

# BUILDING A BILLET TEMPERATURE DISTRIBUTION MODEL TO DEFINE THE TEMPERATURE SET POINT FOR THE CONTINUOUS FURNACE

Dao Duy Yen<sup>1</sup>, Nguyen Nam Trung<sup>2</sup>

<sup>1,2</sup>Thai Nguyen University of Technology, Thai Nguyen city, Viet Nam

## ABSTRACT

*In the steel rolling industry, the continuous furnace is the largest energy consumer object and has the most complex dynamics. By making discrete for spatial variable, a nonlinear distributed parameter system is transformed into ordinary differential equations by using finite element method (FEM). The author also provided proof of model equivalence between continuous and static furnaces in specific modes to facilitate model testing. Simulation and experimental results performed on a static furnace have proven the correctness for the proposed method, the appropriateness to object of study and the capability for applying to metal burning process which is before the final processing stages such as rolling, forging, shaping etc.*

**Keyword:** slab temperature control, metal heating problem, temperature set-point.

---

## 1. INTRODUCTION

The object of research in the article is a metal furnace. This is a complex control problem for a system that consumes a lot of energy, so it has attracted the attention of many scientists around the world. The results achieved [1, 2] using the linear laplace transform and nonlinear programming method, the optimal solution algorithm (uDEAS) also only considers the case of temperature conductivity coefficient is a constant. In cases where the nonlinearity of the coefficient  $a$  in some steel grades is significant. It is very difficult to design advanced controllers to ensure the technical requirements for steel rolling technology and often cannot respond. Therefore, the contribution of this article is to supplement the above limited problems: It is the use of the finite element method to approximate the distributed parameter system (taking into account the nonlinear of the thermal conductivity coefficient) nonlinearity  $a(Q)$  to convert the system to a centralized system with allowable errors [3, 4] to develop a model that may predict the temperature distribution within the steel billet during heating. This step reduces the complexity of the model, bringing applicability high in practice while still ensuring the necessary accuracy.

## 2. THE OBJECT MODEL [5, 6, 7]

Burning metal in furnace is a continuous process, where materials are conveyed through some heat areas in succession. Steel slabs are directed by pushing motors and moved from low temperature area to high temperature one in the furnace space. They move through different temperature zones along the furnace.

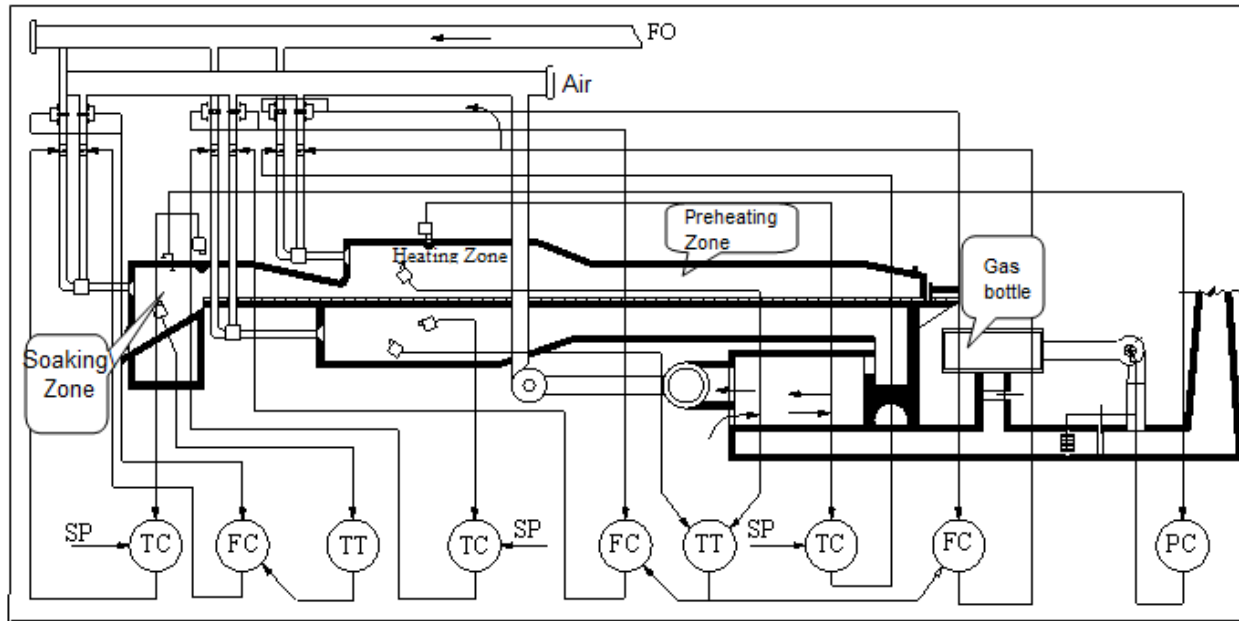


Fig.1. Geometry of the model continuous reheating furnace.

