Application Of Scheffe's Third Degree Regression Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC)

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ABSTRACT

This research work is aimed at applying an optimization model based on Scheffe's Third Degree Regression Model for five component mixture, Scheffe's (5,3), developed by Nwachukwu and others (2022a) to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). It is in comparison with the results of the previous work done on PFRC based on Scheffe's Second Degree Polynomial (5,2) Model by Nwachukwu and others (2022c). Through the use of Scheffe's Simplex method, the compressive strengths of PFRC with respect to Scheffe's third degree model were determined for different mix ratios/proportion. Control experiments were also carried out as a check, and the compressive strengths determined. The adequacy of the model was evaluated using the Student's t-test and the test statistics confirmed the adequacy of the model. The maximum compressive strength of PFRC based on the Scheffe's (5, 3) model was 27.25 N/mm^2 .This is slightly higher than 25.23 N/mm^2 , being the maximum value obtained by Nwachukwu and others (2022c) for the previous work done on PFRC based on the Scheffe's (5,2) model. However, the optimum strengths obtained from both models are higher than the minimum value specified by the American Concrete Institute (ACI), as 20N/mm^2 .Thus PFRC based on both Scheffe's models can produce the required compressive strength needed in major construction projects such as bridges and light-weight structures at the best possible economic and safety advantages.

***Keywords**: PFRC, Scheffe's (5,3) Regression/Polynomial Model, Optimization, Compressive Strength.*

1.INTRODUCTION

According to Jackson and Dhir (1996), concrete mix design is the procedure by which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. In this context, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. Based on the above criteria, the methods proposed by Hughes(1971) and DOE(1988) can be time consuming as they involve a lot of trial mixes and deep statistical works before the desired strength of the concrete can be attained. Therefore, optimization of the concrete mixture design remains the best option and the most efficient way of selecting concrete mix /proportion for better efficiency and performance of concrete such as workability, strength and durability. It is systematic and far better than the usual empirical method which involves rigorous and time consuming process. A typical example of optimization model is Scheffe's Regression Models. The popular Scheffe's models are the Scheffe's Second Degree model and the Scheffe's Third Degree model. Though, work has been done on PFRC based on Scheffe's Second Degree model, the experience gathered from the works Obam (2006) and Nwachukwu and others (2022a) shows that the third degree model usually has an advantage over the second degree model. Thus in this recent study, Scheffe's Third Degree Polynomial for five components mixtures (namely cement, fine aggregate, coarse aggregate, water and polypropylene fibre) will be on focus.

In general, concrete is a very important material widely used in the construction industry and only second to water in terms of usage since ancient time. Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance. However conventional concrete has two major limitations, which are low tensile strength and a destructive and brittle failure .Concrete, is a brittle material with low tensile strength and low strain capacity that result in low resistance to cracking. As a result of this, many new technologies of concrete and some modern concrete specification approaches have been adopted. One of the technologies introduced for concrete was the addition of steel bars to make up for its low tensile strength. This enables concrete gain an amount of tensile strength and thus reducing its brittle nature. However, these types of reinforced concrete structures still have their own shortcomings. They experience deterioration when exposed to deleterious environment which often reduce the service life of the structure. Based on several further researches over the years, the reinforcement (usually steel bars) has been replaced with other materials like fibre (which may include glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre etc.) to further increase both its tensile strength and compressive strength and also, produce light weighted reinforced concrete unlike when reinforced with steel bars.

Fibre reinforced concrete (FRC) is defined as composite materials made with cement, aggregate, and incorporation of any of discrete discontinuous fibres as listed above. The main purpose of incorporating the fibrous materials remains to increase the concrete’s durability and structural integrity and at the same time save costs. The last purpose, cost efficiency is achievable because, all fibres reduce the concrete’s need for steel reinforcements. And since fibre reinforcement tends to be less expensive than steel bars (and less likely to corrode), it makes FRC more cost-effective. In general , fibres can improve the concrete’s workability, flexibility, tensile strength, durability, ductility, cohesion, freeze-thaw resistance, resistance to plastic shrinkage while curing, resistance to cracking, shrinkage at an early age, fire resistance, homogeneity, to mention but a few.

Polypropylene Fibre Reinforced Concrete (PFRC) is one form of FRC and is concrete mixture where the conventionally steel reinforcement in concrete production is replaced with polypropylene fibre. Polypropylene fibre is a kind of linear polymer synthetic fibre obtained from propylene polymerization. It is a light fibre, its density (0.91 gm/cm^3) being the lowest of all synthetic fibres. It is manufactured from propylene gas in the presence of a catalyst such as titanium chloride. In addition, polypropylene fibre (PF) is a by- product of oil refining process and at the same, relatively inexpensive. A typical sample of PF has been shown in the previous related wok by Nwachukwu and others (2022c). PF has excellent chemical resistance to acids and alkalis and high abrasion resistance. PF has some advantages which are not limited to light weight, high strength, high toughness and corrosion resistance. And because of its superior performance characteristics and comparatively low-cost, PF finds extensive use as construction material in asphalt manufacturing, industrial pavements, and highly resistant concrete production. Compressive strength of PFRC is the Strength of hardened PFRC measured by the compression test. And in general, compressive strength is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine. The compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete in question.

The present study therefore focuses on the application of Scheffe’s Third Degree Regression Model to optimize the compressive strength of PFRC. Some related works have been done by many researchers, but none has addressed the real subject matter. For instance, Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC. In recent years, many researchers have used Scheffe’s method to carry out one form of optimization work or the other. Nwakonobi and Osadebe (2008) used Scheffe’s model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe’s model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe’s model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe’s simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe’ mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe’s (4,2) and Scheffe’s (4,3). Nwachukwu and others (2017) developed and employed Scheffe’s Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe’s Third Degree Polynomial model, Scheffe’s (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). And finally, Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). From the forgoing, it can be envisaged that no work has been done on the use of Scheffe's method to optimize the compressive strength of PFRC except, the work by Nwachukwu and others (2022c) which is based on Scheffe's Second Degree Polynomial. Henceforth, the need for this recent research work.

