CALCULATION OF THE SPEEDS OF SOME TIDAL HARMONIC CONSTITUENTS

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ABSTRACT

In this paper we have investigated some aspects of astronomical tide. We have discussed about constants and constituents and their effects on tide. We have also computed the speeds of Some Tidal Harmonic Constituents. The results are found to be in good agreement with the predicted data of others. By this work we can calculate the speeds of some constituents over all locations of Bangladesh where observed data are available and we think our work may be helpful for those people who live in coastal area.

Keyword: - *Harmonic constituents, speed, tide, horizontal amplitude, vertical amplitude.*

1. INTRODUCTION

The vertical excursion of the cosine curve is the range that the tide-generating forces are trying to cause in the waters from this component of the total tide. Each one of the tide-generating motions, represented by a simple harmonic cosine curve, is known as a tidal component, tidal constituent, or harmonic constituent. Each constituent represents a periodic change or variation in the relative positions of the Earth, Moon and Sun. A single constituent is usually written in the form

$$y(t) = A\cos(\omega t + \Phi).$$

In which $y(t)$ is a function of time as expressed by the symbol *t* and is reckoned form a specific origin, A is the constituent amplitude. The argument for the cosine function includes two terms, ωt and Φ. The term ωt represents the constituent speed multiplied by time t and Φ is the constituent phase. The period of the constituent is the time required for the phase to change through 360 degrees and is the cycle of the astronomical condition represented by the constituent [1-4].

2. CALCULATION OF SPEEDS OF CONSTITUENTS

There are many types of tidal constituents that govern tides. The constituents that are the main players in determining the types of tide are discussed below. The principal solar and lunar semidiurnal constituents are designated as S_2 and M_2 respectively. S is for Sun and M is for Moon and the subscripts mean that there are two complete tidal cycles for each astronomic cycle. The principal solar semidiurnal constituent S_2 represents the Earth spinning relative to the Sun. The Earth rotates once in 24 mean solar hours, it is going at the rate of 360° / 24 = 15° / hr. However, there is a maximum in the solar tide producing force under the Sun and again on the opposite side (midnight). So, the period (maximum to maximum) of the constituent is 12 mean solar hours and the speed of S_2 is 360° / 12 = 30° / hr. The principal lunar semidiurnal constituent, M_2, represents the Earth spinning relative to the Moon. Since the Moon is moving eastward, it takes 24.8412 mean solar hours to bring the Moon back overhead. Again, there are two maximums in this lunar day, so the period is only 12.4206 mean solar hours and the speed of M_2 is 360° / 12.4206 = 28.984° / hr. S_2 and M_2 get into phase and out of phase to produce spring and neap tides, respectively [5-8]. Spring tides occur at the times of full Moon and new Moon while neap tides occur at the times of the first and third quarter Moons. The revolution of the Moon around the Earth relative to the Sun takes 29.5306 days (called the synodic month or one lunation). Since there are two maximum, spring tides occur every 29.5306 / 2 = 14.765 days and neap tides occur 7.383 days later than the spring tides. There are another two constituents, namely the larger lunar elliptic semidiurnal constituent, N_2 and the smaller lunar elliptic semidiurnal constituent, L_2. These are completely artificial constituents in contrast with S_2 and M_2 that have realistic relationships to the solar and lunar envelopes of the tide-generating forces. Perigee to perigee occurs every 27.5546 days (the anomalistic month) or 661.31 mean solar hours. The speed of perigee to perigee is thus 360° / 661.31 = .544° / hr. This is a lunar event and the speed of M_2 is 28.984° / hr. The constituent speeds are, therefore [9-14]:

N_2 =28.984 – .544 = 28.440° / hr.

L_2 = 28.984 + .544 = 29.528° / hr.

There are also another two constituents, namely the luni-solar declinational diurnal constituent, K_1 and the principal lunar declinational diurnal constituent, O_1. North to maximum north occurs every 27.3216 days (the tropical month) or 655.72 mean solar hours. However, both north and south declinations produce the same results. The north to south (and south to north) cycle is 655.72 / 2 = 327.86 hrs. The speed is 360° / 327.86 = 1.098° / hr. The speeds of the constituents, as they modify M_2, will be the speed of M_2 plus and minus the speed of the north to south cycle. Since the maximum is only felt once per day as the Earth spins, the constituent speeds are half the sum and difference:

K_1 = (28.984 + 1.098) / 2 = 15.041° / hr.