This kind of motion is the same as the case that slabs are kept immobile but temperature varies in time. Thus, a static heating furnace can be used in place of a continuous reheating furnace for experimental purposes, such as testing model to calculate the temperature distribution of the object. Therefore, this process is similar to the moving of slabs in the continuous reheating furnace. The static furnace considered as a one-sided heat transfer system is described by the parabolic-type partial differential equation, as follows in (1)

$$\left. \begin{aligned} \frac{\partial Q(x, t)}{\partial t} &= a(Q) \frac{\partial^2 Q(x, t)}{\partial x^2} & (1) \\ Q(x, 0) &= Q_0 & (2) \\ -\frac{\partial Q}{\partial x}(0, t) &= \frac{\alpha}{\lambda(Q)} [u(t) - Q(0, t)] & (3) \\ \frac{\partial Q}{\partial x}(\delta, t) &= 0 & (4) \end{aligned} \right\}$$

where  $Q(x, t)$ , the temperature distribution in the slab, depending on the spatial coordinate  $x$  with  $0 \leq x \leq L$  and the time  $t$  with  $0 \leq t \leq t_f$ ,  $a$  is the temperature-conducting factor ( $m^2/s$ ),  $\delta$  is the thickness of object in the  $x$  direction (m),  $T$  is the allowed burning time (s), Where  $\lambda$ ,  $a$ ,  $\alpha$  and  $u(t)$  are the heat-conducting factor, temperature conductivity factor, the as the heat-transfer coefficient between the furnace space and the object ( $W/m^2 \cdot ^\circ C$ ) and  $u(t)$  as the temperature of the furnace respectively ( $^\circ C$ ).

The initial condition (2) and the boundary conditions (3), (4).The temperature  $u(t)$  of the furnace is controlled by voltage  $v(t)$ . Therefore, the temperature distribution  $Q(x, t)$  will depend on voltage  $u(t)$ .

The relationship between the provided voltage for the furnace  $u(t)$  and the temperature of the furnace  $v(t)$  is usually the first order inertia system with time delay as the following equation in (5).

$$T \dot{u}(t) + u(t) = k \cdot v(t - \tau) \tag{5}$$

Where,  $T$  is the time constant,  $\tau$  is the time delay;  $k$  is the static transfer coefficient;  $u(t)$  is the temperature of the furnace and  $v(t)$  is the provided voltage for the furnace.

### 3. MODEL TO CALCULATE THE TEMPERATURE DISTRIBUTION OF THE OBJECT [3, 4]

The equations (1), (2), (3) is drawn from the condition ( $\delta J = 0$ ). Equations (1), (2), (3) are also used in the corresponding variational problem: find  $Q(x,t)$  such that the functional:

$$J = \frac{1}{2} \int_0^\delta G \, dx + G_0 \rightarrow \min \tag{6}$$

Functions  $G, G_0$  are determined by comparing the Euler-Lagrange equations (7) with (1),(2),(3).

$$\frac{\partial G}{\partial Q} - \frac{\partial}{\partial x} \left( \frac{\partial G}{\partial Q_x} \right) = 0 \tag{7}$$

$$J = \frac{1}{2} \int_0^\delta \left[ a(Q) \left( \frac{\partial Q}{\partial x} \right)^2 + \frac{\partial(Q)}{\partial t} \right] dx + \frac{\alpha}{2c\rho} [2Q(0,t)u(t) - Q^2(0,t)] \rightarrow \min \tag{8}$$

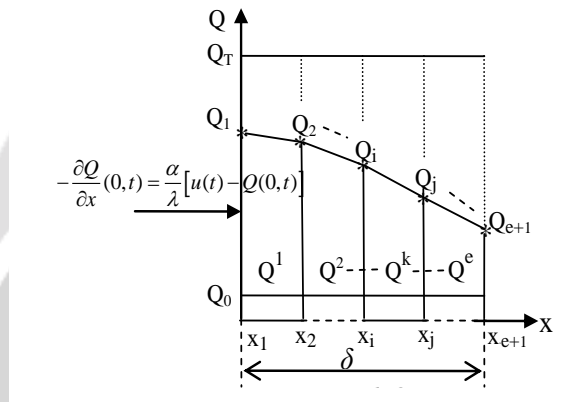


Fig. 2.

