

# COMPARISON OF GAIN MARGIN AND PHASE MARGIN ALONG WITH SEVERAL OTHER PARAMETERIC COEFFICIENTS OF A SECOND ORDER SYSTEM UNDER DIFFERENT DAMPING CONDITIONS- A MATLAB SYNTAX BASED APPLICATION OF CONTROL SYSTEM ENGINEERING

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[ **ABSTRACT** : This paper is an application of control system where we have used second order system in laplacian domain to find out the gain and phase margin. But before realising this we have calculated several parameters like Rise time, Peak time, natural and damping frequency under different damping conditions. Here we have used for types of damping factors; Underdamping, Critical damping, Over-damping and Oscillatory wave. For each cased we have used different transfer functions to generate gain phase margin and in whole paper we have displayed a comparison between above mentioned parameters how are they varying as per different damping conditions. For stability analysis we have used bode plot; all mat-lab syntaxes are available try to use it scientifically.]

## INTRODUCTION:

Before discussing about the main topic of the paper let's discuss about the basic of control system engineering technology. Transfer function is a key factor of control system programming. The main definition of " Transfer function" is something like that;

The TRANSFER FUNCTION is defined by

$C(s)/R(s) = \text{LAPLACE TRANSFORM OF OUTPUT FUNCTION} / \text{LAPLACE TRANSFORM OF INPUT FUNCTION}$

$C(s)/R(s) = G(s) / (1 + G(s)H(s))$  Where  $G(S)$  = forward path gain in laplacian-domain ,  $H(S)$  = feedback path gain in laplacian domain.

The transfer function has two important terminologies ; poles and zeroes,

**POLES:** Let us consider a transfer function

$$C(s)/R(s) = (S-Z1)*(S-Z2)*(S-Z3)*(S-Z4) / (S-P1)*(S-P2)*(S-P3)*(S-P4)$$

So pole means the value or the values for which the entire numerical value of the transfer function of a control system unit will tend to infinity.

**ZEROES:** Let us consider a transfer function

$$C(s)/R(s) = (S-Z1)*(S-Z2)*(S-Z3)*(S-Z4) / (S-P1)*(S-P2)*(S-P3)*(S-P4)$$

Therefore, zero means the value or the values for which the entire numerical value of the entire transfer function of a control-system unit will tend to zero.

Now we should know the terminologies applied in this paper regarding a second order system;

One by one we are discussing these terminologies;

**RISE TIME:** The rise time is the time needed for the response to reach from 10% to 90% or 0% to 100% of the desired value of the output at the very instant;

**PEAK TIME:** It is the instant of occurring the maximum overshoot

$\omega_n$  is the natural frequency of oscillation

$\omega_d$  is called damping frequency of oscillation  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$

$\zeta$  = So called damping constant determining the nature of damping

$\zeta > 1$  OVER-DAMPED OSCILLATION

$\zeta < 1$  UNDERDAMPED OSCILLATION

$\zeta = 1$  CRITICALLY DAMPED

Based on those damping factor we have designed the step plot analysis;

Lets talk about the stability analysis of the entire second order system under different damping conditions for which we have used BODE PLOT analysis. Now the question is that there are NYQUIST PLOT or NICHOLE'S CHART ANALYSIS but still we have used BODE PLOT because mat-lab syntaxes are easy to obtain and use in this case as well as we are determining and comparing the GAIN MARGIN and PHASE MARGIN analysis for which we need to know the technology of BODE PLOT. BODE PLOT is a method of ABSOLUTE STABILITY. Which judges whether the system is absolutely stable or unstable. So you need to know the necessity of 'GAIN MARGIN' and 'PHASE MARGIN'.

**GAIN MARGIN:** The gain margin refers to the amount of gain which can be increased or decreased without making the system unstable. It is expressed in DB.

**PHASE MARGIN:** Phase margin. Phase margin is defined as the amount of change in open-loop phase needed to make a closed-loop system unstable. The phase margin is the difference in phase between  $-180^\circ$  and the phase at the gain cross-over frequency that gives a gain of 0 dB.

