Comparison of Duane Growth Model & Crow Amsaa Model

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ABSTRACT

The reliability of the software represents one of the most important attributes of software quality, and the estimation of the reliability of the software is a problem hard to solve with accuracy. Nevertheless, in order to manage the quality of the software and of the standard practices in an organization, it is important to achieve an estimation of the reliability as accurate as possible. In the present work there are described the principles and techniques which underlie the estimation of the reliability of the software, starting from the definition of the concepts which express the attributes of software quality. It is taken into account the issue of the estimation of a software part. The presumed objective of the estimation of the reliability consists in the analysis of the risk and of the reliability of the software-based systems. Supposedly, a documented opinion of the expert exists regarding the reliability of the software and an update of the defined estimation of the reliability is tried with the information contained in the records of the operational data.

Keyword: *-Model Formulation, Failure Rate and Failure Intensity*, *Duane Growth Model & Crow Amsaa Model.*

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1. Introduction

In general, the first Prototypes produced during the development of a new complex system will contain design, manufacturing and engineering deficiencies. Because of these deficiencies the initial reliability of the prototypes may be below the system's reliability goal or requirement. In order to identify and correct these deficiencies, the prototypes are often subjected to a rigorous testing program. During testing, problem areas are identified and appropriate corrective actions (or redesign) are taken. Reliability growth [1] is the improvement of a product (component, subsystem or system) over a period of time due to changes in the products design and the manufacturing process. A reliability growth program is a well-structured process of finding reliability problems by testing, incorporating corrective actions and monitoring the increase of the products reliability throughout the test phases. The term growth is used since you assume that the reliability of the product will increase over time as design changes and fixes are implemented. However, in Practice no growth or negative growth may occur.

Reliability goals are generally associated with a reliability growth program. A program may have more than one reliability goal. For example, there may be a reliability goal associated with failures resulting in unscheduled maintenance actions and a separate goal associated with those failures causing a mission abort or catastrophic failure. Other reliability goals may be associated with failures modes that are safety related. The monitoring of the increase of the product's reliability through successive phases in a reliability growth analysis (RGA) concerns itself with the quantification and assessment of parameters (or metrics) relating to the products reliability growth over time. Reliability growth management addresses the attainment of the reliability objectives through planning and controlling of the reliability growth process.

Reliability growth Occurs From Corrective and Preventive actions based on experience gained from failures and analysis of the equipment's design, production and operation processes. The reliability growth test analysis and fix concept in design is applied by uncovering weakness during the testing stages and performing appropriate corrective action before full scale production. A corrective action takes place at the problem and root cause level Therefore a failure mode is a failure and root cause. Reliability growth addresses failure modes. For example, a problem such as a seal leak may have more than one cause. Each problem and cause constitutes a separate failure mode and, if necessary separate corrective actions. Consequently there may be several failure modes and design corrective actions corresponding to a seal leak problem. The formal procedure and manuals associated with the maintenance and support of the product are part of the system design and may require improvement. Reliability growth is due to permanent improvements in the reliability of a product (component, subsystem or system) that result from changes in the product design and the manufacturing process. Rework, repair and temporary fixes do not constitute reliability growth.

Screening addresses the reliability of an individual unit and not the inherent reliability of the design. If the population of devices is heterogeneous then the high failure rate items are naturally screened out through operational use or testing. Such screening can improve the mixture of the heterogeneous population. Generating an apparent growth phenomenon when the devices are not improving themselves. This is not considered reliability growth. Screening is a form of rework. Reliability growth is concerned with permanent corrective action focused on prevention of problems.

1.1 Why Reliability Growth

It is typical in the development of a new technology or complex system to have reliability goals. Each goal will generally be associated with a failure definition. The Attainment of the various reliability goals usually involves implementing a reliability program and performing reliability tasks. These tasks will vary from program to program. A reference of common reliability tasks is MIL-STD785. It is widely used and readily available.

Table 1.1: MIL-STD 785 reliability tasks

Program Surveillance and Control		Design and Evaluation		Development and Production Testing	
101	Reliability Program Plan	201	Reliability Modeling	301	Environmental Stress Screening (ESS)
102	Monitor/Control of Subcontractors and Suppliers	202	Reliability Allocations	302	Reliability Development / Growth Test (RDGT)
103	Program Reviews	203	Reliability Predictions	303	Reliability Qualification Test (RGT) Program
104	Failure Reporting, Analysis and Corrective Action System(FRACAS)	204	Failure Modes Effects and Criticality Analysis (FMECA)	304	Production Reliability Acceptance Test (PRAT) Program
105	Failure Review Board (FRB)	205	Sneak Circuit Analysis		
		206	Electronic Parts / Circuit Tolerance Analysis		
		207	Parts Program		
		208	Reliability Critical Items		
		209	Effects of Functional Testing, Storage, Handling, Packaging, Transportation and Maintenance		

The Program Surveillance, control, design and evaluation tasks can be combined into a group called basic reliability tasks. These are basics tasks in the sense that many of these tasks are included in a comprehensive reliability program of the MIL-STD785 300 series tasks. Only the RDGT reliability growth testing task is specially directed toward finding and correcting reliability deficiencies.
For discussion Purposes, consider the reliability metric mean time between failures (MTBF). This term is used for continuous system as well as one shot systems
The MTBF of the prototypes immediately after the basics reliability tasks are completed is called the initial MTBF. This is the key Basics Reliability tasks output parameter. If the system is tested after the completion of the basic reliability tasks then the initial MTBF is the mean time between failures as demonstrated from actual data. The initial MTBF is the reliability achieved by the basic reliability tasks and would be the system MTBF if the reliability program were stopped after the Basic reliability tasks have been completed.