Segment the space  $[0, \delta]$  by partition  $x_1 \dots x_{e+1}$  (Fig. 2). The corresponding time functions at the partition points are:  $Q_1(x_1, t) = Q_1(t) \dots Q_{e+1}(x_{e+1}, t) = Q_{e+1}(t)$ ; Finding the extremum of the function  $J$  is replaced by finding the extremum of this function approximating the function by approximating the integral by replacing  $Q(x, t)$  by the sum of a finite number consisting of  $e+1$  function  $Q^k(x, t), k = 1, e$  in elements  $x_i, x_j (i = 1, e; j = i + 1)$  and consider the temperature function  $Q^k$  in elements  $x_i, x_j$  to be linear (Fig. 2).

Specifically, we have the function approximation:

$$Q(x, t) = \sum_{k=1}^e Q^k \approx \sum_{k=1}^e b_1^k(t) + b_2^k(t)x \tag{9}$$

Coefficients  $b_1^k(t), b_2^k(t)$  determined according to system (10):

$$\begin{cases} Q_i = b_1^k(t) + b_2^k(t)x_i \\ Q_j = b_1^k(t) + b_2^k(t)x_j \end{cases} \tag{10}$$

Define:  $\underline{Q} = [Q_1 \ Q_2 \ \dots \ Q_{e+1}]^T$ .

At any time  $t$  the functional  $J$  is a function of  $e+1$  node temperature  $J = J(\underline{Q})$  so the necessary condition to minimize  $J$  is:

$$\frac{\partial J}{\partial \underline{Q}} = 0 \tag{11}$$

Assume  $a$  is a quadratic function with the temperature  $Q^k$  of element  $k$  :

$$a^k = a_0^k + a_1^k Q^k + a_2^k (Q^k)^2$$

The average temperature conductivity coefficient at each time t of element k or (xi, xj) is:

$$a_{ib}^k = \frac{1}{x_j - x_i} \int_{x_i}^{x_j} a_0^k + a_1^k Q^k + a_2^k (Q^k)^2 dx = a_0^k + a_1^k \frac{1}{2}(Q_i + Q_j) + a_2^k \frac{1}{3}(Q_i^2 + Q_i Q_j + Q_j^2) \tag{12}$$

Transform (11) to get the system of nonlinear differential equations:

$$\underline{\dot{Q}}(t) = -C^{-1} \left( A(Q) - \frac{\alpha}{c\rho} H \right) \underline{Q} - C^{-1} \underline{1} \frac{\alpha}{c\rho} u(t) \tag{13}$$

Where c is the specific heat of the calcined object, C is the heat capacity matrix:

$$C = \frac{1}{6} \sum_{k=1}^e x_{ij} \begin{bmatrix} i & j \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{matrix} i \\ j \end{matrix}; \quad x_{ij} = x_j - x_i = \text{const} \quad k = \overline{1, e} \Leftrightarrow \begin{cases} i = \overline{1, e} \\ j = i + 1 \end{cases} \tag{14}$$

A(Q) is the temperature conductivity matrix:

$$A(Q) = \sum_{k=1}^e \frac{1}{x_{ij}} \begin{bmatrix} i & j \\ a_{ib}^k & -a_{ib}^k \\ -a_{ib}^k & a_{ib}^k \end{bmatrix} \begin{matrix} i \\ j \end{matrix} + \sum_{k=1}^e \frac{1}{2x_{ij}} \begin{bmatrix} i & j \\ \frac{a_1^k}{2}(Q_i - Q_j) + \frac{a_2^k}{3}(2Q_i^2 - Q_i Q_j - Q_j^2) & -\frac{a_1^k}{2}(Q_i - Q_j) - \frac{a_2^k}{3}(2Q_i^2 - Q_i Q_j - Q_j^2) \\ \frac{a_1^k}{2}(Q_i - Q_j) + \frac{a_2^k}{3}(Q_i^2 + Q_i Q_j - 2Q_j^2) & -\frac{a_1^k}{2}(Q_i - Q_j) - \frac{a_2^k}{3}(Q_i^2 + Q_i Q_j - 2Q_j^2) \end{bmatrix} \begin{matrix} i \\ j \end{matrix} \tag{15}$$