So for some extra work we have calculated the phase and gain cross over frequency  $\omega_{co}$  related to gain or phase margin respectively.

[NOTE: Before realising the process of programming the reviewer must note that for all four different damping conditions we have fixed the damping constants in programming for the corresponding transfer functions by using mat-lab syntaxes in each case for each table so that you can understand the transfer functions values based on the nature of damping.

**PROCESS OF PROGRAMMING:**

**STEP 1:** Define the necessary terminologies of a second order system in mat-lab

**STEP 2:** Fixed the values of natural frequency of oscillation ( $\omega_n$ ) and damping factor

**STEP 3:** Arrange the transfer function of Over-damped, Under damped, Critically damped and An pure Oscillatory system through different damping factors as per definitions.

**STEP 4:** Use subplot technology to find out the STEP analysis of types of damping system of second order.

**STEP 5:** Find out the numerical values terminologies that you have mentioned as mat-lab syntaxes to prepare a table;

**STEP 6:** Same way find out Gain and Phase margin with gain and phase crossover frequency to prepare another table.

STEP 7: Find out the pole zero plot for all the damping conditions in a single window by using "Subplot" technology and prepare a table which contains the values of poles under different damping constants with locations in complex 'S' plane.

### 3. MATLAB CODES OF PROGRAMMING:

[NOTE: All throughout the programming we have used different damping constants for four different conditions of damping but same natural frequency of oscillation; if you want to change values of damping constants in each case you can do it but four different damping conditions must be maintained depending on damping constant geta; different mat-lab syntaxes can be used to find out the step plots.]

#### 3.1 PROGRAMMING CODE OF TIME RESPONSE OF SECOND ORDER CONTROL SYSTEM

```
subplot(2,2,1)
syms wn1 s wd1 tp1 tr1 phi1 sys1 geta1
wn1=10;
geta1=.5;
wd1=wn1*sqrt(1-geta1^2);
tp1=pi/wd1;
sys1 = tf(10,[1 2*.5*10 10^2]);
phi1=atan(sqrt(1-geta1^2)/geta1);
tr1=(pi-phi1)/wd1;
step(sys1);
title('underdamped response');
subplot(2,2,2)
syms wn2 s wd2 tp2 tr2 phi2 sys2 geta2
geta2=0;
wn2=10;
wd2=wn2*sqrt(1-geta2^2);
tp2=pi/wd2;
sys2 = tf(10,[1 2*0*10 10^2]);
phi2=atan(sqrt(1-geta2^2)/geta2);
tr2=(pi-phi2)/wd2;
step(sys2);
title('oscillatory wave');
subplot(2,2,3)
syms wn3 s wd3 tp3 tr3 phi3 sys3 geta3
wn3=10;
geta3=1;
wd3=wn3*sqrt(1-geta3^2);
tp3=pi/wd3;
```

```

sys3 = tf(10,[1 2*1*10 10^2]);
phi3=atan(sqrt(1-geta3^2)/geta3);
tr3=(pi-phi3)/wd3;
step(sys3);
title('critical response');
subplot(2,2,4)
syms wn4 s wd4 tp4 tr4 phi4 sys4 geta4
wn4=10;
geta4=1.5;
wd4=wn4*sqrt(1-geta4^2);
tp4=pi/wd4;
sys4 = tf(10,[1 2*1.5*10 10^2]);
phi4=atan(sqrt(1-geta4^2)/geta4);
tr4=(pi-phi4)/wd4;
step(sys4);
title('ovardamped response');

