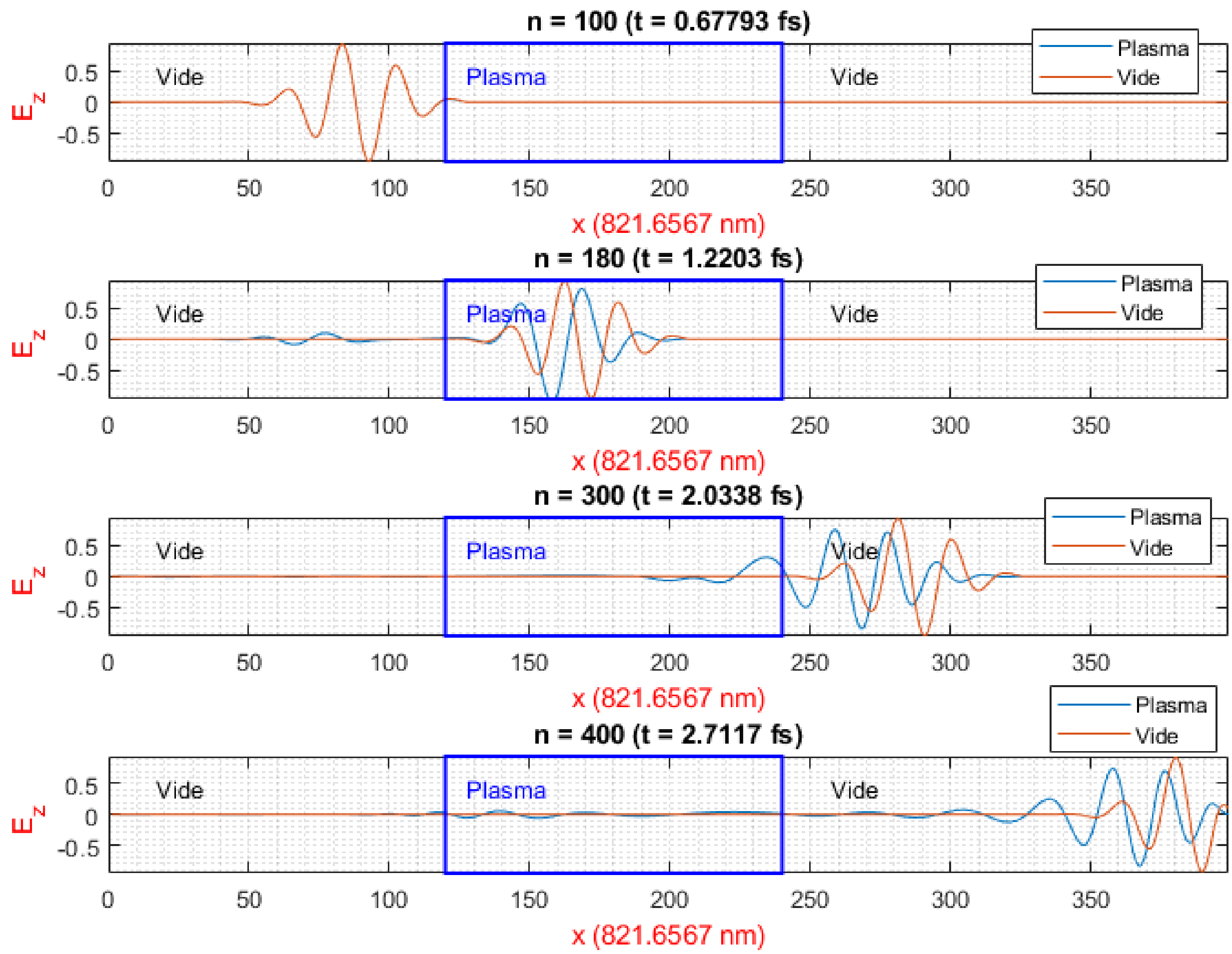

Fig.2: Simulation of a modulated Gaussian with frequency $f_{src} = 7\,302.3\, THz$ passing through aluminum plasma