The initial MTBF after the completion of the basic reliability tasks will generally be lower than the goal. If this is the case then the reliability growth program is appropriate. Formal reliability growth testing is usually conducted after the basic reliability tasks have been completed for a system subjected to RDGT, the initial MTBF is the system reliability at the beginning of the test. The objective of the testing is to find problems, implement corrective action and increase the initial reliability. During RDGT, failures are observed and an underlying failure mode is associated with each failure. A failure mode is defined by a problem and cause. When a new failure mode is observed during testing, management makes a decision to either not correct or correct the failure mode in accordance with the management strategy. Failures modes that are not corrected are called A modes and failure modes that receive a corrective action are called B modes. If the corrective action is effective for a B mode, then the failure intensity for the failure mode will decrease. The effectiveness of the corrective action is part of the overall management strategy. If the RDGT testing and corrective action processes are conducted long enough the system MTBF will grow to a mature MTBF value is called the growth potential. It is the direct function of the design and management strategy. The system growth potential MTBF is the MTBF that would be attained at the end of the basic reliability tasks if all the problem failure modes were uncovered early design and corrected in accordance with the management strategy.

In Summary, The initial MTBF is the value actually achieved by the basic reliability tasks. The growth potential is the MTBF with the current management strategy that can be attained if the test is conducted long enough see figure.1

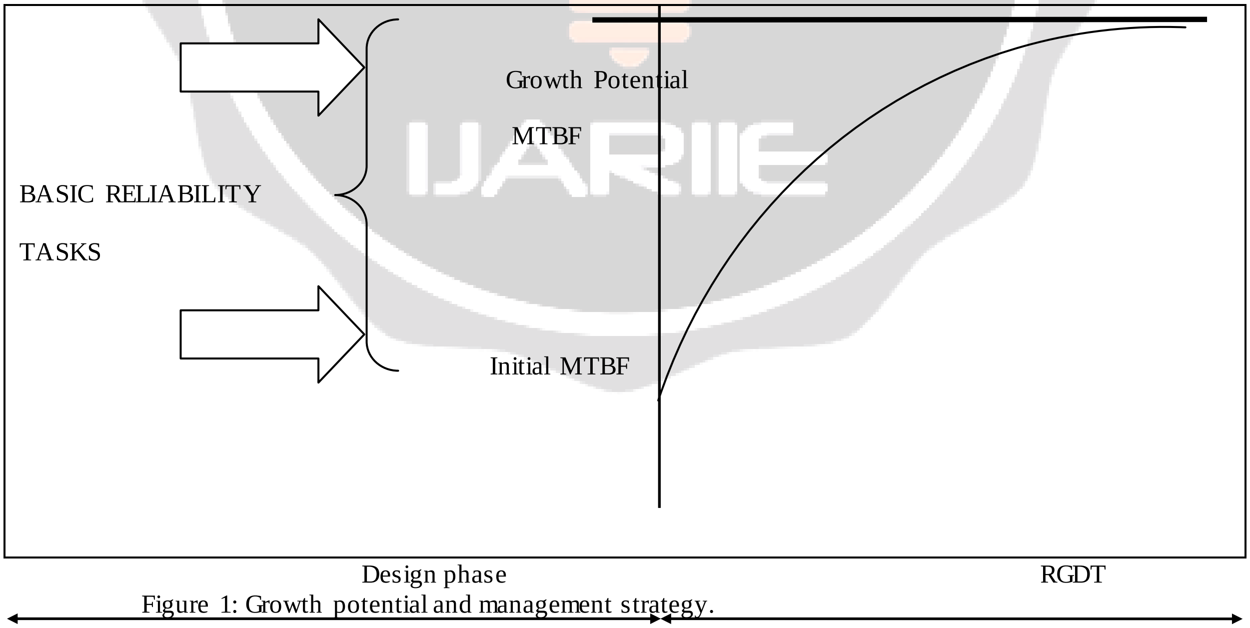

Figure 1: Growth potential and management strategy.

1.2 Elements of a Reliability Growth Program

In a formal reliability growth program, a reliability goal (or goals) is set and should be achieved during the development- testing program with the necessary allocation or reallocation of resources. Therefore planning and evaluating are essential factors in a growth process program. A comprehensive reliability growth program needs well-structured planning of the assessment techniques. A reliability growth program [3] differs from a conventional reliability program there is a more objectively developed growth standard against which assessment techniques are compared. A compression between the assessment and the planned value provides a good estimate of whether or not the program is progressing as scheduled. If the program does not progress as planned, then new strategy should be considered for example, a reexamination of the problem areas may result in changing the management strategy so that more problem failure modes surfaced during the testing actually receive a corrective action instead of a repair. Several important factors for an effective reliability growth program are:

- Management: the decisions are made regarding the management strategy to correct problems or not correct problems and the effectiveness of the corrective actions.
- Testing: provides opportunities to identify the weakness and failure modes in the design and manufacturing process.
- Failure mode root cause identification: funding, personnel, and procedures are provided to analyze ,isolate and identify the cause of failures
- Corrective action effectiveness : design resources to implement corrective actions that are effective and support attainment of the reliability goals
- Valid reliability assessments

The management strategy may be driven by budget and schedule but it is defined by the actual action of management in correcting reliability problems. If the reliability of a failure mode is known through analysis or testing, then management makes the decision either not to fix (no corrective action) or to fix (implement a corrective action) that failure mode. Generally if the reliability of the failure modes meets the expectation of management then no corrective actions would be expected. If the reliability of the failure mode is below expectation, then the management strategy would generally call for the implementation of a corrective action.