```

### 3.2 PROGRAMMING CODE OF BODE PLOT OF SECOND ORDER CONTROL SYSTEM WITH DIFFERENT DAMPING

```

subplot(2,2,1)
syms wn1 s wd1 tp1 tr1 phi1 sys1 geta1
wn1=10;
geta1=.5;
wd1=wn1*sqrt(1-geta1^2);
sys1 = tf(10,[1 2*.5*10 10^2]);
bode(sys1);
title('bode plot of underdamped response');
subplot(2,2,2)
syms wn2 s wd2 tp2 tr2 phi2 sys2 geta2
geta2=0;
wn2=10;
wd2=wn2*sqrt(1-geta2^2);
sys2 = tf(10,[1 2*0*10 10^2]);
bode(sys2);
title('bode plot of oscillatory wave');
subplot(2,2,3)
syms wn3 s wd3 tp3 tr3 phi3 sys3 geta3

```

```

wn3=10;
geta3=1;
wd3=wn3*sqrt(1-geta3^2);
sys3 = tf(10,[1 2*1*10 10^2]);
bode(sys3);
title('bode plot of critical response');
subplot(2,2,4)
syms wn4 s wd4 tp4 tr4 phi4 sys4 geta4
wn4=10;
geta4=1.5;
wd4=wn4*sqrt(1-geta4^2);
sys4 = tf(10,[1 2*1.5*10 10^2]);
bode(sys4);
title('bode plot of overdamped response');

```

### 3.3 PROGRAMMING CODE OF POLE ZERO PLOT OF SECOND ORDER CONTROL SYSTEM WITH DIFFERENT DAMPING

```

subplot(2,2,1)
syms wn1 s sys1 geta1
wn1=10;
geta1=.5;
sys1 = tf(10,[1 2*.5*10 10^2]);
pzmap(sys1);
title('pole zero plot of underdamped response');
subplot(2,2,2)
syms wn2 s sys2 geta2
wn2=10;
geta2=0;
sys2 = tf(10,[1 2*0*10 10^2]);
pzmap(sys2);
title('pole zero plot of oscillatory response');
subplot(2,2,3)
syms wn3 s sys3 geta3
wn3=10;
geta3=1;
sys3 = tf(10,[1 2*1*10 10^2]);