In case  $a_2^k = 0$  then A(Q) is :

$$A(Q) = \sum_{k=1}^e \frac{1}{x_{ij}} \begin{bmatrix} i & j \\ a_0^k + a_1^k \frac{1}{2}(Q_i + Q_j) & -a_0^k + a_1^k \frac{1}{2}(Q_i + Q_j) \\ -a_0^k + a_1^k \frac{1}{2}(Q_i + Q_j) & a_0^k + a_1^k \frac{1}{2}(Q_i + Q_j) \end{bmatrix} \begin{matrix} i \\ j \end{matrix} + \sum_{k=1}^e \frac{1}{2x_{ij}} \begin{bmatrix} i & j \\ \frac{a_1^k}{2}(Q_i - Q_j) & -\frac{a_1^k}{2}(Q_i - Q_j) \\ \frac{a_1^k}{2}(Q_i - Q_j) & -\frac{a_1^k}{2}(Q_i - Q_j) \end{bmatrix} \begin{matrix} i \\ j \end{matrix} \tag{16}$$

$$H = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}_{e+1} \quad \underline{1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{e+1}$$

Set variable:

$$\underline{q} = (u \ Q_1 \ Q_2 \ \dots \ Q_{e+1})^T = (u \ \underline{Q})^T \tag{17}$$

$$(13) \Leftrightarrow \underline{\dot{q}} = A(q)\underline{q} + Bu(t - \tau) = \underline{f} \tag{18}$$

In which, Control variable:  $v(t - \tau) \in R^1$ ; State variable:  $\underline{q} = (u, \underline{Q})^T \in R^{(e+2) \times 1}$

$$A(q) = \begin{bmatrix} \frac{1}{T_1} & \underline{0}^T \\ C^{-1} \underline{1} \frac{\alpha}{c\rho} & A(Q) - \frac{\alpha}{c\rho} H \end{bmatrix}_{e+2} \in R^{(e+2) \times (e+2)}; \underline{0}^T = \underbrace{[0 \ \dots \ 0]}_{e+1}; B = [k/T_1 \ 0 \ \dots \ 0]^T \in R^{(e+2) \times 1}$$

With binding conditions (19):

$$\begin{aligned} u &\leq [u]; \quad Q_1 \leq [Q_1] \\ (Q_1 - Q_{e+1}) &\leq [\Delta Q] \\ 0 &\leq v_{\min} \leq v(t - \tau) \leq v_{\max} \end{aligned} \tag{19}$$

#### 4. SIMULATION AND EXPERIMENTAL RESULTS DISCUSSION [8, 10, 11]

Burning metal in furnace is a continuous process, where materials are conveyed through some heat areas in succession. Steel slabs are directed by pushing motors and moved from low temperature area to high temperature one in the furnace space. They move through different temperature zones along the furnace. This kind of motion is

the same as the case that slabs are kept immobile but temperature varies in time. Thus, a static reheating furnace can be used in place of a continuous reheating furnace for experimental purposes, such as testing model to calculate the temperature distribution of the object. Therefore, this process is similar to the moving of slabs in the continuous reheating furnace.

In conditions a continuous furnace has not for testing, we can replace it with a static furnace. First, the static furnace can arrange the firing slab so that it can have a firing model similar to one in the continuous furnace.

The first is bio standards.

The second is the temperature field in the furnace.

As mentioned above, for slabs heated in a continuous furnace, for example, for a steel billet with a thickness of  $\delta = 20\text{cm}$ , the thermal conductivity of the steel is an average  $\lambda = 30$  and the heat exchange coefficient in the firing zone is  $\alpha = 300$ .

$$Bi = \frac{\alpha \cdot \delta}{\lambda} = \frac{300 \cdot 0,2}{30} = 2$$

We can calculate the bio standard: Hence, the flat-slab of steel is a thick object because the coefficient  $Bi > 0.5$

To implement the temperature distribution model mentioned in the previous section, in the absence of a real continuous furnace, we use a static furnace, in a static furnace the billet does not move. We can replace the heat regime in a continuous furnace by subjecting the slabs to different temperatures over time (similarly, the billet in a furnace continuously moves through different temperature ranges). Regarding the billet, to ensure similarity in the model of the heat in the furnace, we use a slab of Diatomite ( $\delta = 2\text{cm}$ ,  $\lambda = 0,2$  and  $\alpha = 20$ ) that can calculate the bio standard.

$$Bi = \frac{\alpha \cdot \delta}{\lambda} = \frac{20 \cdot 0,02}{0,2} = 2$$

Based on  $Bi$  standards, the diatomite billet used has the same thickness as the steel billet mentioned in the continuous furnace. Thus, the experiment of controlling the temperature of the billet in the continuous furnace is converted to the temperature control experiment of the slab in the resistance chamber furnace. The temperature variation according to the layers of the fired billet (diatomite sample) also reflects the temperature change of the billet in the continuous furnace.