Another part of the management strategy is the effectiveness of the corrective actions. A mode corrective action typically does not eliminate a failure mode from occurring again. It simply reduces its rate of occurrence a corrective action, or fix a problem failure mode typically removes a certain amount of the failure modes failure intensity, but a certain amount will remain in the system. The fraction decreases in the problem mode failure intensity due to the corrective action is called the effectiveness factor (EF). The EF will vary from failure mode to failure mode but a typical average for government and industry system has been reported to be about0.70.with an EF equal to 0.70, a corrective action for a failure mode removes about 70% of the failure intensity, but 30% remains in the system.
Corrective action implementation raises the following question; “what if some of the fixes cannot be incorporate during testing?” it is possible that only some fixes can be incorporated into the product during testing. However others may be delayed until the end of the test since it may be too expensive to stop and then restart the best, or the equipment may be too complex for performing a complete teardown. Implementing delayed fixes usually results in a distinct jump in the reliability of the system at the end of the test phase. For corrective action implemented during testing, the additional follow-on testing provides feedback on how effective the corrective actions are, and provides opportunity to cover additional problem to correct.

Evaluation of the delayed corrective actions is provided by projected reliability values. The demonstrated reliability is based on the actual current system performance and estimates the system reliability due to corrective action incorporated during testing. The projected reliability is based on the impact of the delayed fixes that will be incorporated at the end of the test or between test phases.
When does a reliability growth program take place in the development process? Actually there is more than one answer to this question. The modern approach to reliability realize that typical reliability tasks obtain do not yield a system that has attained the reliability goals or attained the cost effective reliability potential in the system. There for reliability growth may start very early in a program utilizing integrated reliability growth testing (IRGT). This approach recognized better reliability problems obtain surface early in engineering test. The focus of these engineering tests is typically in performance and not reliability. IRGT simply piggybacks reliability failure reporting, in an informal fashion, on all engineering test when a potential reliability problem is observed, reliability

engineering is notified and appropriated design action is taken. IRGT will usually be implemented at the same time as the basic reliability tasks. In addition to IRGT , reliability growth may take place during early prototype testing ,during dedicated system testing, during production testing, and from feedback from any manufacturing or quality testing or inspections .the formal dedicated testing or RGDT will typically take place after the basic reliability tasks have been completed .

Note that when testing and assessing against a products specification, the test environment must be consistent with the specified environmental condition under which the product specifications are defined. In addition, when testing subsystem it is important to realize that interaction failure modes may not be generated until the subsystems are integrated in to the total system

1.3 Why Are Reliability Growth Models Needed?

In order to effectively manage reliability growth program and attain the reliability goals. It is imperative that valid reliability assessments of the system be available. Assessments of interest generally include estimating the current reliability of the system configuration under test and estimating the projected increase in reliability if proposed corrective actions are incorporated into the system. These and other metrics give management information on what actions to take in order to attain the reliability goals. Reliability growth assessments are made in a dynamic environment where are reliability is changing due to corrective actions. The objective of most reliability growth models is to account for this changing situation in order to estimate the current and future reliability and other metrics of interest. The decision for choosing a particular growth model is typically based on how well it is expected to provide useful information to management and engineering. Reliability growth can be quantified by looking at various metrics of interest such as the increase in the MTBF, the decrease in the failure intensity or the increase in the mission success probability, which are generally mathematically related and can be derived from each other. Key estimates used in reliability growth management such as demonstrated reliability , projected reliability and estimates of the growth potential can generally all be expressed in terms of the MTBF, failure intensity, or mission reliability. Change in these values, typically as a function of test time, are collectively called reliability growth trends and are usually presented as reliability growth curves. These curves are often constructed based on certain mathematically and statistical models called reliability growth models. The ability to accurately estimate the demonstrated reliability and calculate projections to some point in the future can help determine the following:

- Whether the stated reliability requirements will be achieved
- The associated time for meeting such requirements
- The associated costs of meeting such requirements
- The correlation of reliability changes with reliability activities

In addition, demonstrated reliability and projections assessments aid in:

- Establishing warranties
- Planning for maintenance resources and logistics activities
- Life-cycle-costs analysis

1.4 Reliability Growth Analysis

Reliability growth analysis [4] is the process of collecting, modeling, analyzing and interpreting data from the reliability growth development test program (Development testing). In addition, reliability growth analysis can be done for data collected from the field (Fielded system). Fielded system also includes the ability to analyze data of complex repairable system. Depending on the metric of interest and the data collection method, different models can be utilized (or Developed) to analyze the growth processes.

1.5 Fielded Systems

When a complex system with new technology is fielded and subjected to a customer use environment, there is often considerable interest in assessing it’s reliability and other related performance parameters, such as availability. This interest in evaluating the system reliability based and actual customer use failure data may be motivated by a number of factors. For example, the reliability, which is generally measured during development, is typically related to the systems inherent reliability capability. This inherent capability may differ from actual use experience because of different operating condition or environments, different maintenance policies different levels of experience of maintenance personnel, etc. Although operational tests are conducted for many systems during development, it is generally recognized that in many cases these tests may not yield complete data representative of an actual use environment. Moreover, the testing during development is typically limited by the usual cost and schedule constraints, which prevent obtaining a systems reliability profile over and extended portion of its life. Other interests in measuring the reliability of a fielded system may Centre on, for example, logistics and maintenance policies, quality and manufacturing issues, burn in, wear out, mission reliability or warranties.

Most complex systems are repaired, not replaced, when they failed. For example, a complex communication system or truck would the repaired upon failure, not thrown away and replaced by a new system. A number of books and papers in literature have stressed that the usual non repairable reliability methodology, such as the weibull distribution, are not appropriate for repairable system reliability analyses and have suggested the use of non-homogeneous poison process models. The homogeneous process is equivalent to the widely used poison distribution and exponential times between system failures can be modeled appropriately when the systems failure intensity is not affected by the systems age. However, to realistically consider burn-in, wear out, useful life maintenance policies, warranties, mission reliability, etc. will often require an approach recognizes that the failure intensity these system may not be constant over the operating life of interest but may change with system age. A useful, and generally pratical, extension of the homogeneous poison process is the non-homogeneous process, which allow of the system failure intensity to change with system age. Typically, the reliability analyses of a repairable system under customer use will involve data generated by multiple systems. Crow proposed the weibull process or power law non homogeneous position process for these types of analysis and developed appropriate statistically procedure for maximum likely hood estimation. Goodness-of-fit and confidence bonds.