```

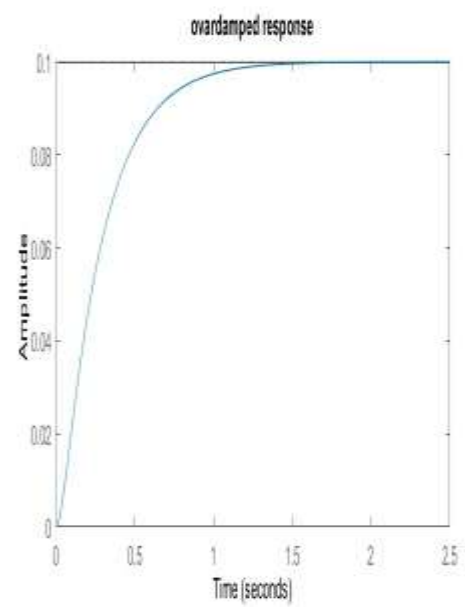
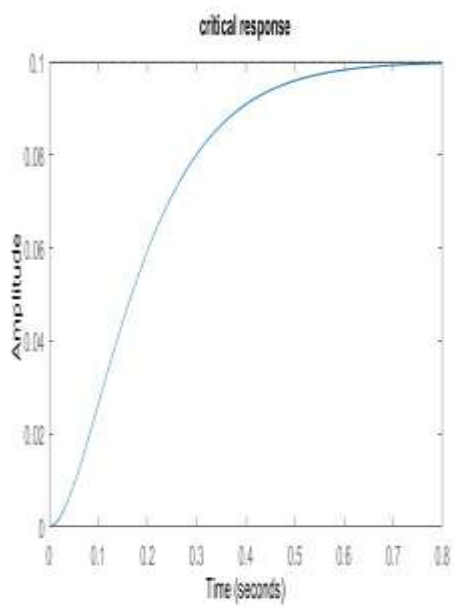
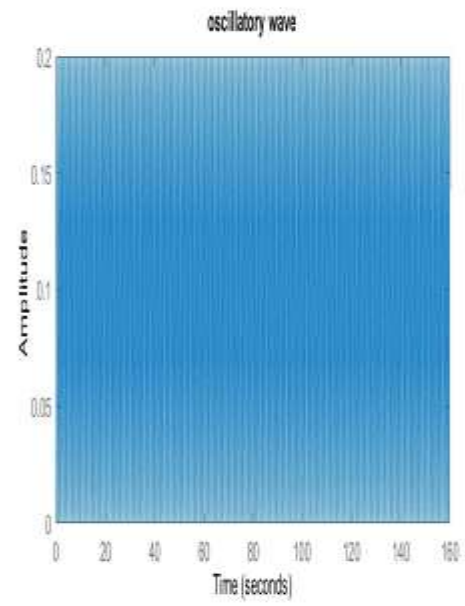
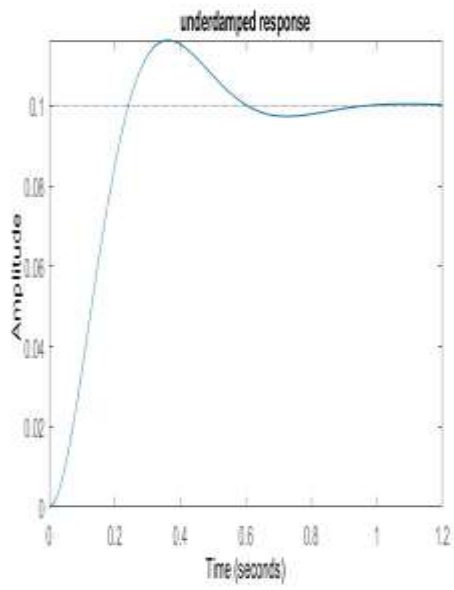
```
pzmap(sys3);  
title('pole zero plot of critically damped response');  
subplot(2,2,4)  
syms wn4 s sys4 geta4  
wn4=10;  
geta4=1.5;  
sys4 = tf(10,[1 2*1.5*10 10^2]);  
pzmap(sys4);  
title('pole zero plot of over-damped response');
```