2. SCHEFFE'S THIRD DEGREE, SCHEFFE'S (5, 3) REGRESSION EQUATION

A simplex lattice, according to Aggarwal (2002), remains a structural representation of lines joining the atoms of a mixture, whereas these atoms are constituent components of the mixture. For PFRC mixture, the constituent elements are these five components, water, cement, fine aggregate, coarse aggregate and polypropylene fibre. Thus, a simplex of five-component mixture is a four-dimensional solid. An imaginary space showing a four dimensional factor space with respect to Scheffe's third degree model has been shown in the work of Nwachukwu and others (2022a). According to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \;; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\ldots q$, and q = the number of mixtures.

2.1. THE SIMPLEX LATTICE DESIGN

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 3.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. Thus, the possible design points for Scheffe's (5,3) lattice can be as follows:

A_1 (1,0,0,0,0); A_2 (0,1,0,0,0); A_3 (0,0,1,0,0); A_4 (0,0,0,1,0), A_5 (0,0,0,0,1); A_{112} (2/3. 1/3, 0, 0, 0,); A_{122} = (1/2, 2/3, 0,0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{113} (2/3, 0, 1/3, 0,0); A_{133} (1/3, 0, 0, 2/3, 0, 0); A_{114} (2/3, 0,0,1/3,0); A_{114} (1/3, 0, 0, 2/3, 0); A_{115}, (2/3, 0, 0, 0, 1/3); A_{115} (1/3, 0,0,0, 2/3); A_{223} (0, 2/3, 1/3, 0,0); A_{223} (O, 1/3, 0,0); A_{224} (0,0 2/3, 0, 1/3, 0); A_{224} (0, 1/3, 0, 2/3,0); A_{225} (0, 2/3, 0,0, 1/3); A_{255} (0, 1/3, 0, 0, 2/3); A_{334} (0,0, 2/3, 1/3, 0); A_{344} (0,0,1/3, 2/3,0), A355 (0,0,2/3,0, 1/3); A_{355} (0,0,1/3,0, 2/3); A_{445} (0,0,0, 2/3, 1/3); A_{445} (0,0,0, 1/3, 2/3); A_{123} (1/3, 1/3, 1/3, 0,0); A_{124} (1/3, 1,3, 0, 1/3, 0); A_{125} (1/3, 1/3, 0,0, 1/3); A_{134} (134 (1/3, 0, 1/3, 1/3, 0); A_{135} (1/3, 0, 1/3, 0, 1/3); A_{145} (1/3, 0, 0,1/3,1/3); A_{234} (0,1/3, 1/3,1/3, 0); A_{235} (0,1/3, 1/3, 0, 1/3); A_{245} (0, 1/3, 0, 1/3, 1/3); A_{345} (0,0,1/3,1/3, 1/3). **(2)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \ldots X_q$ is given in the form of Eqn.(3)

$$Y = b_0 + \sum bi\,xi + \sum bij\,xj + \sum bijxjxk + + \sum bi^{j}{}_{2} + \ldots i_n x i_2 x i_n \qquad \textbf{(3)}$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response which represents the property under investigation, which ,in this case is the compressive strength.

This research work is based on the Scheffe's (5, 3) simplex and the actual form of Eqn. (3) for five component mixture , degree three (5, 3) has been developed by Nwachukwu and others (2022a) and will be applied subsequently.

2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mix design, there exist a relationship between the pseudo components and the actual components. It has been established as Eqn.(4):

$$Z = A * X \tag{4}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (4) yields:

$$X = A^{-1} * Z \tag{5}$$

2.3. FORMULATION OF REGRESSION EQUATION FOR SCHEFFE'S (5, 3) LATTICE

The regression equation by Scheffe (1958), otherwise known as response is given in Eqn.(3) .Hence, for Scheffe's (5,3) simplex lattice, the regression equation for five component mixtures has been formulated from Eqn.(3) by Nwachukwu and others (2022a) and is given as follows:

$$\begin{aligned} Y = {} & b_0 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 \\ & + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112}X_1^2X_2 + b_{113}X_1^2X_3 + b_{114}X_1^2X_4 + b_{115}X_1^2X_5 + b_{22}X_2^2 + b_{23}X_2X_3 \\ & + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222}X_2^3 + b_{223}X_2^2X_3 + b_{224}X_2^2X_4 + b_{225}X_2^2X_5 + b_{33}X_3^2 + b_{34}X_3X_4 \\ & + b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3^2X_4 + b_{335}X_3^2X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4^2X_5 + b_{55}X_5^2 + b_{555}X_5^3 \end{aligned} \tag{6}$$

$$\begin{aligned} = {} & b_0X_1 + b_0X_2 + b_0X_3 + b_0X_4 + b_0X_5 + b_1 X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11} X_1 - b_{11}X_1X_2 - b_{11}X_1X_3 \\ & - b_{11}X_1X_4 - b_{11}X_1X_5 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{111}X_1^3 + b_{112} X_1 X_2 - b_{112} X_1 X_2^2 \\ & - b_{112} X_1 X_2X_3 - b_{112} X_1 X_2X_4 - b_{112} X_1 X_2X_5 + b_{113}X_1 X_3 - b_{113}X_1X_2 X_3 - b_{113}X_1 X_3^2 - b_{113}X_1 X_3X_4 - b_{113}X_1 X_3X_5 \\ & + b_{114}X_1 X_4 - b_{114}X_1X_2 X_4 - b_{114}X_1X_3 X_4 - b_{114}X_1 X_4^2 - b_{114}X_1 X_4X_5 + b_{115} X_1 X_5 - b_{115}X_1X_2 X_5 \\ & - b_{115}X_1X_3 X_5 - b_{115}X_1X_4 X_5 - b_{115}X_1 X_5^2 + b_{22}X_2 - b_{22}X_1X_2 - b_{22}X_2X_3 - b_{22}X_2X_4 - b_{22}X_2X_5 \\ & + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{222} X_2^3 + b_{223}X_2 X_3 - b_{223}X_1X_2 X_3 - b_{223}X_2 X_3^2 - b_{223}X_2 X_3X_4 \\ & - b_{223}X_2 X_3X_5 + b_{224} X_2 X_4 - b_{224} X_1X_2 X_4 - b_{224} X_2X_3 X_4 - b_{224} X_2 X_4^2 - b_{224} X_2 X_4X_5 + b_{225}X_2 X_5 \\ & - b_{225}X_1X_2 X_5 - b_{225}X_2X_3 X_5 - b_{225}X_2X_4 X_5 - b_{225}X_2 X_5^2 + b_{33}X_3 - b_{33}X_1X_3 - b_{33}X_2X_3 - b_{33}X_3X_4 - b_{33}X_3X_5 \\ & + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{333}X_3^3 + b_{334}X_3 X_4 - b_{334}X_1X_3 X_4 - b_{334}X_2X_3 X_4 - b_{334}X_3 X_4^2 - b_{334}X_3 X_4X_5 \\ & + b_{335}X_3 X_5 - b_{335}X_1X_3 X_5 - b_{335}X_2X_3 X_5 - b_{335}X_3X_4 X_5 - b_{335}X_3 X_5^2 + b_{44}X_4 - b_{44}X_1X_4 - b_{44}X_2X_4 - b_{44}X_3X_4 \\ & - b_{44}X_4X_5 + b_{45}X_4X_5 + b_{444}X_4^3 + b_{445}X_4 X_5 - b_{445}X_1X_4 X_5 - b_{445}X_2X_4 X_5 - b_{445}X_3X_4 X_5 - b_{445}X_4 X_5^2 \\ & + b_{55} X_5 - b_{55} X_1X_5 - b_{55} X_2X_5 - b_{55} X_3X_5 - b_{55} X_4X_5 + b_{555}X_5^3 \end{aligned} \tag{7}$$