O_1 = (28.984 – 1.098) / 2 = 13.943° / hr.

The speeds of the tidal harmonic constituents also may be derived by combining the speeds of certain fundamental astronomic elements. The classic description of the tide-producing forces uses a reference frame for which the Earth is the center and projections of the movements of the Sun and Moon are made upon the celestial sphere. The fundamental astronomic elements are [15-20]:

i) Mean rotation of Earth relative to Sun, T = 15° / mean solar hr.
ii) Rate of change of Moon, s = 0.549° / mean solar hr.
iii) Rate of change of Sun, h = 0.041° / mean solar hr.
iv) Rate of change of lunar perigee, p = 0.005° / mean solar hr.

The speeds, n of the constituents described above can also be computed by the method described in Table 1.

Table-1: Calculation of speeds of some constituents

Name	Constituent	Speed (n)	n =
Principal solar semidiurnal constituent	S_2	$n = 2T$	30°/hr.
Principal lunar semidiurnal constituent	M_2	$n = 2T - 2s + 2h$	28.984°/hr.
Larger lunar elliptic semidiurnal constituent	N_2	$n = 2T - 3s + 2h + p$	28.440°/hr.
Smaller lunar elliptic semidiurnal constituent	L_2	$n = 2T - s + 2h - p$	29.528°/hr.
Luni-solar declinational diurnal constituent	K_1	$n = T + h$	15.041°/hr.
Principal lunar declinational diurnal constituent	O_1	$n = T - 2s + h$	13.943°/hr.
Principal solar declinational diurnal constituent	P_1	$n = T - h$	14.959°/hr.

Using the method discussed above, we have calculated speeds of some major 38 constituents and they are given in Table 2.

Table-2: Speeds of some major constituents

Constituent name	Constituent	Speed
Principal lunar semidiurnal constituent	M_2	28.98
Principal solar semidiurnal constituent	S_2	30.00
Larger lunar elliptic semidiurnal constituent	N_2	28.439
Lunar diurnal constituent	K_1	15.04
Shallow water over tides of principal lunar constituent	M_4	57.968
Lunar diurnal constituent	O_1	13.94
Shallow water over tides of principal lunar constituent	M_6	86.952
Shallow water terdiurnal	MK_3	44.025
Shallow water over tides of principal solar constituent	S_4	60.00
Shallow water quarter diurnal constituent	MN_4	57.423
Larger lunar evectional constituent	NU_2	28.525
Shallow water over tides of principal solar constituent	S_6	90.00
Variational constituent	MU_2	27.968
Lunar elliptical semidiurnal second-order constituent	$2N_2$	27.895
Lunar diurnal	OO_1	16.139
Smaller lunar evectional constituent	LM_2	29.455
Solar diurnal constituent	S_1	15.00
Smaller lunar elliptic diurnal constituent	M_1	14.496
Smaller lunar elliptic diurnal constituent	J_1	15.585
Lunar monthly constituent	MM	0.544
Solar semiannual constituent	SAA	0.0821
Solar annual constituent	SA	0.0410
Lunisolar synodic fortnightly constituent	MSF	1.0158
Lunisolar fortnightly constituent	MF	1.098
Larger lunar evectional diurnal constituent	RHO	13.4715
Larger lunar elliptic diurnal constituent	Q_1	13.3986
Larger solar elliptic constituent	T_2	29.958
Smaller solar elliptic constituent	R_2	30.041
Larger elliptic diurnal	$2Q_1$	12.854
Solar diurnal constituent	P_1	14.958
Shallow water semidiurnal constituent	$2SM_2$	31.0158
Lunar diurnal constituent	M_3	43.476
Smaller lunar elliptic semidiurnal constituent	L_2	29.528

Shallow water diurnal constituent	$2MK_3$	42.927
Lunisolar semidiurnal constituent	K_2	30.0821
Shallow water eighth diurnal constituent	M_8	115.936
Shallow water quarter diurnal constituent	MS_4	58.984

The constituents are classified as semi-diurnal, diurnal, and mixed. They are given accordingly in Tables 3, 4 and 5. The amplitudes may vary from those listed within several percent.