### 3.4 PROGRAMMING CODE OF GAIN AND PHASE MARGIN

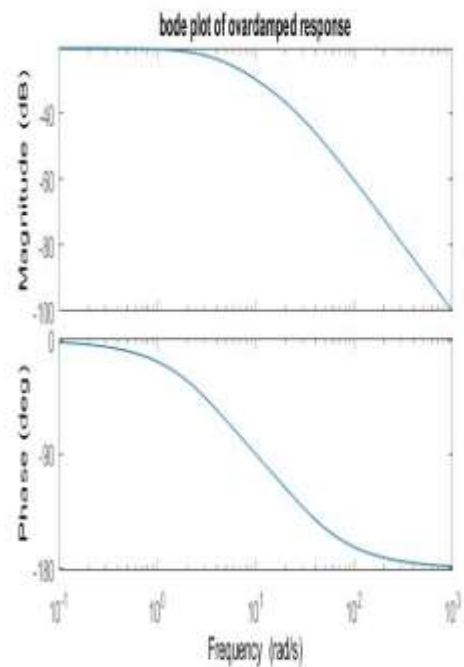
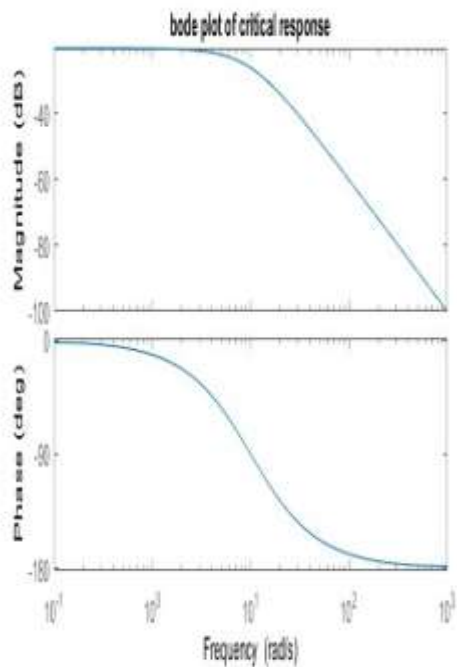
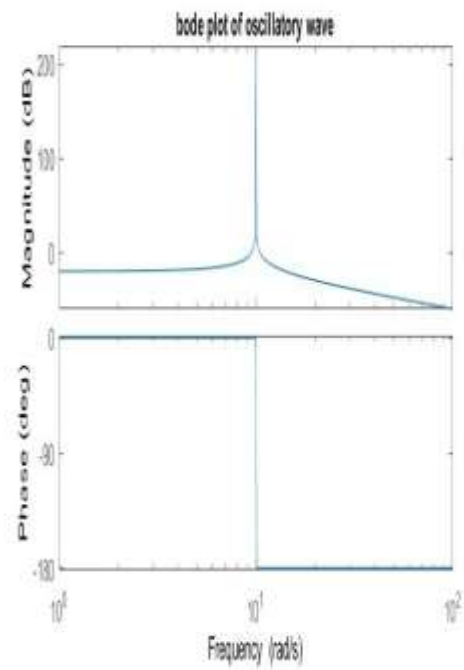
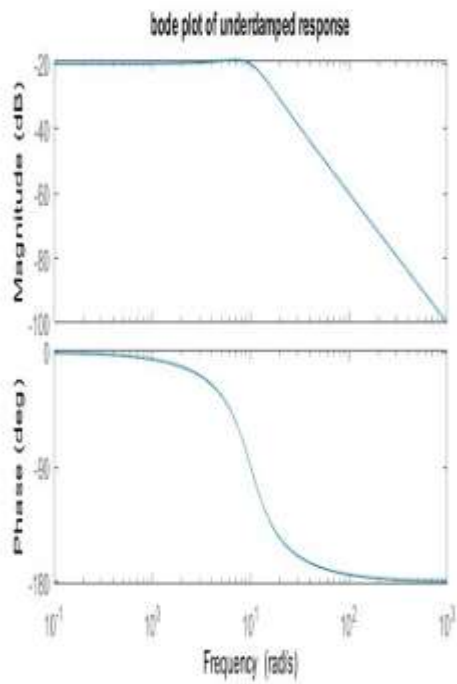
```
syms wn1 s sys1 geta1  
wn1=10;  
geta1=.5;  
sys1 = tf(10,[1 2*.5*10 10^2]);  
[Gm,Pm,Wcg,Wcp] = margin(sys1)
```