$$\begin{aligned} Y = {} & \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14}X_1 X_4 + \beta_{15}X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24}X_2 X_4 \\ & + \beta_{25}X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35}X_3 X_5 + \beta_{45}X_4 X_5 + \gamma_{12}X_1 X_2^2 + \gamma_{13} X_1 X_3^2 + \gamma_{14} X_1 X_4^2 + \gamma_{15} X_1 X_5^2 \\ & + \gamma_{23} X_2 X_3^2 + \gamma_{24}X_2 X_4^2 + \gamma_{25} X_2X_5^2 + \gamma_{34} X_3 X_4^2 + \gamma_{35} X_3 X_5^2 + \gamma_{45} X_4 X_5^2 + \beta_{123} X_1 X_2 X_3 + \beta_{124} X_1 X_2 X_4 \\ & + \beta_{125}X_1 X_2 X_5 + \beta_{134}X_1 X_3 X_4 + \beta_{135}X_1 X_3 X_5 + \beta_{145}X_1 X_4 X_5 + \beta_{234}X_2 X_3 X_4 + \beta_{235}X_2 X_3 X_5 \\ & + \beta_{245} X_2 X_4 X_5 + \beta_{345}X_3 X_4 X_5 \end{aligned} \tag{8}$$

Where

$\beta_1 = [b_0 + b_1 + b_{11}]$; $\beta_2 = [b_0 + b_2 + b_{22}]$; $\beta_3 = [b_0 + b_3 + b_{33}]$; $\beta_4 = [b_0 + b_4 + b_{44}]$; $\beta_5 = [b_0 + b_5 + b_{55}]$;

$\beta_{12} = [b_{12} – b_{11} - b_{22} + b_{112}]$.; $\beta_{13} = [b_{13} – b_{11} - b_{33} + b_{113}]$; $\beta_{14} = [b_{14} – b_{14} - b_{11} – b_{44} + b_{114}]$;

$\beta_{15} = [b_{15} – b_{11} – b_{55} + b_{115}]$; $\gamma_{12} = [- b_{112}]$; $\gamma_{13} = [- b_{113}]$; $\gamma_{14} = [- b_{114}]$; $\gamma_{15} = [- b_{115}]$;

$\beta_{123} = [-b_{112}- b_{113} –b_{223}]$; $\beta_{124} = [-b_{112}- b_{114} – b_{224}]$; $\beta_{125} = [-b_{112}- b_{115} –b_{225}]$; $\beta_{134} = [-b_{113}- b_{114} –b_{334}]$;

$\beta_{135} = [-b_{113}- b_{115} –b_{335}]$; $\beta_{145} = [-b_{113}- b_{115} –b_{445}]$; $\beta_{23} = [b_{23} – b_{22} - b_{33} + b_{223}]$; $\beta_{24}= [b_{24} – b_{22} - b_{44} + b_{224}]$;

$\beta_{25} = [b_{25} – b_{22} - b_{55} + b_{225}]$; $\gamma_{23} = [- b_{223}]$; $\gamma_{24} = [- b_{224}]$; $\gamma_{25} = [- b_{225}]$; $\beta_{234} = [-b_{223}- b_{224} –b_{334}]$;

$\beta_{235} = [-b_{223}- b_{225} –b_{335}]$; $\beta_{245} = [-b_{224} – b_{225} –b_{445}]$; $\beta_{34} = [b_{34} – b_{33} – b_{44} + b_{334}]$; $\beta_{35} =[b_{35} – b_{33} – b_{55} + b_{335}]$;

$\gamma_{34} = [- b_{334}]$; $\gamma_{35} = [- b_{335}]$; $\beta_{345} = [-b_{334} – b_{335} –b_{445}]$; β_{45}; $= [b_{45} – b_{44} – b_{55} + b_{445}]$; $\gamma_{45} = [- b_{445}]$ **(9)**

Equation (8) is the regression equation for Scheffe's (5, 3) simplex

2.4 . COEFFICIENTS OF THE SCHEFFE'S (5, 3) POLYNOMIAL

From the work of Nwachukwu and others (2022a), the coefficients of the Scheffe's (5, 3) polynomial have been determined as under. :

$\beta_1 = Y_1$; $\beta_2 = Y_2$; $\beta_3 = Y_3$; $\beta_4 = Y_4$; and $\beta_5 = Y_5$ **10(a-e)**

$\beta_{12} = 9/4(Y_{112} + Y_{122} – Y_1 – Y_2)$; $\beta_{13} = 9/4 (Y_{113} + Y_{133} - Y_1 - Y_3)$; $\beta_{14} = 9/4 (Y_{114} + Y_{144} - Y_1 - Y_4)$; **11(a-c)**