In Fig.2, the frequency of the wave is twice that of the plasma. A large majority of the wave passes through the plasma and a small part is reflected at the vacuum / plasma interface. The wave reflected by this interface is visible to the left of it interface at the 180th time step ($t = 1.22\, fs$). At this time step, an acceleration of the wave is observed, which is consistent with the theory of wave propagation in plasma [6]. In addition, at this frequency the plasma has a non-zero conductivity value (Fig.1), which leads to the fact that it behaves like a conductive medium by reflecting an incident wave. As the phase speed of the wave packet is increased, so does its wavelength. So at the outlet of the plasma ($t = 2.03\, fs$), the wave packet is much larger than that of in the vacuum.

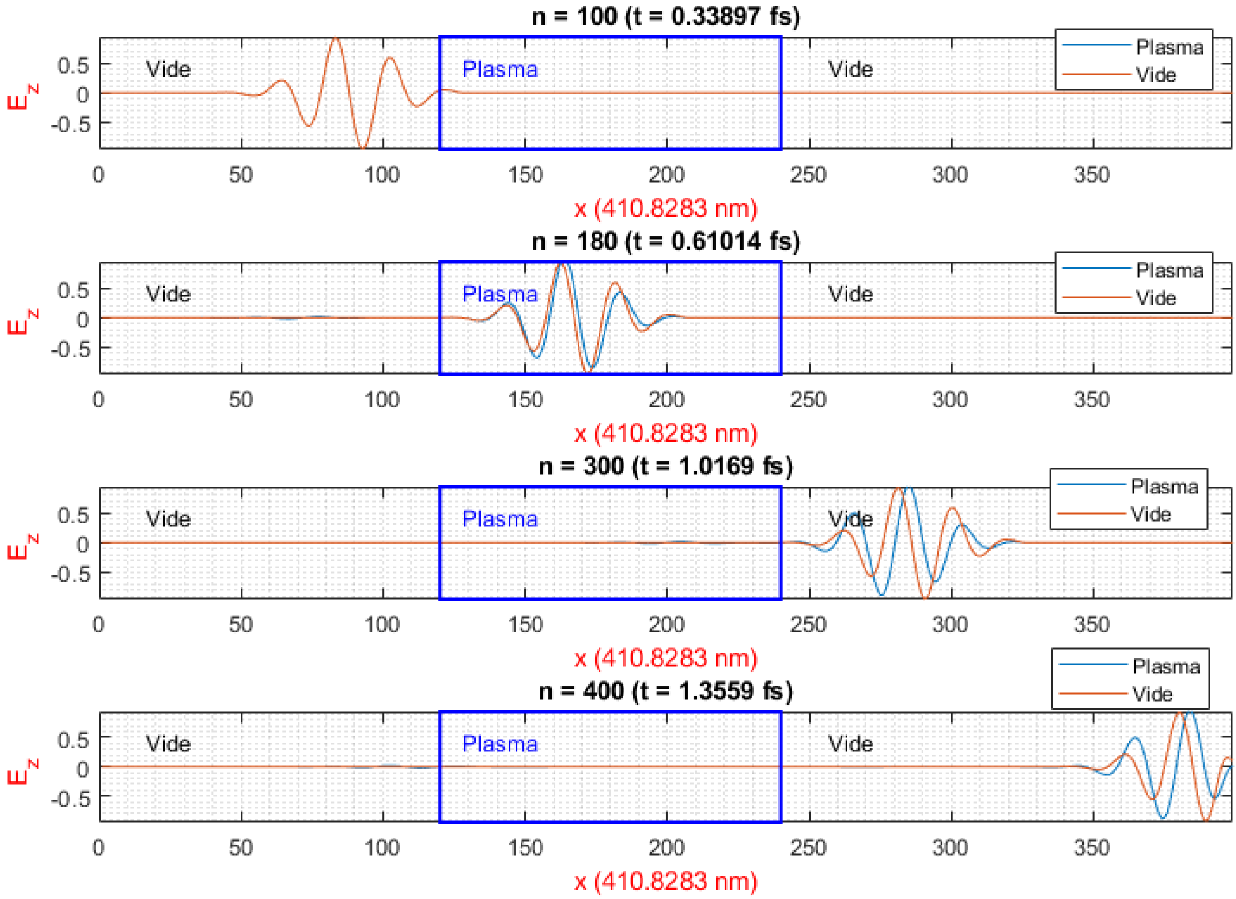

Fig.3: Simulation of a modulated Gaussian with frequency $f_{src} = 14\ 605\ THz$ passing through aluminum plasma

In Fig.3, the frequency of the wave is four times that of the plasma. Almost all of the wave passes through the plasma, and a tiny part is reflected at the vacuum / plasma interface. The wave reflected by this interface is barely visible to the left of the interface at the 180th time step ($t = 0.61\ fs$). Fig.1 indicates that the conductivity of the plasma is less important than for the previous simulation, hence the less reflection. Likewise, the observed acceleration of the wave is lower compared to that of the previous simulation. The phase velocity of the wave packet being only slightly increased, at the output of the plasma ($t = 1.01\ fs$), the width of the wave packet is almost equal to that of the vacuum.

In Fig. 4, the frequency of the wave is 142 times higher than that of the plasma. No reflection is visible to the left of the vacuum / plasma interface. Referring to Fig.1, at this frequency the relative permittivity is equal to $\varepsilon_r = 1$, which is the permittivity of vacuum. In addition, the conductivity of the plasma (Fig.1) is almost zero at this frequency. So at this source frequency, the wave passes through the plasma as if it were simply traveling through a vacuum medium. This is why in Fig. 4, only the propagation of a wave packet is observed as the two wave packets are perfectly superimposed.