## 4. STEP PLOT, BODE-PLOT AND POLE-ZERO PLOT OF SECOND ORDER SYSTEM WITH DIFFERENT DAMPING FACTORS

### 4.1 STEP PLOT OUTCOMES

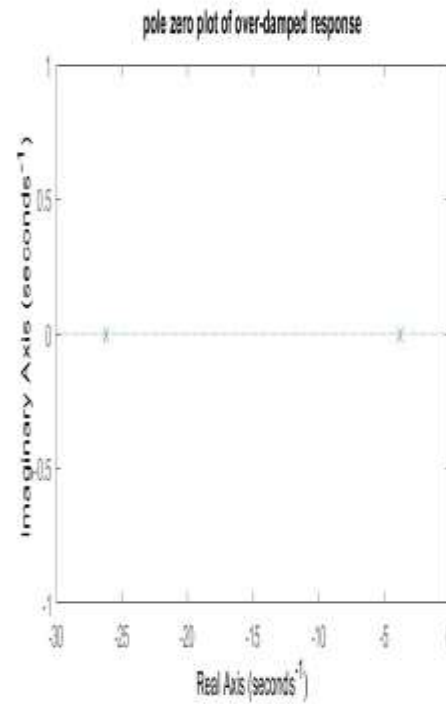
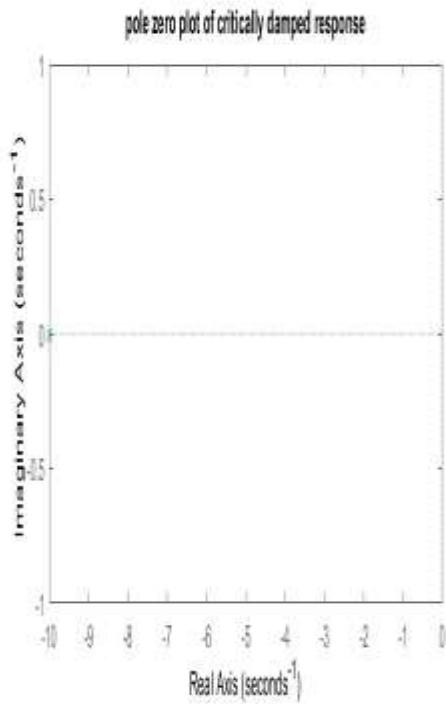
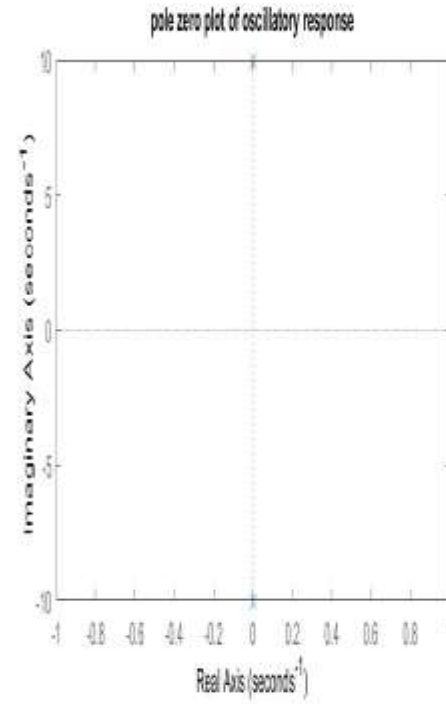
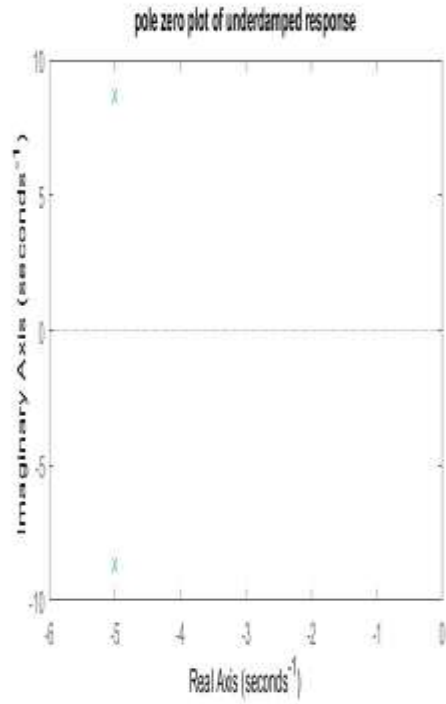


BODE PLOT OUTCOMES





POLE ZERO PLOT



## TABULAR FORM OF MATLAB REALTED CALCULATIONS

[NOTE: If you change the values of damping constants in each case then the values of transfer functions as well as other parameters will be changed in each case. But be cautious about the four different nature of damping conditions used in this programming. ]