$\beta_{15} = 9/4 (Y_{115} + Y_{155} - Y_1 - Y_5)$; $\beta_{23} = 9/4 (Y_{223} + Y_{233} - Y_2 - Y_3)$; $\beta_{24} = 9/4 (Y_{224} + Y_{244} - Y_2 - Y_4)$ **12(a-c)**

$\beta_{25} = 9/4(Y_{225} + Y_{255} - Y_2 - Y_5)$; $\beta_{34} = 9/4(Y_{334} + Y_{344} - Y_3 - Y_4)$; $\beta_{35} = 9/4(Y_{335} + Y_{355} - Y_3 - Y_5)$ **13(a-c)**

$\beta_{45} = 9/4(Y_{445} + Y_{455} - Y_4 - Y_5)$; $\gamma_{12} = 9/4(3Y_{112} + 3Y_{122} - Y_1 + Y_2)$; $\gamma_{13} = 9/4(3Y_{113} + 3Y_{133} - Y_1 + Y_3)$ **14(a-c)**

$\gamma_{14} = 9/4(3Y_{114} + 3Y_{144} - Y_1 + Y_4)$; $\gamma_{15} = 9/4(3Y_{115} + 3Y_{115} - Y_1 + Y_5)$; $\gamma_{23} = 9/4(3Y_{223} + 3Y_{233} - Y_2 + Y_3)$ **15(a-c)**

$\gamma_{24} = 9/4 (3 Y_{224} + 3 Y_{244} - Y_2 + Y_4)$; $\gamma_{25} = 9/4(3Y_{225} + 3Y_{255} - Y_2 + Y_5)$; $\gamma_{34} = 9/4(3Y_{334} + 3Y_{344} - Y_3 + Y_4)$ **16(a-c)**

$\gamma_{35} = 9/4(3Y_{335} + 3Y_{355} - Y_3 + Y_5)$; $\gamma_{45} = 9/4(3Y_{445} + 3Y_{455} - Y_4 + Y_5)$ **17(a-b)**

$\beta_{123} = 27Y_{123} - 27/4(Y_{112} + Y_{122} + Y_{113} + {}_{133} + Y_{223} + Y_{233}) + 9/4(Y_1 + Y_2 + Y_3)$ **(18)**

$\beta_{124} = 27Y_{124} - 27/4(Y_{112} + Y_{122} + Y_{114} + Y_{144} + Y_{224} + Y_{244}) + 9/4(Y_1 + Y_2 + Y_4)$ **(19)**

$\beta_{125} = 27Y_{125} - 27/4(Y_{112} + Y_{122} + Y_{115} + Y_{155} + Y_{225} + Y_{255}) + 9/4(Y_1 + Y_2 + Y_5)$ **(20)**

$\beta_{134} = 27Y_{134} - 27/4(Y_{113} + Y_{133} + Y_{114} + Y_{144} + Y_{334} + Y_{344}) + 9/4(Y_1 + Y_3 + Y_4)$ **(21)**

$\beta_{135} = 27Y_{135} - 27/4(Y_{113} + Y_{133} + Y_{115} + Y_{155} + Y_{335} + Y_{355}) + 9/4(Y_1 + Y_3 + Y_5)$ **(22)**

$\beta_{145} = 27Y_{145} - 27/4(Y_{114} + Y_{144} + Y_{115} + Y_{155} + Y_{445} + Y_{455}) + 9/4(Y_1 + Y_4 + Y_5)$ **(23)**

$\beta_{234} = 27Y_{234} - 27/4(Y_{223} + Y_{233} + Y_{224} + Y_{244} + Y_{334} + Y_{344}) + 9/4(Y_2 + Y_3 + Y_4)$ **(24)**

$\beta_{235} = 27Y_{235} - 27/4(Y_{223} + Y_{233} + Y_{225} + Y_{255} + Y_{335} + Y_{355}) + 9/4(Y_2 + Y_3 + Y_5)$ **(25)**

$\beta_{245} = 27Y_{245} - 27/4(Y_{224} + Y_{244} + Y_{225} + Y_{255} + Y_{445} + Y_{455}) + 9/4(Y_2 + Y_4 + Y_5)$ **(26)**

$\beta_{345} = 27Y_{345} - 27/4(Y_{334} + Y_{344} + Y_{335} + Y_{355} + Y_{445} + Y_{455}) + 9/4(Y_3 + Y_4 + Y_5)$ **(27)**

Where Y_i = Response Function (Compressive Strength) for the pure component, *i*

2.5. SCHEFFE'S (5, 3) MIXTURE DESIGN MODEL

Substituting Eqns. (10)-(27) into Eqn. (8), yields the mixture design model for the Scheffe's (5, 3) lattice.

2.6. ACTUAL AND PSEUDO MIX RATIOS OF SCHEFFE'S (5, 3) DESIGN LATTICE

The requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4, 1:3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions are always chosen for the five vertices of Scheffe's (5, 3) lattice. See the works of Nwachukwu and others (2022a), for the figure showing the vertices of a Scheffe's (5, 3) lattice for both actual and pseudo mix ratios.

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), which represent water/cement ratio, cement, fine aggregate, coarse aggregate and polypropylene fibre respectively.

For the pseudo mix ratio, the following corresponding mix ratios at the vertices for five component mixtures are always chosen: A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1)

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (4) and (5) are used.

Substituting the mix ratios from point A_1 into Eqn. (4) gives:

$$\begin{pmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \textbf{(28)}$$

Transforming the R.H.S matrix and solving, we obtain

A_{111}= 0.67; A_{221}= 1; A_{331}= 1.7; A_{441}= 2; A_{551}= 0.5

The same approach is used to obtain the remaining values as shown in Eqn. (29)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \qquad \textbf{(29)}$$

Considering mix ratios at the mid points from Eqn.(2) and substituting these pseudo mix ratios in turn into Eqn.(29) will yield the corresponding actual mix ratios.