Table-3: List of some major semi-diurnal constituents along with their speeds and amplitudes

Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude(mm)
M_2	12.421 hr	384.83	53.84
S_2	12.000 hr	179.05	25.05
N_2	12.658 hr	73.69	10.31
K_2	11.967 hr	48.72	6.82

Table-4: List of some major diurnal constituents along with their speeds and amplitudes

Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude(mm)
K_1	23.934 hr	191.78	32.01
O_1	25.819 hr	158.11	22.05
P_1	24.066 hr	70.88	10.36
φ_1	23.804 hr	3.44	0.43
ψ_1	23.869 hr	2.72	0.21
S_1	24.000 hr	1.65	0.25

Table-5: List of some major mixed constituents along with their speeds and amplitudes

Tidal constituent	Period	Vertical amplitude (mm)	Horizontal amplitude(mm)
M_f	13.661 days	40.36	5.59
M_m	27.555 days	21.33	2.96
S_{sa}	0.50000 yr	18.79	2.60
lunar node	18.613 yr	16.91	2.34
S_a	1.0000 yr	2.97	0.41

3. CONSTITUENTS EFFECTS ON TIDE

Constants and hence constituents play a vital role on tide. Different types of constituents have different types of effects on tide. As for example, in spring tide, the effect of M_2 constituent is similar to that of S_2 constituent. But when both M_2 and S_2 constituents affect combinedly on spring tide, then the amplitude is higher than that of the individual constituent. Effects of M_2, S_2 and $M_2 + S_2$ are shown in Figs.1, 2 and 3 respectively.