## 5.1 TABLE NO.1 CALCULATIONS OF SECOND ORDER SYSTEM RELATED PARAMETERS

Geta= Damping factor

tr = Rise time

tp = Peak time

wn= Natural frequency of oscillation

wd=Damping frequency of oscillation

phi= Tangential angle of damping

NATURE OF DAMPING	TRANSFER FUNCTION	geta	tr	tp	wn	wd	phi
Under damped	$\frac{10}{s^2 + 10s + 100}$	0.5000	0.2418	0.3628	10	8.6603	1.0472
Oscillatory	$\frac{10}{s^2 + 100}$	0	0.1571	0.3142	10	10	1.5708
Critically damped	$\frac{10}{s^2 + 20s + 100}$	1	Infinity	Infinity	10	0	0
Over damped	$\frac{10}{s^2 + 30s + 100}$	1.5	-0.0861 - 0.2810i	0.0000 - 0.2810i	10	0.0000 + 11.1803i	0.0000 + 0.9624i

## 5.2 TABLE NO.2 TABLE OF CALCULATIONS OF GAIN AND PHASE MARGIN USING MATLAB CODES

NATURE OF DAMPING	TRANSFER FUNCTION	Gain margin	Phase margin	Gain crossover frequency	Phase crossover frequency
Under damped	$\frac{10}{s^2 + 10s + 100}$	Infinity	Infinity	Infinity	NaN
Oscillatory	$\frac{10}{s^2 + 100}$	Infinity	0	Infinity	10.4882
Critically damped	$\frac{10}{s^2 + 20s + 100}$	Infinity	Infinity	Infinity	NaN
Over damped	$\frac{10}{s^2 + 30s + 100}$	Infinity	Infinity	Infinity	NAN

	$s^2 + 30s + 100$				
--	-------------------	--	--	--	--

5.3 TABLE NO.3 DETERMINE THE VALUES AND LOCATION OF POLES IN 'S' PLANE OR IN COMPLEX PLANE FOR ALL TYPES OF DAMPING CONSTANTS

NATURE OF DAMPING	TRANSFER FUNCTION	POLE	Location of poles
Under damped	$\frac{10}{s^2 + 10s + 100}$	$-5.0000 + 8.6603i$ $-5.0000 - 8.6603i$	Left half of 'S' plane $S = \sigma + j\omega$
Oscillatory	$\frac{10}{s^2 + 100}$	$0.0000 + 10.0000i$ $0.0000 - 10.0000i$	On imaginary axis of 'S' plane
Critically damped	$\frac{10}{s^2 + 20s + 100}$	$-10, -10$	Left half of 'S' plane on X-axis
Over damped	$\frac{10}{s^2 + 30s + 100}$	$-26.1803,$ $-3.8197$	Left half of 'S' plane on X-axis

#### DISCUSSIONS OF RESULTS:

Look at the table 1 very carefully Rise time and Peak time both are maximum in a critically damped system but if you look at table 2 very carefully you can see that gain and phase margin all are infinity in most of the cases. Now depending on the value of geta or damping factor the transfer function changes. Mat-lab syntaxes are available you may find out the settling time for each damping factors.

#### CONCLUSIONS:

Here we have used BODE PLOT analysis to define the stability of the transfer function. But if you want you can use NYQUIST PLOT or NICHOLE'S PLOT analysis. In NYQUIST PLOT there is a point called 'Critical Point' which is  $-1+j0$ ; you will have to observe that how many times the plot encircles the critical point. Exactly the same number of poles lie in the right half of s-plane and the entire system becomes unstable otherwise if not encircles the critical point then the system is stable. Here in this paper we have not taken any help from other journals to write the codes. We have followed a book on LINEAR CONTROL SYSTEM- composed by author B.S.MANKE to define the second order system related terminologies. So you are requested to write your codes by your own techniques. But you cannot detect gain and phase margin from NYQUIST PLOT or NICHOLE'S PLOT.

#### FUTURE SCOPE OF WORK:

We have done step response analysis of each transfer function; try to implement this concept by using 'impulse response' and see the changes in the outcomes; if you want you can change the values of natural frequency( $\omega_n$ ) or damping factor( $\zeta$ ) but the conditions of all types of damping must remain same; therefore choose the damping constants very carefully.

#### REFERENCES:

a. 'LINEAR CONTROL SYSTEM'-with mat-lab application- Prof. B.S.Manke