For point A_{112}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.63 \\ 1 \\ 1.67 \end{Bmatrix} \qquad \textbf{(30)}$$

Z_4	2.0	1.8	1.7	1.8	1.2	0	1.90
Z_5	0.5	0.8	1.0	1.2	1.5	0	1.60

Solving, Z_1 = 0.63; Z_2 = 1.00; Z_3 = 1.67' Z_4 = 1.90; Z_5 = 1.60

The same approach goes for the remaining mid-point mix ratios.

Hence, to generate the regression coefficients, 35 experimental tests will be carried out and the corresponding mix ratios are depicted in Table 1.

Table 1: Actual and Pseudo Mix Ratio for the Scheffe's (5,3) Lattice.

Points	**Pseudo Component**					**Response**	**Actual Component**				
	X_1	**X_2**	**X_3**	**X_4**	**X_5**	**Symbol**	**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**
1	1	0	0	0	0	Y_1	0.67	1.00	1.70	2.0	0.5
2	0	1	0	0	0	Y_2	0.56	1.00	1.60	1.8	0.8
3	0	0	1	0	0	Y_3	0.50	1.00	1.20	1.7	1.0
4	0	0	0	1	0	Y_4	0.70	1.00	1.00	1.8	1.2
5	0	0	0	0	1	Y_5	0.75	1.00	1.30	1.2	1.5
112	0.67	0.33	0	0	0	Y_{112}	0.63	1.00	1.67	1.9	1.6
122	0.33	0.67	0	0	0	Y_{122}	0.60	1.00	1.63	1.8	0.7
113	0.67	0	0.33	0	0	Y_{113}	0.61	1.00	1.54	1.9	0.6
133	0.33	0	0.67	0	0	Y_{133}	0.56	1.00	1.37	1.8	0.8
114	0.67	0	0	0.33	0	Y_{114}	0.68	1.00	1.47	1.9	0.7
144	0.33	0	0	0.67	0	Y_{144}	0.69	1.00	1.23	1.8	0.9

115	0.67	0	0	0	0.33	Y_{115}	0.70	1.00	1.57	1.7	0.8
155	0.33	0	0	0	0.67	Y_{115}	0.72	1.00	1.43	1.4	1.1
223	0	0.67	0.33	0	0	Y_{223}	0.55	1.00	1.40	1.7	0.8
233	0	0.33	0.67	0	0	Y_{233}	0.52	1.00	1.20	1.7	0.9
224	0	0.67	0	0.33	0	Y_{224}	0.61	1.00	1.67	1.8	0.9
244	0	0.33	0	0.67	0	Y_{244}	0.66	1.00	1.73	1.8	1.0
225	0	0.67	0	0	0.33	Y_{225}	0.63	1.00	1.50	1.6	0.7
255	0	0.33	0	0	0.67	Y_{255}	0.69	1.00	1.40	1.4	0.6
234	0	0	0.67	0.33	0	Y_{334}	0.57	1.00	1.13	1.7	1.0
344	0	0	0.33	0.67	0	Y_{344}	0.64	1.00	1.07	1.7	1.1
335	0	0	0.67	0	0.33	Y_{355}	0.58	1.00	1.23	1.5	1.1
355	0	0	0.33	0	0.67	Y_{335}	0.67	1.00	1.27	1.3	1.3
445	0	0.33	0	0	0.67	Y_{445}	0.72	1.00	1.10	1.6	1.3
455	0	0	0	0.67	0.33	Y_{445}	0.73	1.00	1.20	1.4	1.4
123	0.33	0.33	0.33	0	0	Y_{123}	0.57	1.00	1.49	1.8	0.7
124	0.33	0.33	0	0.33	0	Y_{124}	0.64	1.00	1.09	1.8	0.8
125	0.33	0.33	0	0	0.33	Y_{125}	0.66	1.00	1.52	1.6	0.9
134	0.33	0.33	0	0.33	0	Y_{134}	0.62	1.00	1.29	1.8	0.8
135	0.33	0	0.33	0	0.33	Y_{135}	0.63	1.00	1.39	1.6	0.9
145	0.33	0	0	0.33	0.33	Y_{145}	0.70	1.00	1.32	1.6	1.0

234	0	0.33	0.33	0.33	0	Y_{234}	0.58	1.00	1.25	1.7	0.9
235	0	0.33	0.33	0	0.33	Y_{235}	0.60	1.00	1.32	1.5	1.0
245	0	0.33	0	0.33	0.33	Y_{245}	0.67	1.00	1.29	1.5	1.1
345	0	0	0.33	0.33	0.33	Y_{345}	0.64	1.00	1.6	1.5	1.2

2.7. FOR THE CONTROL POINTS

Thirty five (35) different controls were predicted which according to Scheffe's (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2 : Actual and Pseudo Component of Scheffe (5,3) Lattice for Control Points

Points	**Pseudo Component**					**Control Points**	**Actual Component**				
	X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	0.25	0.25	0.25	0.25	0	C_1	0.61	1	1.38	1.83	0.5
2	0.25	0.25	0.25	0	0.25	C_2	0.62	1	1.45	1.68	0.8
3	0.25	0.25	0	0.25	0.25	C_3	0.67	1	1.40	1.70	1
4	0.25	0	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.2
5	0	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.5
112	0.20	0.20	0.2	0.20	0.20	C_{112}	0.64	1	1.36	1.70	0.65
122	0.30	0.30	0.30	0.10	0	C_{122}	0.59	1	1.45	1.83	0.75
113	0.30	0.30	0.30	0	0.10	C_{113}	0.59	1	1.48	1.77	0.85
133	0.30	0.30	0	0.30	0.10	C_{133}	0.65	1	1.42	1.80	1
114	0.30	0	0.30	0.30	0.10	C_{114}	0.64	1	1.30	1.77	0.9
144	0	0.30	0.30	0.30	0.10	C_{144}	0.60	1	1.27	1.71	1