1.6 Failure Rate and Failure Intensity

Failure rate and failure intensity are very similar term [5]. The term failure intensity typically refers to process such as reliability growth program. The system age when a system is first put into service is time zero. Under the non-homogeneous poison process (NHPP), the first failure is governed by a distribution F(x) with a failure rate r(x). Each succeeding failure is governed by the intensity u(t) of the processes. Let t be the age of the system and Δt is very small the probability that a system of age t fails between t and $t+\lambda t$ is given by the intensity function u(t). Δt Notice that this probability is not conditioned on not having any system failures up to time t, as is the case for a failure rate. The failure intensity u(t) for the NHPP has the same functional form as the failure rate governing the first system failure. Therefore, u(t)= r(t), where r(t) is the failure rate for the distribution function of the first system failure. If the first system failure followed the weibull distribution, the failure rate is:

$$r(x) = \lambda\beta x\beta\text{-}1$$

Under minimal repair, the system failure intensity is:

$$u(t) = \lambda\beta t\beta\text{-}1$$

This is the power law model. It can be viewed as an extension of the weibull distribution. The weibull distribution governs the first system failure and the power law model governs each succeeding system failure.

2. Comparison of Two Models for Managing Reliability Growth during Product Design

2.1 Growth Model Formulation

2.1(A) Modified Power Law Model

The modified power law (MPL) model aims to support planning decision in the product design phase. it assumes design reliability improvements are implemented successfully to mitigate a failure mode or to reduce its probability of occurrence, and requires information about the time from the beginning of design to occurrence of the improvement, this model can estimate the number, or the magnitude of improvements in the original design to increases its reliability from that initially assessed to its goal value. These assumptions of a power law are justified by the fact that the early improvements will be those that contribute most to the reliability improvement. The actual reliability values achieved in the course of the design are plotted corresponding to the design time when they were realized and compared to the model. This model is use to plan the strategies necessary for reliability improvement of a design during the available time period from the initial design revision until the design is completed and released for production the model can the formulated as follows the initial product reliability for the predetermined product operational life at time T is denoted by R0 (T). Assuming that the distribution of the time to failure exponentially distributed then the initial hazard rate of that product is

$$\lambda a0= - In[R0(T)]/T \qquad ....(1)$$

Under the assumption that growth in reliability follows a power function, the product hazard rate decreases as modification are made . We can represent the hazard rate after a period of design time t as a function of the accumulated number of modification made by t, d(t). Therefore, we can express the hazard rate of the product at time t during the design period as

$$\lambda a(t)=\lambda a0[1+d(t)]-\alpha D \qquad ....(2)$$

Assuming the number of design modification is a function of time distributed linearly over the design

$$d(t)=Dt/tD \qquad ....(3)$$

Where D is the number of modification that will be made during specified design period tD and αD is the reliability growth rate resultant from fault mitigation the hazard rate as a function of design time becomes

$$\lambda a(t)= \lambda a0(1+Dt / tD) -\alpha D \qquad ....(4)$$

Denoting the product reliability goal by RG(T), then the goal hazard rate at the end of design period tD, is

$$\lambda aG (tD)= - ln[RG(T)] / T \qquad ....(5)$$

Equating goal and projection from initial reliabilities gives

$$\lambda aG(tD) = \lambda a0(1+DtD / tD)-\alpha D= - In [Ro(T)] / T x (1+D)-\alpha D \qquad ....(6)$$

Substituting λaG(tD) in to the expression for goal reliability and solving for D Gives

$$D = exp[-In(In[RG(T)] / ln [R0(T) / \alpha D] – 1 \qquad ....(7)$$

Solving the same equation for the growth rate, expressed as a function of the design modification initial and goal reliability, gives

$$\alpha D= In(In[Ro(T)] /In [RG(T) / In (1+D) \qquad ....(8)$$

During the design period, the product reliability at operational time T can be expressed as a function of design time t (the reliability growth model in the time period from 0 to tD) as follows :

$$R(t , T) = exp(- \lambda a(t) T) \qquad ....(9)$$

Substituting the expression for the average failure rate, the modified power low model for the design phase 0<t<tD, is derived as follows:

$$R (t,T) =exp [-\lambda a0T(1+Dt / tD)-\alpha D]=exp[In[Ro(T)] / T(1+Dt/tD)-\alpha DT] \qquad ....(10)$$

In the above equation, expressing D in terms of initial and goal reliability, the reliability growth as a function of time in design period, available for design improvements becomes

$$R(t,T) = R_0(T)\left\{[t_D]^{-1}\left[t_D + t\left(\frac{ln[R_G(T)]}{ln[R_0(T)]}\right)^{\frac{-1}{\propto_D}} - t\right]\right\}^{-\propto_D} \qquad .... \ (11)$$

2.1(B) Modified Ibm Model

This modification of the IBM –Rosner reliability growth model (MIBM) [7] was initially motivated by the analysis of test data. This version is adopted for supporting planning decisions during the product design phase. The model is based on a Bayesian approach the combines a prior distribution for the number of design weaknesses in the new product design with empirical data for the reliability of similar product design to produce posterior distribution for estimating the reliability of the new product design like the IBM –rosner model, this model assumes a fixed number of weaknesses or potential faults are inherent in the product design and that within the period between design modifications the rate at which failures occur is constant. It is further assumed that modifications to the design to remove weaknesses are perfect.