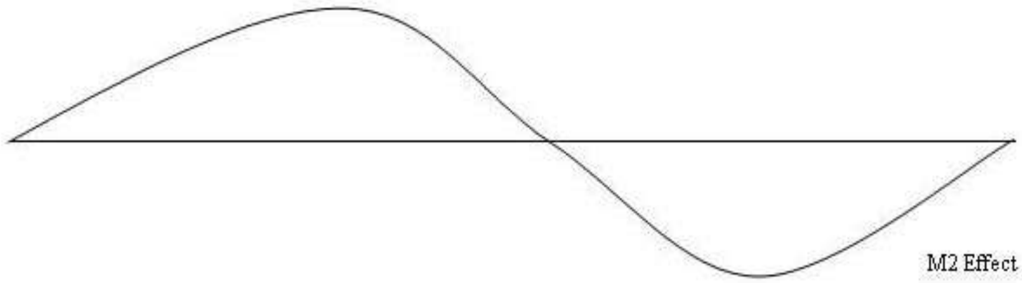

Fig-1: Effect of M2 on spring tide

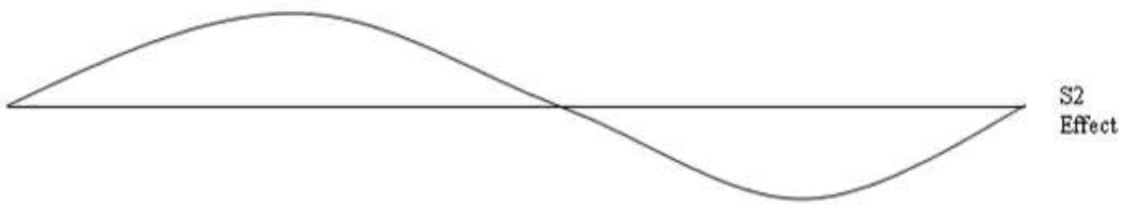

Fig-2: Effect of S2 on spring tide

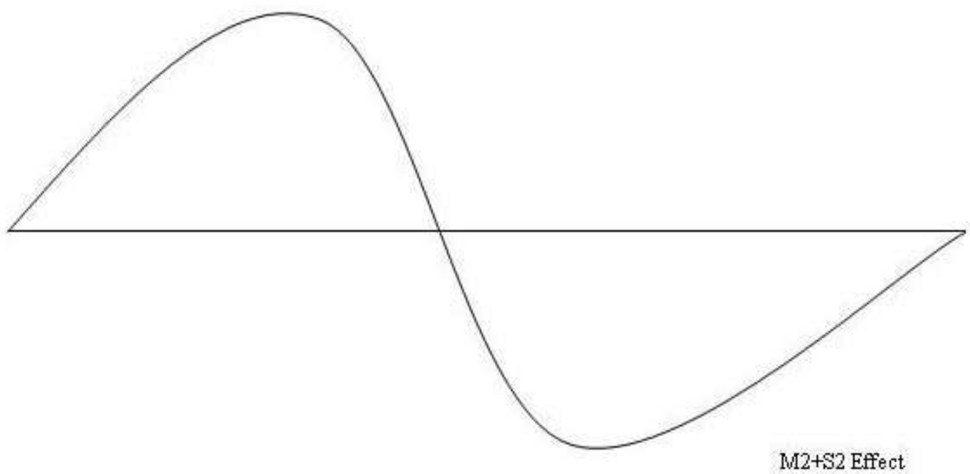

Fig-3: Effect of M2+S2 on spring tide

On the other hand, for neap tide, we observe that the effect of M_2 constituent is laager and opposite than that of S_2 constituent. But when both M_2 and S_2 constituents affect combinedly on neap tide, then the average amplitude is lower than that of the individual constituent. Effects of M_2, S_2 and $M_2 + S_2$ are shown in Figs.4, 5 and 6 respectively.

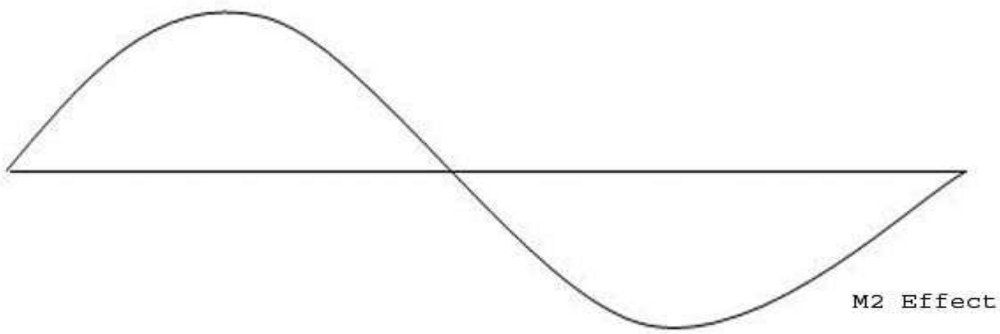

Fig-4: Effect of M2 on neap tide

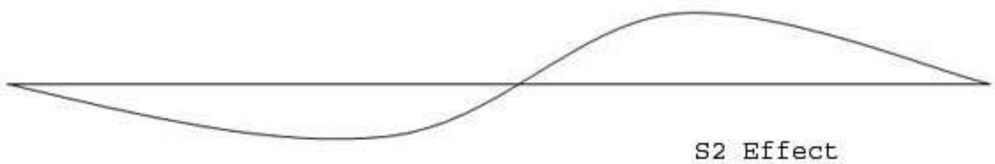

Fig-5: Effect of S2 on neap tide

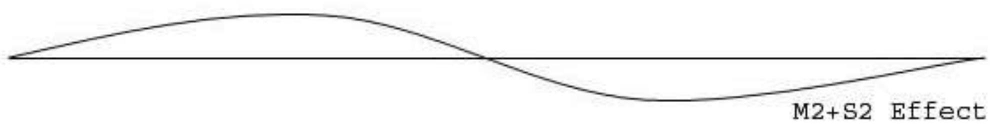

Fig-6: Effect of M2 +S2 on neap tide

4. CONCLUSION

Here are the calculations of the speeds of some constituents. It is obvious that the results are in good agreement with the predicted data of others. In this process, the speeds of some constituents can be calculated over all stations of Bangladesh whether the sample observed data are available for those stations. We hope our work will be helpful to compute the speeds over all those stations where observed data are available and this work may also be helpful for those who live in maritime area.

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