115	0.10	0.30	0.30	0.30	0	C_{115}	0.60	1	1.31	1.79	1.55
155	0.30	0.10	0.30	0.30	0	C_{155}	0.62	1	1.33	1.83	1.1
223	0.30	0.10	0.30	0.30	0	C_{223}	0.63	1	1.41	1.85	1.25
233	0.10	0.20	0.30	0.40	0	C_{233}	0.61	1	1.25	1.79	1.35
224	0.30	0.20	0.10	0.40	0	C_{224}	0.64	1	1.35	1.85	0.89
244	0.20	0.20	0.10	0.10	0.40	C_{244}	1.40	1	1.04	1.59	1.08
225	0.30	0.10	0.30	0.20	0.10	C_{225}	0.62	1	1.36	1.77	0.92
255	0.25	0.25	0.15	0.15	0.20	C_{255}	0.61	1	1.51	3.16	0.91
334	0.30	0.30	0.20	0.10	0.10	C_{334}	0.68	1	1.56	1.96	0.98
344	0.10	0.30	0.30	0.30	0	C_{344}	1.30	1	1.31	1.79	0.95
335	0.25	0.15	0.20	0.20	0.20	C_{335}	0.65	1	0.96	1.05	0.97
355	0.15	0.25	0.20	0.20	0.20	C_{355}	0.64	1	1.37	1.71	0.79
445	0.10	0.20	0.30	0.40	0	C_{445}	0.61	1	1.25	1.79	0.99
455	0.30	0.10	0.20	0.30	0.10	C_{455}	0.61	1	1.31	1.72	1.03
123	0.25	0.10	0.40	0	0.25	C_{123}	0.61	1	1.39	1.66	0.98
124	0.30	0.20	0.40	0.10	0	C_{124}	0.58	1	1.41	1.82	0.83
125	0.15	0.15	0.20	0.10	0.40	C_{125}	0.65	1	1.36	1.57	1.11
134	0.10	0.30	0	0.30	0.30	C_{134}	0.67	1	1.34	1.65	1.10
135	0.25	0.20	0.20	0.20	0.15	C_{135}	0.74	1	1.38	2.08	0.88
145	0.10	0.10	0.10	0.30	0.40	C_{145}	0.68	1	1.27	1.57	1.19

234	0.40	0.20	0.10	0.10	0.20	$\mathbf{C_{234}}$	0.73	1	1.61	1.87	1.03
235	0.25	0.25	0.15	0.25	0.10	$\mathbf{C_{235}}$	0.63	1	1.39	1.78	0.93
245	0.15	0.20	0.10	0.25	0.30	$\mathbf{C_{245}}$	0.66	1	1.34	1.64	1.09
345	0.30	0.10	0.20	0.25	0.15	$\mathbf{C_{345}}$	0.64	1	1.34	1.75	0.96

The actual component as transformed from Eqn. (29) , Table (1) and (2) were used to measure out the quantities of water (Z_1), cement (Z_2), fine aggregate as sand (Z_3), coarse aggregate (Z_4) and polypropylene fibre (Z_5) in their respective ratios for the concrete cube strength test.

3. MATERIALS AND METHODS

3.1 MATERIALS

The materials under investigation in this research work are cement, water, fine and coarse aggregate and polypropylene fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. The same size and nature of polypropylene fibre used previously by Nwachukwu and others (2022c) is the same as the one being used in this present work. Also, potable water drawn from the clean water source was used in the experimental investigation.

3.2. METHOD

3.2.1. SPECIMEN PREPARATION / BATCHING/ CURING

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 70 mix ratios were to be used to produce 140 prototype concrete cubes. Thirty five (35) out of the 70 mix ratios were as control mix ratios to produce 70 cubes for the conformation of the adequacy of the mixture design given by Eqn. (8), whose coefficients are given in Eqns. (10) – (27). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank.

3.2.2. COMPRESSIVE STRENGTH TEST

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline .Two samples were crushed for each mix ratio and in each case, the compressive strength was then calculated using Eqn.(31)

$$\text{Compressive Strength} = \frac{\text{Average failure Load (N)}}{\text{Cross- sectional Area (mm}^2)} \qquad \frac{P}{A} \tag{31}$$

4. RESULTS AND DISCUSSION

4.1. COMPRESSIVE STRENGTH RESULTS FOR THE INITIAL EXPERIMENTAL TESTS.

The results of the compressive strength ($R_{esponse,}$ Y_i) based on a 28-days strength is presented in Table 3. These are calculated from Eqn.(31)

Table 3: 28th Day Compressive Strength Test Results for PFRC Based on Scheffe's (5, 3) Model for the Initial Experimental Tests.

Points	Replicate	Response Y_i, N/mm^2	Response Symbol	$\sum Y_i$	Average Response Y, N/mm^2
1	1A 1B	20.98 21.34	$\mathbf{Y_1}$	42.32	21.16
2	2A 2B	20.43 20.24	$\mathbf{Y_2}$	40.67	20.34
3	3A 3B	19.43 19.38	$\mathbf{Y_3}$	38.81	19.41
4	4A 4B	20.12 20.08	$\mathbf{Y_4}$	40.20	20.10
5	5A 5B	18.46 19.06	$\mathbf{Y_5}$	37.52	18.76
112	6A 6B	20.33 20.24	$\mathbf{Y_{112}}$	40.57	20.29
122	7A 7B	27.26 27.24	$\mathbf{Y_{122}}$	54.50	27.25
113	8A 8B	17.84 18.08	$\mathbf{Y_{113}}$	35.98	17.96
133	9A 9B	20.11 20.16	$\mathbf{Y_{133}}$	40.27	20.14
114	10A 10B	18.98 18.86	$\mathbf{Y_{114}}$	37.84	18.92
144	11A 11B	20.33 20.34	$\mathbf{Y_{144}}$	40.67	20.34
115	12A 12B	21.56 21.42	$\mathbf{Y_{115}}$	42.98	21.49
155	13A 13B	16.75 17.32	$\mathbf{Y_{155}}$	34.07	17.04
223	14A 14B	20.22 20.26	$\mathbf{Y_{223}}$	40.48	20.24
233	15A 15B	15.98 16.08	$\mathbf{Y_{233}}$	32.06	16.03