In this model we additionally decompose the inherent failure rate of product design in to appropriate systematic classes ,such as build environment , and degradation in such a way that groups of like faults are together for example , major faults of the same types with low rates of occurrence or minor faults of the same types with high rates of occurrence . We also include a non-systematic failure category to represents those noise failures, such as no founds that cannot be attributed to a particular cause. This allows us to modify the reliability growth profile as the systematic failure rate changes when design modifications are implemented, while always taking in to account the impact of noise failures on the estimated reliability at a given time.

The non-systematic failures are assumed to occur at a constant rate (λNS) and can be estimated using data from similar product designs or using engineering judgment. The systematic failures are assessed through a combination of expert judgment about the design weaknesses and the failure rates associated with fault classes from engineering experience.

To assess the effect of systematic failures on the reliability of the product design, all potential design weaknesses (D) should be identified and may be allocated to one of K fault classes as appropriate. The probability of each design weakness within each fault class resulting in failure during the specified life of the product should be estimated using for example, engineering judgment. Procedures for identifying design weaknesses and estimating there probability of resulting in failure using engineering judgment may be required.

The expected number of design weaknesses in fault class k (η_k) likely to result in failure if the design is not modified can be calculated using.

$$\eta_k = \sum_{j=1}^{D_k} P_{kj} \qquad \text{……………………………… (1)}$$

Where Dk is the total number of design weaknesses expected in fault class k and Pkj is the probability of the Jth design weaknesses in fault class k being realized. This calculation is based on the assumption that the number of design weaknesses for each fault class is a Poisson random variable.

Systematic failure rates can be estimated for each fault class using empirical or generic data on relevant existing product designs of similar complexity.

Given that the input data have been specified, the (posterior) estimator of the reliability of the initial product design can be found. This is the product of the reliabilities of the non-systematic and the systematic failures. The rate of the former is the product of the (prior) distribution for the no. of design weaknesses and the (empirical) data for the systematic failures. Thus the reliability of the initial product design can be written as:

$$R_I(T) = \exp\{-[\lambda_{NS}T + \sum_{k=1}^{k} \eta k(1 - e^{-\lambda_k T})]\} \qquad \text{…………… (2)}$$

Given that modifications will be implemented to remove design weaknesses, the reliability of the product design will grow. Therefore, to take into account the rate of reliability growth (αD). The reliability of the modified product design at time T is given by

$$R(t, T) = \exp\left\{-\left[\lambda_{NS}T + \sum_{k=1}^{k} \eta k e^{-\alpha Dt}\left(1 - e^{-\lambda_k T}\right)\right]\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{3}$$

To estimate the rate of growth, replace the goal reliability (RG(T)) and the specified time of the design period (tD) with R(T) and time index T on the growth rate (αD), respectively, in the previous equation. Rearranging gives

$$\propto_D = \frac{\ln\left[\dfrac{\sum_{k=1}^{K} \eta k\left(1 - e^{-\lambda_k T}\right)}{-\ln\left[R_G(T)\right] - \lambda_{NS}T}\right]}{t_D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{4}$$

If a growth rate has been specified or estimated then, similarly, an estimate of the expected time to reach the goal reliability is given by

$$t_G = \frac{\ln\left[\dfrac{\sum_{k=1}^{K} \eta k\left(1 - e^{-\lambda_k T}\right)}{-\ln\left[R_G(T)\right] - \lambda_{NS}T}\right]}{\propto_D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \tag{5}$$

2.2 Achieved Growth and Relationship

Figure 1, also shows the observed reliability growth profiles during design. The actual design changes, whose impact on the system reliability was estimated using fault tree analysis, result in three step change improvement market as modification 1-3. Note that similar type of modification where included in the same time. The first modification involved in changing capacitor types to those with much better dielectric properties are higher reliability (106 capacitor of various values and various contribution to unreliability where considered on design modification). The second change was the introduction of parts with higher ratings, as there were some parts (capacitors on semiconductors) that, because of improper electrical rating demonstrated high unreliability. The third change was also a completion of several modifications: more reliable IC component, some switching field effect transistor (FETs) to be obtained from a more reliable vendor reduction in some discrete semi-conductor components. Following each design modification, product reliability was re-estimated using fault tree analysis. Further improvement was not considered cost effective, and the final reliability estimate was accepted satisfactory. The view of the reliability manager in this application, and for the other products for which the approach was adopted, is that duration of the development test programs was shortened in composition was previous practice. In part, this is because effort to improve the design through the analysis leads to successful modification and in part, because the test was conditioned upon information gained through design analysis. Consequently, the time to full production ramp-up was reduced.

Returning to fig. 1, both models present growth profiles that so pathways to achieving the goal reliability of 0.95 by the end of the specified design period. Despite showing more early growth than was observed, in part due to the accumulation of faults to be addressed during one modification, the MPL model fits the observed profile well in the latter stages of the design and did help support the decision that goal reliability was achievable. The profile of the MIBM model is less steep than the MPL and so presents a more conservative estimate of reliability growth. Statistically this model matches the observed profile better than the modified power law. However, this is partly due to the retrospective nature of the analysis, although it may also be argued that delayed modifications may lead to less immediate growth and so the MIBM model may represent a more realistic profile.

The foregoing analysis examines the fit of the models to the design data as if they were being used in the same manner as those statistical models in reliability growth testing. This is not wholly appropriate. For example, in test, the models are often used to estimate the rate of growth and so inferences about this parameter are used to informed decision about the effectiveness of test in flushing out potential faults. In contrast, during design the models may be used to provide to a benchmark against which the observed path of the growth (or decay) may be tracked. This provides information about the efficiency of the modifications and the chance of a blueprint design being achievable with the required project duration. Hence the reliability manager may be more interested assessing whether the observed data consistence with the assumed model rather than vice versa as was the case in the above example.