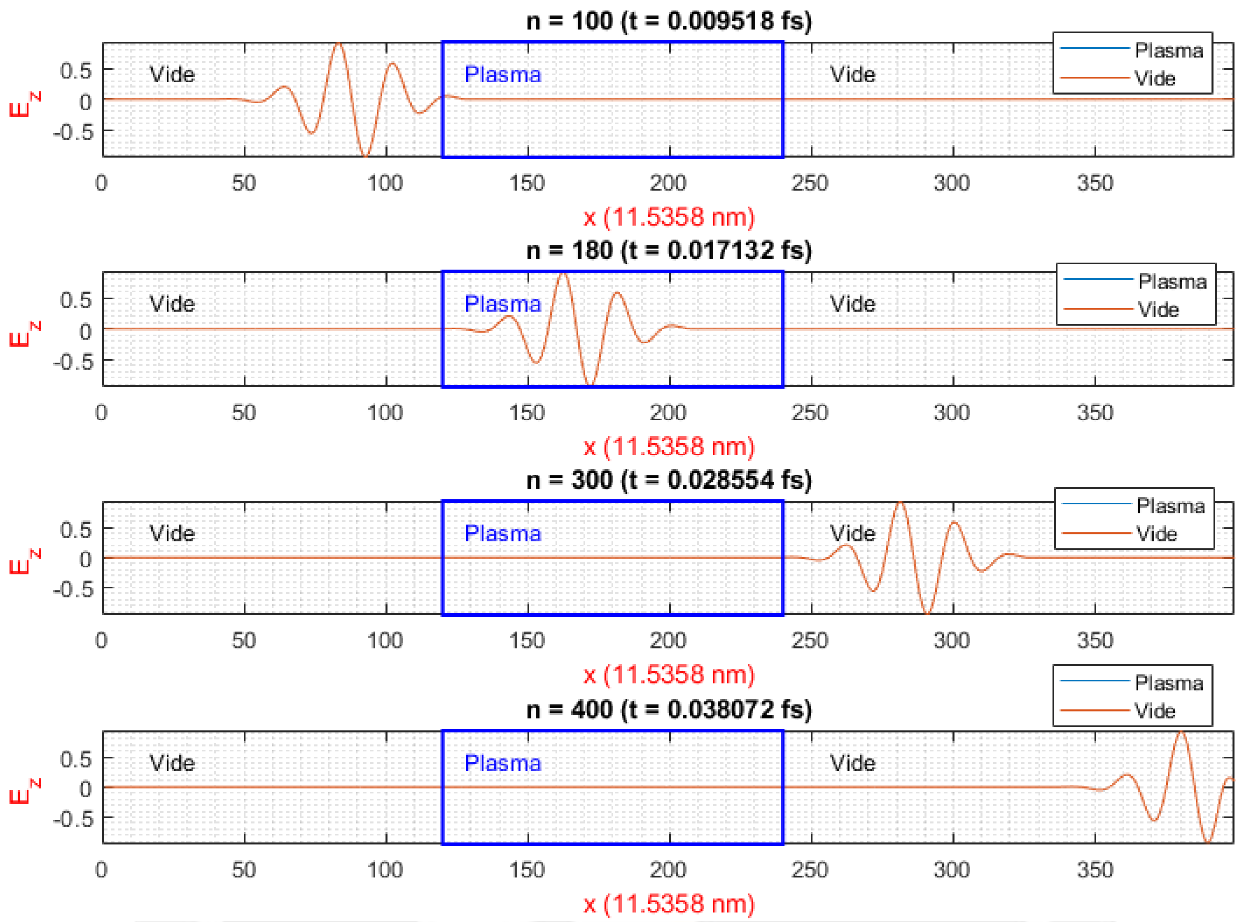

Fig.4: Simulation of a modulated Gaussian with frequency $f_{src} = 520\,120\,THz$ passing through aluminum plasma

4. COMPARISONS WITH ANALYTICAL CALCULATIONS

In order to calculate the speed of an EM wave in plasma, the Klein-Gordon equation is used (Eq.6) [6]. This equation gives the dispersion relation for a wave passing through plasma. From this equation, Eq.7 (giving the phase velocity) is obtained [7]. Tab. 2 shows the different calculated phase velocity with simulations frequencies. More the frequency of the wave increases, more the velocity of the wave in the plasma decreases. These calculations are consistent with the observations during simulations with the FDTD method.

$$k^2 = \frac{\omega^2 - \omega_p^2}{c_0^2} \tag{6}$$

$$v_\varphi(\omega) = \frac{\omega}{k} = \frac{1}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} c_0 \tag{7}$$

Tab.2: Phase velocity of the EM wave in plasma at different angular frequencies

$\omega_{src}\ (rad.s^{-1})$	4.5882×10^{15}	9.1764×10^{16}	3.268×10^{18}
$v_\varphi\ (m.s^{-1})$	3.46×10^8	3.09×10^8	3.0×10^{18}

5. CONCLUSION

The FDTD method makes possible a numerical implementation of a dispersive medium such as, in the case of this work, the plasma. Numerical results indicate that a medium consisting of plasma increases the propagation speed of an EM wave passing through it. This article focuses on the propagation of waves at frequencies above that of plasma. At these frequencies the medium becomes transparent to the wave up to certain measurements. For frequencies relatively large compared to the plasma frequency, the medium is not completely transparent and reflects part of the wave. Higher as the frequency, than less as the reflection of the wave. In addition, higher as the wave frequency, than the wave celerity in the plasma tends towards to the light celerity in vacuum. Using aluminum plasma for the simulations, it was seen that the results of the FDTD simulations correspond with the results of the analytical calculations. Even if the plasma allows high frequency waves to pass, they are accelerated in the plasma. In addition, part of the wave incident to the plasma is reflected by the medium.

6. REFERENCES

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