224	16A 16B	20.77 21.04	$\mathbf{Y_{224}}$	41.81	20.91
244	17A 17B	20.45 20.23	$\mathbf{Y_{244}}$	40.68	20.34
225	18A 18B	17.86 18.09	$\mathbf{Y_{225}}$	35.95	17.98
255	19A 19B	20.55 21.11	$\mathbf{Y_{255}}$	41.66	20.83
334	20A 20B	19.76 19.80	$\mathbf{Y_{334}}$	39.56	19.78
344	21A 21B	24.87 25.22	$\mathbf{Y_{344}}$	50.09	25.05
335	22A 22B	18.88 19.09	$\mathbf{Y_{335}}$	37.97	18.99
355	23A 23B	20.66 20.45	$\mathbf{Y_{355}}$	41.11	20.56
445	24A 24B	19.77 19.76	$\mathbf{Y_{445}}$	39.53	19.77
455	25A 25B	18.83 18.90	$\mathbf{Y_{455}}$	37.73	18.86
123	26A 26B	20.22 19.89	$\mathbf{Y_{123}}$	40.11	20.06
124	27A 27B	25.64 25.44	$\mathbf{Y_{124}}$	51.08	25.54
125	28A 28B	20.33 20.42	$\mathbf{Y_{125}}$	40.75	20.38
134	29A 29B	16.77 17.12	$\mathbf{Y_{134}}$	33.89	16.95
135	30A 30B	21.33 21.34	$\mathbf{Y_{135}}$	42.67	21.34
145	31A 31B	22.43 22.34	$\mathbf{Y_{145}}$	44.77	22.39
234	32A 32B	19.88 20.14	$\mathbf{Y_{234}}$	40.02	20.01
235	33A	18.98	$\mathbf{Y_{235}}$	38.19	19.10

	33B	19.21			
245	34A 34B	20.33 20.38	**Y_{245}**	40.71	20.36
345	35A 35B	20.43 20.54	**Y_{345}**	40.97	20.49

4.2 COMPRESSIVE STRENGTH RESULTS FOR THE EXPERIMENTAL (CONTROL) TEST.

Table 4 shows the 28th day Compressive strength results for the Experimental (Control) Test

Table 4: 28TH Day Compressive Strength Values for PFRC Based on Scheffe's (5, 3) Model for the Experimental (Control) Tests.

Control Points	Replicate	Response N/mm^2	Average Response N/mm^2
C_1	1A 1B	21.44 21.36	21.40
C_2	2A 2B	20.12 20.00	20.06
C_3	3A 3B	18.55 18.32	18.44
C_4	4A 4B	20.67 20.65	20.66
C_5	5A 5B	19.86 19.74	19.80
C_{112}	6A 6B	20.02 19.68	19.85
C_{122}	7A 7B	26.34 26.58	26.46
C_{113}	8A 8B	18.23 18.21	18.22
C_{133}	9A 9B	20.78 20.87	20.83
C_{114}	10A 10B	19.08 18.99	19.04

C_{144}	11A 11B	22.11 21.88	22.00
C_{115}	12A 12B	20.33 20.98	20.66
C_{155}	13A 13B	18.35 18.12	18.24
C_{223}	14A 14B	20.78 20.58	20.68
C_{233}	15A 15B	16.98 17.22	21.10
C_{224}	16A 16B	21.56 21.44	21.50
C_{244}	17A 17B	19.75 19.88	19.82
C_{225}	18A 18B	18.22 18.18	18.20
C_{255}	19A 19B	21.09 21.11	21.10
C_{334}	20A 20B	20.43 19.80	20.12
C_{344}	21A 21B	26.35 25.89	26.12
C_{335}	22A 22B	19.12 19.10	19.11
C_{355}	23A 23B	19.76 19.68	19.72
C_{445}	24A 24B	19.76 19.45	19.61
C_{455}	25A 25B	19.08 19.10	19.09
C_{123}	26A 26B	19.66 19.89	19.76
C_{124}	27A 27B	25.00 24.49	24.75
C_{125}	28A	20.98	21.06

	28B	21.13	
C_{134}	29A 29B	17.44 17.38	17.41
C_{135}	30A 30B	21.82 22.02	21.92
C_{145}	31A 31B	22.78 22.43	22.61
C_{234}	32A 32B	19.14 19.23	19.19
C_{235}	33A 33B	18.04 18.22	18.13
C_{245}	34A 34B	21.34 21.42	21.38
C_{345}	35A 35B	20.76 20.74	20.75

4.3 SCHEFFE'S (5,3) REGRESSION MODEL FOR COMPRESSIVE STRENGTH OF PFRC

By substituting the values of the responses from Table 3 into Eqns. (10) through (27), the coefficients of the Scheffe's third degree polynomial were determined as follows:

$\beta_1 = 21.16$; $\beta_2 = 20.34$; $\beta_3 = 19.41$; $\beta_4 = 20.10$; $\beta_5 = 18.76$; $\beta_{12} = 13.59$; $\beta_{13} = -5.56$; $\beta_{14} = -4.50$; $\beta_{15} = -3.13$; $\beta_{23} = -7.83$; $\beta_{24} = 1.82$; $\beta_{25} = -3.67$; $\beta_{34} = 11.97$; $\beta_{35} = 3.11$; $\beta_{45} = -0.52$; $\gamma_{12} = 319.05$; $\gamma_{13} = 253.24$; $\gamma_{14} = 262.62$; $\gamma_{15} = 254.68$; $\gamma_{23} = 242.73$; $\gamma_{24} = 277.90$; $\gamma_{25} = 258.41$; $\gamma_{34} = 304.16$; $\gamma_{35} = 264.89$; $\gamma_{45} = 257.74$; $\beta_{123} = 410.58$; $\beta_{124} = -140.16$; $\beta_{125} = -159.53$; $\beta_{134} = -330.57$; $\beta_{135} = -74.55$; $\beta_{145} = -46.27$; $\beta_{234} = -150.93$; $\beta_{235} = -126.40$; $\beta_{245} = -118.24$; $\beta_{345} = .-145.98$ **(32)**

Substituting the values of these coefficients in Eqn.(32) into Eqn. (8), we obtain the mathematical/regression model for the optimization of the compressive strength of the concrete cubes made using polypropylene fibre (PFRC) based on Scheffe's (5,3) polynomial given in Eqn.(33).