3. Comparison of Models

A more general comparison of the two models is given by examining, for example, their assumptions, data requirements, model implementation, and support for planning decisions, discussions with analysts and managers involved in reliability improvement identified many criteria against which the models can be compared. In our discussion we group these into two classes corresponding to the internal model theoretical assumptions and the external model support in terms of practical data inputs and output information.

3. (A) Assumptions and Model Formulations

The MPL assumes that there are fixed number of design modifications (D), that will be conducted over know design time period (t_D). Moreover, this number knows a priori. The MIBM differs in show far as the number of design modification is assumed unknown a priori, but modeled as a Poisson random variable with a known mean. Both modeling approaches utilize subjective assessments to support inference regarding the number of design modifications that will be made. The MPL requires a point estimate of the number of modifications while the MIBM demands a prior distribution, describing the uncertainty associated with the number of faults that exists within the design.

Each model makes deferent assumptions concerning the rate at which this modification is implemented. The MPL assumes that modification are evenly spaced throughout the design phase, while the MIBM assumes that it is more likely to conduct modifications early in design and they become fewer as time progresses.

Both models agree that the hazard rate decreases more substantially in early design by assuming a power law and equally spaced modifications; the MPL implies that earlier modifications have a greater impact on the hazard rate then later. The MIBM focuses more on the underlying fault, whereby it is assumed that each fault within a class contributes to the overall hazard rate of the design. By assuming more modifications in early design, the MIBM captures the decreasing rate of failures.

Both models assume that when design is terminated, the product has a constant hazard rate. However, in the MPL this hazard rate assumed known, while in the MIBM it is composed of an unknown number of underlying faults for which we are in possession of a prior distribution. As such this latter model while not characterize the time to failure with an exponential distribution but model the time to failure throw an average of exponential distributions but models the time to failure an average of exponential distribution weighted against the prior distributions.

In theory neither model explicitly considers imperfect fault mitigation, or the introduction of new fault throughout design. However re-assessing the input parameter D or the MPL and re-eliciting the prior distribution for MIBM can address this and allows the models to be updated. In practice the analyst may assess the impact of the modifications in this way since changes often lead to more complex designs and hence greater potential for reduced reliability. Therefore, the assumption that reliability must improve is not a constraint when applying these models dynamically through the design cycle.

3. (B) Data Requirements and Model Outputs

Both models require a clear specification of the goal reliability and an initial assessment of the reliability. Using these as inputs they explicate the relationship between the number of design modifications and effectiveness of the design phase. The MIBM requires as input a mean number of faults within the initial design and therefore the effectiveness of design time in mitigating faults is an output, selected to achieve the goal reliability. However, with the MPL the number of modifications drives the reliability enhancement and is determined assuming an effectiveness parameter, which would be characteristic of the product and the anticipated type's modifications. The data used to populate the models are based open failure events on components and assemblies provided by their manufacturer's life test data, from accelerated test on components with unknown reliability as well as historical data from similar products of the some complexity. Such data will provide either failure rates as input for the fault tree, or related analysis for MPL or the failure rates or intensities in the MIBM. Expert engineering judgment prevails in both the selection and treatment of the an aforementioned data to ensure appropriate classes and types of failure event are considered, but more importantly in gauging the expectations of the design team to elicit and quantify the potential design weaknesses.

Neither model is complex. Each can be supported using a spreadsheet and can provide a useful guide to growth profiling in design practice. Both models have formulations that are intuitive to project management teams. The MIBM requires more inputs, is grounded in a stochastic process theory and so requires slightly deeper understanding to implement. However, it can formally account for uncertainty in reliability growth through a prior distribution, which can be updated mathematically through Bayesian methods or through re-elicitation, during design. All through not shown in this example, the MIBM will support the generation of interval estimates. The MPL has a simpler formulation that is intuitive and integrates with standard reliability techniques use in the design phase. However, it is only able to support point estimates of reliability growth.

4. Comparison Duane Growth Model & Crow Amsaa Model

4.1 (A) Duane Growth Model

The earliest developed and most frequently used reliability growth model was first proposed by Duane (1964)[8,9], who observed that a plot of the logarithm of the cumulative number of the failures per test time versus the logarithm of test time during growth testing was approximately linear Fig 4.1 (a). This observation can be expressed mathematically and then extrapolated to predict the growth in MTBF while the test –fix-test-fix cycle continues. This model assumes the underline failure process in exponential (constant failure rate).

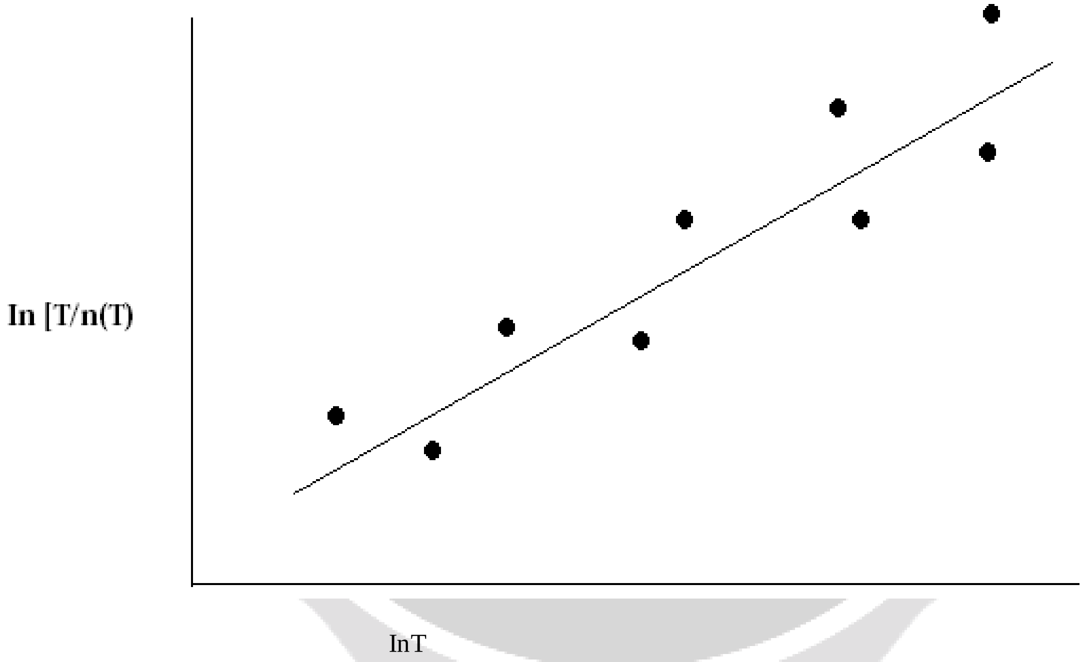

Fig. 4.1 (a)