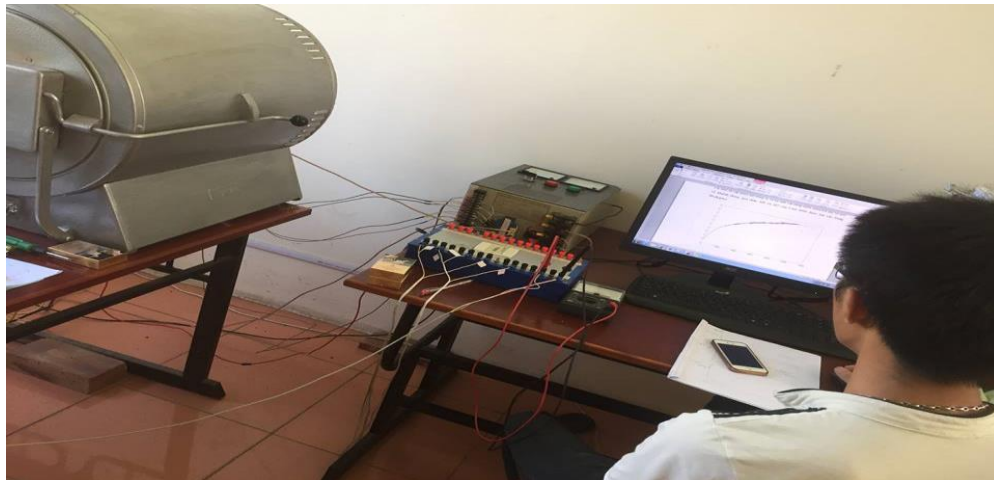


Fig.3. Diatomite sample furnace experimental system.

**4.1. The temperature distribution simulation for a slab when knowing the furnace temperature.**

The system uses NI USB-6008 interface card to output the voltage signal to the furnace control and collects the temperature signal in the furnace. The voltage passed from the computer to the NI card, through the DAC to the voltage converter, thereby changing the voltage supplied to the furnace. The temperature in the furnace is measured by the thermocouples XA (Cromen-Alumen) or NiG-Ni with temperature range (0-1000°C) output voltage (0-40mV), after via the standardization kit is also taken to the NI Card, which converts into digital signals and sends temperature data to computer for the identification of the resistor furnace model:

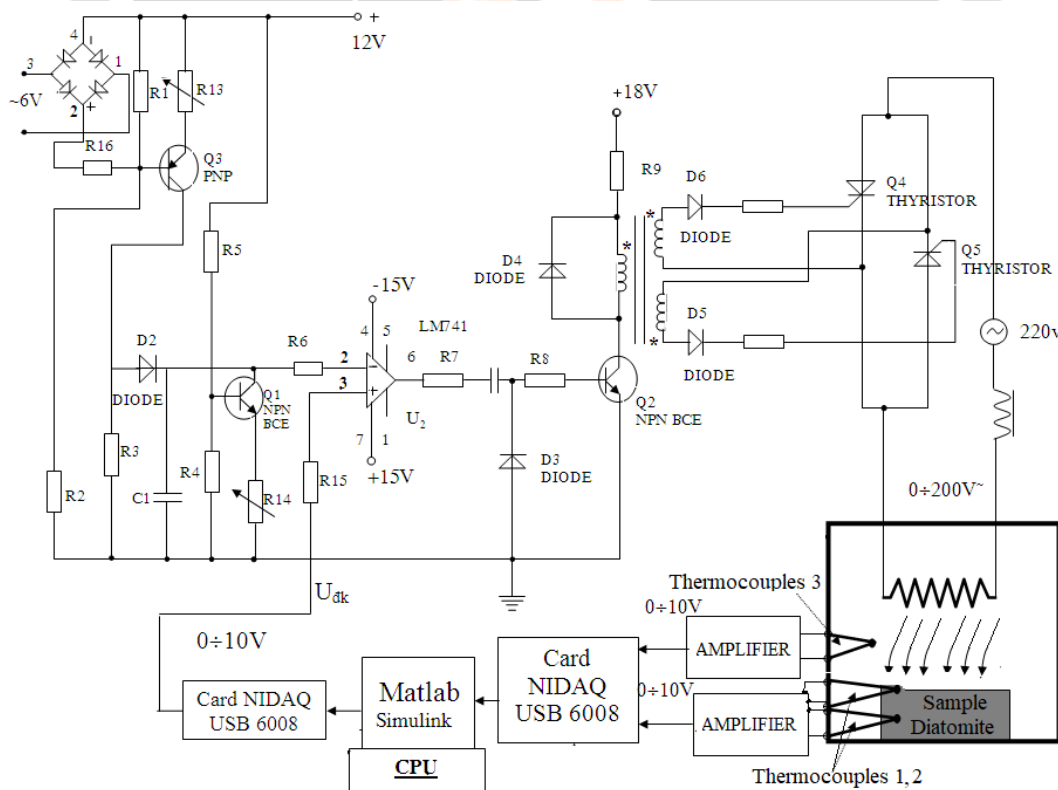


Fig.4 Block diagram of experimental setup

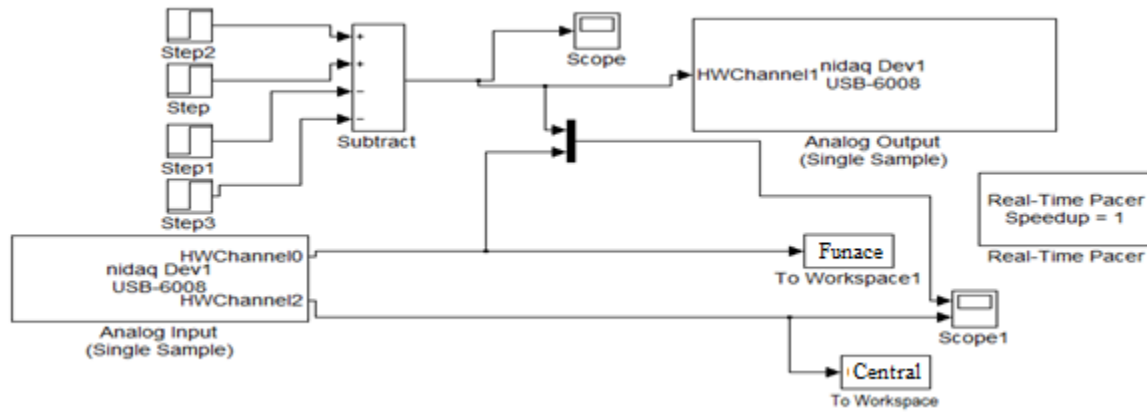


Figure 5 Simulink block diagram for object recognition

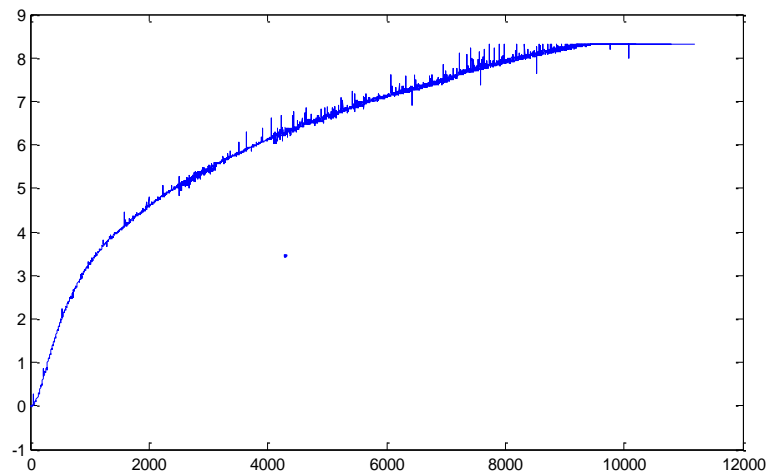


Figure 6 Characteristics of furnace sampling values

The signal from the thermocouple placed in the furnace and at the center of the slab is amplified and read to Matlab through pins AI0 and AI2 of Card 6008 and saved in Workspace. The identification method here is to use the System Identification Tool function (*ident*) included in Matlab. We get the transfer function form:

$$G_{(s)} = \frac{0.578}{452s + 1} e^{-45s} \tag{20}$$

We conduct an experiment for a Diatomite sample with the following parameters:

- The physical parameters of the object
  - The thickness of object:  $\delta = 2$  (cm), density  $\rho = 680\text{kg/m}^3$
  - The heat-transfer factor:  $\alpha = 20$  ( $\text{W/m}^2 \cdot ^\circ\text{C}$ ), specific heat  $c = 0,837\text{kJ/Kg}^\circ\text{C}$
  - The heat-conducting factor:  $\lambda = 0.2$  ( $\text{W/m} \cdot ^\circ\text{C}$ )
  - The temperature-conducting factor:  $a = 4.8 \cdot e^{-7}$  ( $\text{m}^2/\text{s}$ )
- The parameters of the furnace
  - The time constant of the furnace:  $T = 452$  (s)

- The delayed time of the furnace:  $\tau = 45 (s)$ 
  - The period of heating time:  $t_f = 4200 (s)$
  - Limit the temperature of furnace:  $u(t) \leq 500^{\circ}C$
  - Limit the temperature of flat-slab surface:  $Q(0,t) \leq 600^{\circ}C$
  - Limit under voltage:  $v_1=0 (V)$
  - Limit upper voltage:  $v_2=205 (V)$

With these parameters, the coefficient  $Bi$  is calculated as follows:

$$Bi = \alpha \cdot \delta / \lambda = 20.0,02 / 0.2 = 2$$

Hence, the flat-slab of Diatomite is a thick object because the coefficient  $Bi > 0.5$ .

Experimental result as shown in Figure 7

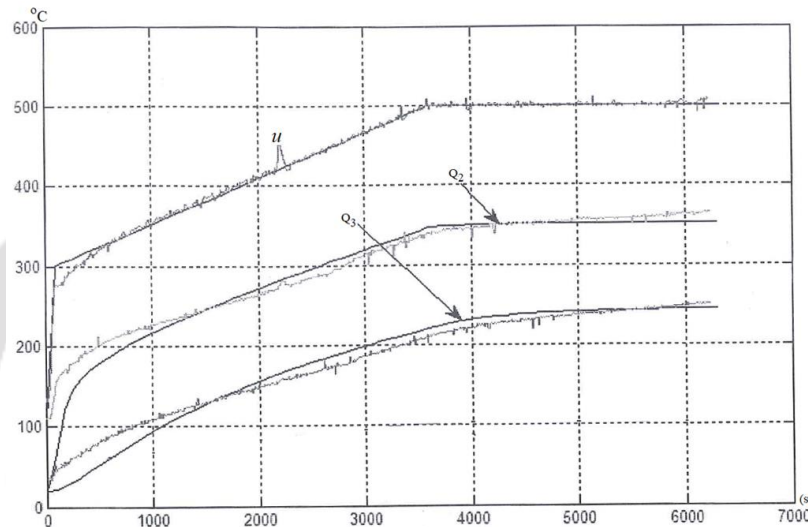


Fig. 7 The temperature curve of the calculated model is close to the real temperature one

Select the heat transfer coefficients, after that run the simulation until the calculated temperature given by the model is equal to the actual measured temperature, then we can determine the actual heat transfer coefficient.

The slab temperature in temperature distribution model approximates the real temperature. The actual surface and middle layer temperatures are measured by two thermocouples.

Figure 7 shows that, at time  $t = 7000s$ , the temperature distribution of the central layers in the  $Q_3$  is approximately  $360^{\circ}C$ , the deviation from the distribution model temperature is small, which proves that the distribution model temperature meets the control requirements for the system.