$$Y = 21.16X_1 + 20.34X_2 + 19.41X_3 + 20.10X_4 + 18.76X_5 + 13.59X_1X_2 - 5.56X_1X_3 - 4.50X_1X_4 - 3.13X_1X_5 - 7.83X_2X_3 + 1.82X_2X_4 - 3.67X_2X_5 + 11.97X_3X_4 + 3.11X_3X_5 - 0.52X_4X_5 + 319.05X_1X_2^2 + 253.24X_1X_3^2 + 262.62X_1X_4^2 + 254.68X_1X_5^2 + 242.73X_2X_3^2 + 277.90X_2X_4^2 + 258.41X_2X_5^2 + 304.16X_3X_4^2 + 264.89X_3X_5^2 + 257.74X_4X_5^2 + 410.58X_1X_2X_3 - 140.16X_1X_2X_4 - 159.53X_1X_2X_5 - 330.57X_1X_3X_4 - 74.55X_1X_3X_5 - 46.27X_1X_4X_5 - 150.93X_2X_3X_4 - 126.40X_2X_3X_5 - 118.24X_2X_4X_5 - 145.98X_3X_4X_5 \quad (33)$$

4.4. RESPONSE OF CONTROL POINTS FROM SCHEFFE'S (5, 3) REGRESSION MODEL.

By substituting the pseudo mix ratio of points c_1, c_2, c_3, c_4, c_5, …….. c_{345} of Table 2 into Eqn.(33) , we obtain the third degree model response as shown in Table 5.

Table 5: PFRC Response of Control Points from Scheffe's (5, 3) Regression Model

Control	X_1	X_2	X_3	X_4	X_5	Response, N/mm^2

Points						
C_1	0.25	0.25	0.25	0.25	0	24.32
C_2	0.25	0.25	0.25	0	0.25	19.65
C_3	0.25	0.25	0	0.25	0.25	21.76
C_4	0.25	0	0.25	0.25	0.25	23.55
C_5	0	0.25	0.25	0.25	0.25	18.89
C_{112}	0.20	0.20	0.20	0.20	0.20	21.98
C_{122}	0.30	0.30	0.30	0.10	0	26.98
C_{113}	0.30	0.30	0.30	0	0.10	20.11
C_{133}	0.30	0.30	0	0.30	0.10	22.65
C_{114}	0.30	0	0.30	0.30	0.10	20.19
C_{144}	0	0.30	0.30	0.30	0.10	19.87
C_{115}	0.1	0.30	0.30	0.30	0	18.98
C_{155}	0.30	0.10	0.30	0.30	0	21.32
C_{223}	0.30	0.30	0.10	0.30	0	23.21
C_{233}	0.10	0.20	0.30	0.40	0	19.23
C_{224}	0.30	0.20	0.10	0.40	0	23.11
C_{244}	0.20	0.20	0.10	0.10	0.40	18.22
C_{225}	0.30	0.10	0.30	0.20	0.10	17.08
C_{255}	0.25	0.25	0.15	0.15	0.20	20.44
C_{334}	0.30	0.30	0.20	0.10	0.10	22.12
C_{344}	0.10	0.30	0.30	0.30	0	25.77
C_{335}	0.25	0.15	0.20	0.20	0.20	20.02
C_{355}	0.15	0.25	0.20	0.20	0.20	21.00
C_{445}	0.10	0.20	0.30	0.40	0	18.87
C_{455}	0.30	0.10	0.20	0.30	0.10	19.08
C_{123}	0.25	0.10	0.40	0	0.25	20.03
C_{124}	0.30	0.20	0.40	0.10	0	23.72
C_{125}	0.15	0.15	0.20	0.10	0.40	20.86

C_{134}	0.10	0.30	0	0.30	0.30	19.43
C_{135}	0.25	0.20	0.20	0.20	0.15	22.88
C_{145}	0.10	0.10	0.10	0.30	0.40	24.65
C_{234}	0.40	0.20	0.10	0.10	0.20	20.42
C_{235}	0.25	0.25	0.15	0.25	0.10	19.68
C_{245}	0.15	0.20	0.10	0.25	0.30	23.98
C_{345}	0.30	0.10	0.20	0.25	0.15	23.01

4.5 VALIDATION AND TEST OF ADEQUACY OF THE MODEL

Here, the Student's – T - test is adopted to check if there is any significant difference between the lab responses (compressive strength results) given in Table 4 and model responses given in Table 5. The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The outcome of the test shows that there is no significant difference between the experimental results and model results. Thus, the model is very adequate for predicting the compressive strength of PFRC based on Scheffe's (5,3) polynomial.

4.6. DISCUSSION OF RESULTS

The maximum PFRC compressive strength of 27.25 N/mm^2 corresponding to mix ratio of 0.60:1:1.63:1.80:0.7 for water, cement, fine aggregate, coarse aggregate and polypropylene fibre respectively was obtained through the Scheffe's third degree lattice. The lowest strength was found to be 16.03 N/mm^2 corresponding to mix ratio of 0.52:1:1.20:1.70:0.9. The maximum strength value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete. Using the model, compressive strength of PFRC of all points in the simplex can be determined based on third degree model**.**

5. CONCLUSION

So far, Scheffe's Third Degree Regression Model, Scheffe's (5,3) was used to predict the mix ratios as well as a model for predicting the compressive strength of PFRC cubes. Using Scheffe's (5, 3) simplex model, the values of the compressive strength were obtained for PFRC. As confirmed through the student's t-test, there is good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The maximum attainable compressive strength of PFRC predicted by the Scheffe's (5, 3) model at the 28th day was 27.25 N/mm$^{2.}$ Although this value is slightly higher than the maximum value (25.23 N/mm^2) obtained by Nwachukwu and others (2022c) for PFRC based on Scheffe's (5,2) model, both values meet the minimum standard requirement stipulated by American Concrete Institute (ACI) of 20N/mm^2 for the compressive strength of good concrete. With the model, any desired strength of Polypropylene Fibre Reinforced Concrete, given any mix proportions can be easily predicted and determined. Thus the problem of having to go through vigorous and laborious mix- design procedures to obtain a desiring strength of PFRC has been reduced by the utilization of this Scheffe's optimization model. More also, stakeholders in the construction industry stand to gain a lot from FRC owing to the fact that replacement(either partially or wholly) of the conventional steel reinforcement with fibre goes a long way to save cost, as steel reinforcements are more costly than fibres.

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