The Duane growth curve

Let T = total test time accumulated on all prototypes

n(T) = accumulated failure through time T

Then n(T)/T is the accumulative failure rate and T/n(T) is the cumulative MTBF. If the graph in

Fig. 4.1 (a) is linear, then we can write

$$\text{In}\,[T / n(T)] = a + b\,\text{In}\,T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

And $$\text{MTBFc} = T/n(T) = e\ a + b\ \text{In}\ T = ea\ Tb = kTb \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Is the cumulative mean time to failure? Observed that b is the rate of the growth, or the slope of the fitted straight line, and a is the vertical intercept typical growth rates for b range from 0.3 to 0.6. Since, from equation [2]

N (T) = (1/k) × T1-b …………………… (3)

And n(T) is the accumulated failures through time T,

dn(T) / dT = λ(T)= (1-b) / k × T-b ……………………………(4)

is the instantaneous failure rate. Assuming a constant failure rate, if growth testing where to stop at time T, the reciprocal would be the instantaneous MTBF, or

MTBFi = k × Tb / 1-b = MTBFc / 1-b …………………. (5)

To use this model, it is necessary to estimate the parameters a and b. This can be done by plotting T / n (T) versus T on log–log graph paper or plotting (In T, In [T / n(T)]) directly. A more accurate method is to fit a straight line to the point (In T, In [T / n(T)]) using the method of least squares. This least- squares equations estimating a and b are

$$\hat{b} = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i{}^2 - n\bar{x}^2}$$ ……………………….. (6)

$$\hat{a} = \bar{y} - b\bar{x}$$ ……… …………….. (7)

Where,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

xi = In(ti)

yi = In[ti/(n(ti)]

ti = cumulative test time associated with n(ti)failures.

From the least-squares estimates of a,b, $\hat{a}$ and $\hat{b}$,

K=e$^{\hat{a}}$ ……………………………………….. (8)

MTBFi = kTb/(1-$\hat{b}$) …………..(9)

Given an MTBFi goal, say Mf then by solving equation (9) for T,

$$T = \left[\frac{(1-\hat{b})M_f}{k}\right]^{\frac{1}{\hat{b}}}$$ …………………………(10)

An estimate for the required time to complete the reliability growth testing may be obtained. The coefficient of determination r2 can be computed as

$$r^2 = \frac{\sum_{i=1}^{n}\left(y_i-\hat{a}-\hat{b}x_i\right)^2}{\sum_{i=1}^{n}(y_i-\bar{y})^2}$$

……………………. (11)

The coefficient of determination measures the strength of the fit of the regression curve and can be interpreted as the proportion of the variation in the y's explained by the x variables. It will have a value between 0 and 1; a value of 1 is a perfect fit. The square root, r, is called the index of fit. If both y and x are random variable, the index of fit would have the same values as the correlation between to variables.

4.1 (B) Crow Amsaa Model

The U.S. Army material system analysis activity (AMSAA) model [10, 11] was developed by crow (1984). This model attempts to track reliability within a series of growth testing cycles, referred to as phases. At the conclusion of each design change (cycle), the failure rate decreases. However, during subsequent testing, the failure rate remains constant, as shown in Fig. 5.1 (b). The staircase behaviour of the failure rates is then approximated with a continuous curve of the from atb . This also leads to linear relationship between cumulative failure rate and time on a log-log scale. As a result, the AMSAA model has the same mathematical form as the Duane model. However, the AMSAA model is often applied to a single test phase where as the Duane model attempts to account for the global change in failure rates and MTBFs over the entire program. In addition, the underlying assumption of the AMSAA model differs considerably from those of the Duane model, which is primarily empirically based. This can be seen from the mathematical development of the AMSAA model.

We begin by letting 0 < s1 < s2 < …..< sk denote cumulative test times at which design changes are made assuming that the failure rate are constant between design changes, and letting Ni (the number of failures during the ith testing period) be a random variable, then Ni has a Poisson probability distribution with a probability function.

$$Pr(Ni=n) = \frac{[\lambda_i(s_i-s_{i-1})]^n e^{-\lambda_i(s_i-s_{i-1})}}{n!}$$

(1)