#### 4.2. Applying the temperature calculation model to the actual heating process.



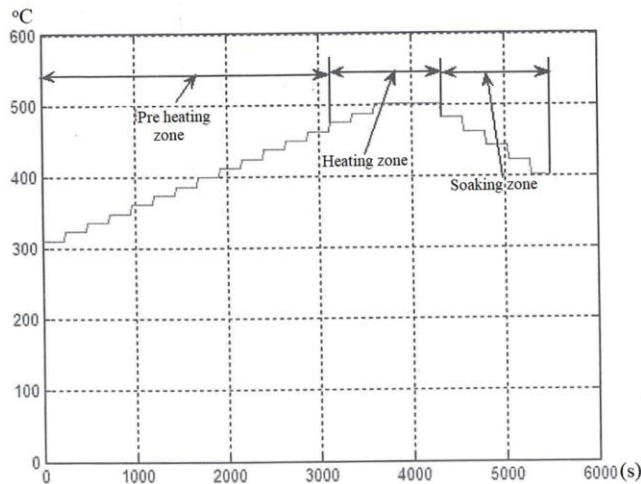


Fig. 8 Define the temperature set point for the continuous furnace

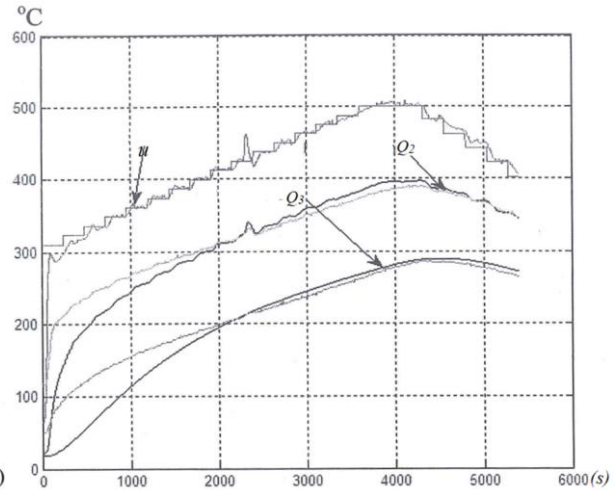


Fig.9 The temperature curve of the calculated model is close to the real temperature one

In short, the above model is used to obtain information about the temperature of the billet fired in a continuous furnace. The model is responsible for calculating the average temperature of the object when knowing the temperature of the air in the furnace, or calculating the furnace temperature distribution according to the required temperature diagram of the fired slab given.

## 5. CONCLUSIONS

The article has built the fastest and most accurate heating control algorithm for heat objects by converting the distributed system model to a centralized system. The simulation results have demonstrated the correctness of the research method that can be applied to control metal smelting processes in industry for both static and dynamic furnaces in the fast response sintering mode (fastest sintering problem) to compensate for productivity loss during rolling mill delays and in working modes with different constant productivity (the most accurate calcination problem). The application of the above algorithms to control the kiln system in synchronization with the rolling mill when there is interference while still ensuring the goals of productivity, product quality and fuel saving will be presented in the article next.

## ACKNOWLEDGEMENT

The work described in this paper was supported by Thai Nguyen University of Technology (<http://www.tnut.edu.vn>).

## 6. REFERENCES

- [1]. Q. Wang and Y. Zu (PRC) - Optimal control of distributed parameter systems based on differentiable orthogonal Functions, Intelligent Systems and Control, Vol. 446, 2004.
- [2]. Yu Jin Jang, Sang Woo Kim - An Estimation of a Billet Temperature during Reheating Furnace Operation, International Journal of Control, Automation, and Systems 5 (1) (2007) 43-50.
- [3]. Joseph E. Flaherty - Finite Element Analysis, Lecture Notes, 2000.
- [4]. O. Axelsson, V. A. Barker - Finite element solution of boundary value problems, Academic Press, 1984.
- [5]. А. А. Шемяков, РВЯ Ясовелва - Управление теловыму объётами с распределёнными параметрами, 1986.
- [6]. А. Г. Бутковский, С. А. Малый - Оптимальное Управление Нагревом Металла, 1972.
- [7]. Е. И. Казанцев - Промышленные печи, Москва "Металлургия", 1975.

- [8]. Nguyen Trong Toan, Nguyen Nam Trung, "Auto-tuning controllers of a class of plants using gradient descent algorithm" International Journal of Engineering and Applied Sciences (IJEAS) ISSN: 2394-3661, Volume-6, Issue-6, June 2019.
- [9]. Nguyen Trong Toan, Nguyen Nam Trung, Design and balanced control for double pendulum system: simulation and experiment, *IJARIE-ISSN(O)-2395-4396*, Vol-10 Issue-3 2024.
- [10]. Mai Trung Thai, Nguyen Nam Trung, "Comparison of two Replacing Methods a delayed Object in Optimal Control Problem for a Distributed Parameter System," SSRG International Journal of Electrical and Electronics Engineering, vol. 7, no. 6, pp. 11-16, 2020.
- [11]. Nguyen Nam Trung (1997), "Controlling the continuous heating process of rolled blanks". Master's thesis in engineering, HaNoi University of science and technology.