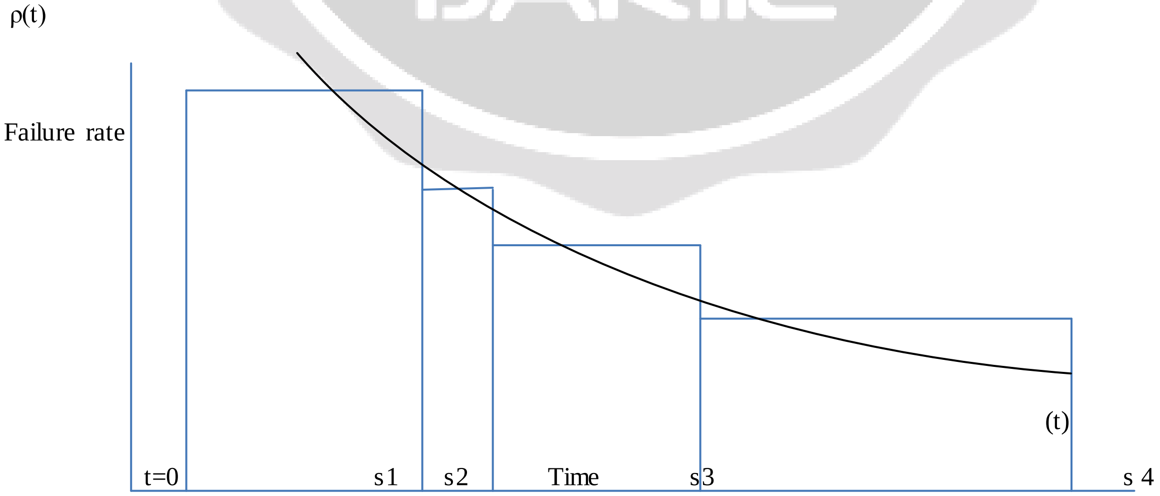

Fig. 4.1(b) The AMSAA reliability growth model

The mean of this distribution is λi (si – si -1). As a result of the relationship between the

Poisson distribution and the exponential distribution, the time to failure during the ith

Cycle is exponential with parameter λi. If t= the cumulative test time and n(t) = the

Cumulative number of failures through t hours of testing, then

$$\Pr\{n(t) = n\} = \frac{\lambda(r)^{e} e^{-\lambda(t)}}{n!} \qquad (2)$$

Where the cumulative failure rate is

$$\lambda(t) = \begin{cases} \lambda 1t & \text{for } 0 \le t < s1 \\ \lambda 1s1 + \lambda 2(t-s1) & \text{for } s1 \le t < s2 \\ \lambda 1\ s1 + \lambda 2s2 + \lambda 3\ (t-s3) & \text{for } s2 \le t < s3 \end{cases}$$

This failure law is the nonhomogeneous poisson process and having an intensity function

$$\rho(t)=\lambda i \text{ for } si-1 < t < si \qquad (3)$$

As long as λ1> λ2 >…> λk (that is , the failure rates are monotonically decreasing) reliability growth is observed.

For the practical implementation of the model, the intensity function is approximated by the power law process as

$$\rho(t) = abtb-1\ t > 0;\ a,b > 0 \qquad (4)$$

Although this is of the same form as a weibull hazard rate function, the underlying failure process is not Weibull. Integrating the intensity function provides the cumulative expected number of failures, m(t):

$$m(t)=\int 0\ abxb-1\ dx= atb \qquad (5)$$

Then with n(t) the observed cumulative number of failure:

$$n(t)=atb$$

and

$$\text{In } n(t)=\text{In } a+b \text{ In } t \qquad (6)$$

Observe that b<1 is necessary for reliability growth. If no further design changes are made after time t0, then future times are assumed to be exponential with an instantaneous MTBF found from

$$MTBFi=[\ abt0b-1]-1 \qquad (7)$$

4.1(C) Parameter Estimation for the Power Law Intensity Function

For the intensity function ρ(t)=abtb-1 , the parameters a and b may be estimated using a least-squares curve fitted to Eq. (6). However, the maximum likelihood estimates [12] are preferred over the least-squares estimates. The formulas for computing the maximum likelihood estimates are as follows

Type I data

Given N successive failure times t1 < t2<.. < tN that occur prior to the accumulated test time or observed system time, T,

$$\hat{b} = \frac{n}{n \ln T - \sum_{i=1}^{n} \ln t_i} \qquad (8)$$

Then

$$\hat{a} = n/Tb \qquad (9)$$

$$\hat{\rho}(T) = \hat{a}\hat{b}T^{\hat{b}-1} \qquad (10)$$

$$\text{MTBFi} = l/\hat{\rho}\ (T) \qquad (11)$$

Two-sided confidence intervals for the MTBF may be obtained from

$$L/\hat{\rho}\ (T) \leq \text{MTBF} \leq U/\rho(T) \qquad (12)$$

Where L and U are confidence interval factors obtained from table A.6 for Type I testing in the appendix.

Type II data

Given N successive failure times t1 < t2 <.. < tN following accumulated test time or observed system time T= tN,

$$\hat{b}_{=} \frac{n}{(n-1)\ln t_n - \sum_{i=1}^{n} \ln t_i} \qquad (13)$$

4. CONCLUSIONS

The choice of model depends on the growth process being modeled. Key driver are the type of system being designed and the project management of the growth process. When the design activities are well understood in advance project workloads can be managed evenly leading to predictably and equally spaced modification ,each of which having similar effect on the reliability of the item ,then the MPL would be the more appropriate model. On the other hand, the MIBM would be more appreciate for more uncertain situation, where the reliability improvement of a design is driven by the removal of faults, which are as yet unknown, and only through further investigation of the design will these be identify. This situations, have less predictable workload and fewer modifications are likely later on in the project. Anecdotal evidence from industrial applications of this model suggests they are useful in supporting reflection upon the potential weaknesses of the design and the impact of modifications on its reliability through analysis thereby allowing reliability engineering to acquire the attributes of a design engineering discipline. The models are simple in comparison with the complexity of the problem they describe; however, this simplicity allows them to provide a heuristic for use by managers tracking reliability growth in design.

Growth modeling in design can be verified by comparing the results with observations from test and use. This is an imperfect verification since the evidence gathered through test to confirm or refute the presence of potential weaknesses, and to flag additional, unexpected problem, only provides partial observations given that only one design option is being examined. In addition, there is a time lag from months to years-between modeling and the generation of observations about the reliability of the system design gathered through test and use. However, such verification is necessary as it allows feedback to be given to the analyst and engineers to re-calibrate the model and their understanding of that family of designs.

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