DYNAMICS AND MANAGEMENT OF THE FOREST AREA BASED ON THE DYNAMICS OF THE HUMAN POPULATION: Case study of the Haute Matsiatra region

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ABSTRACT

Forest degradation in interaction with the ever-increasing world population is part of the various major environmental problems. Therefore, this publication consists of the presentation of a new method using a Markovian approach called zone model. The latter models the dynamics and management of the forest surface in interaction with the dynamics of the human population. For the case of the Haute Matsiatra region, the results of calculations show that the constant growth in the size of the human population amplifies the degradation of the forest surface. On the other hand, with the confidence intervals of the zone model, it is possible to gradually restore the destroyed parts. As the case of the Haute Matsiatra region, by preserving 0.05ha per inhabitant as the surface of housing and infrastructure and 503905ha that of cultivable areas, according to calculations, from 2022, it is necessary to catch up at least 1104.178ha of reforestation per year, to conserve the forest cover of 55126.554ha and to gradually restore the parts destroyed by the fires and by the operations until 2050.

Keyword: *Markov chain, population dynamics, forest dynamics, forest management, area model, confidence interval.*

I. INTRODUCTION

According to the Chinese proverb “one generation plants a tree, the next benefits from its shade” and the World Report on Human Development of 2007/2008 shows that the unceasing increase in consumers multiplies the problem of global warming. The latter can lead to adverse consequences on global socio-economic and socio-environmental organizations. Indeed, good decision-making on the greenhouse gas reduction policy is essential. In the latter, the conservation of forest resources is a fundamental tool and this phenomenon encourages researchers to deduce that the continual increase in the world's population is accelerating this ravage. Thus, does the new method based on the Markov chain facilitate decision-making on forest management? This article starts with the state of the art, then the general methodology, then the theoretical results on the mathematical modeling of human population dynamics, as well as the area model and the confidence interval of forest conservation, and at the end, the case study of the Haute Matsiatra region as well as the general conclusion.

II. STATE OF THE ART

In short, we performed bibliographic and webographic research to define the state of the art and to demonstrate the originality of the ideas. According to bibliographic research on population dynamics and the dynamics of forest surfaces with the Markov chain, most authors focus on the study of the dynamics of particles, cells, animal species and agricultural territories. Some authors accumulate image analyzes using remote sensing and GIS (Geographic Information System) in order to inform the dynamics of land use . In particular, some researchers have studied the dynamics of forest territories by the Markov chain such as:

- Dr. RATIARSON V.'s thesis: He uses a stochastic approach to simulate changes in land use states governed by rules of forest plots from their first clearing. He proposed the period-homogeneous Markov model to simulate the case of the forest corridor of Fianarantsoa, Madagascar ;
- Aurélie Beynier's thesis: She uses Decentralized Markov decision-making processes for task planning in real applications. She invented the OC-DEC-MDPs (DEC-MDP with Occasional Cost) and the concept of occasioned cost.
- El Ghali LAZRAK's thesis: He develops a generic method for modeling the past and current dynamics of the territorial organization of agricultural activity (OTAA). He has developed a method of stochastic modeling based on hidden Markov models which makes it possible to search a corpus of spatio-temporal land cover (OCS) data with a view to segmenting it and revealing hidden agricultural dynamics.
- Mélanie Zetlaoui's thesis: she uses mathematical approaches to estimate stand dynamics using confidence intervals based on the Usher model. She invented the delta-method for obtaining the asymptotic distribution of the maximum likelihood estimators of predictions.

On the other hand, the originality of this thesis project is based on:

- The use of a new theory called the area model which models the dynamics of the forest surface according to the dynamics of the human population (based on the Markov chain of life and death), by considering the dependence of the population with forest resources;
- The use of confidence intervals of the zone model, for the management of forest areas, in the face of population growth.

III. GENERAL METHODOLOGY

3-1. Principle of modeling:

The general principle consists of systemic studies of the interactions of the human species with its surroundings and studies of their actions towards nature.
According to Darwin's theory of natural selection, humans are predators of all kingdoms (animal and plant). On the other hand, the plant kingdoms (trees, fruit trees, plants, ...) play major roles in environmental issues and human life. Thus, the interdependence of the human species with the animal and plant kingdoms is an inevitable parameter in the study of environmental management.
Categorically, the possible actions of humans are distinguished by consumption, destruction and restoration.
In practice, behavioral studies of the target population are based on the results of surveys concerning the trend of consumer habits. As well as the necessary information, for the studies of the previous forest states until its current state of the target region, are collected by written surveys.
In particular, the methodology of these surveys is based on oral and written information, using questionnaires and documentary research respectively.
Thus, the data collected concerns utilitarian individuals and loggers, as well as information in the urban and rural areas of Madagascar (in particular, the Haute Matsiatra region).

3-2. Markov model:

The Markov property describes a property satisfied by many random phenomena, for which the future evolution depends on the past only through its value at the present time. To begin with, we are going to introduce a formal definition: a Markov chain is a random sequence $\{X_n;\ n = 0;\ 1;\ 2;\ \dots\}$, defined on a probability space $(\Omega; F;\ P)$,with values in a set E which can be arbitrary, but which will be here either neither nor countable, and which enjoys the Markov property. Without immediately giving a formal definition, let us indicate that a Markov sequence has the property that knowing X_n, we can forget the past to predict the future. One way to construct such a sequence is to give oneself a sequence of random variables $\{Y_n;\ n \geq 1\}$ which are mutually independent, and globally independent of X_0, with values in F,and a map $f\ :\ IN \times E \times F \longrightarrow E$, such as for $n \geq 1,\ X_n = f(n; X_{n-1}; Y_n)$.
In a way, it is the simplest model of a sequence of non-independent random variables. By definition, the in-valued E stochastic process $\{X_n, n \in \mathbb{N}\}$ is called a Markov chain if for all $n \in \mathbb{N}$, the conditional law of X_{n+1}knowing $X_0, X_1, \dots, X_n$ does equal to the conditional law know X_n : $\forall x_0, x_1, \dots, x_i, x_j \in E$,

$$P(X_{n+1} = x_j / X_0 = x_0; X_1 = x_1; \dots; X_n = x_i) = P(X_{n+1} = x_j / X_n = x_i)$$

Where the law of X_0 is called the initial law of the Markov chain $\{X_n, n \in \mathbb{N}\}$.
We put $q_{ij} = P(X_{n+1} = x_j / X_n = x_i)$ and $Q = (q_{ij}\ ;\ x_i\ ,\ x_j \in E)$. The matrix Q is said to be Markovian, in the sense that it satisfies the property that $\forall x_i \in E$, the row $(q_{ij};\ x_j \in E)$ vector is a probability measure on E $(q_{ij} \geq 0;\ \forall x_j \in E;\ \sum_{j \in E} q_{ij} = 1)$. In other words, the matrix Q is called the “Markov chain transition matrix”.

IV. THEORETICAL RESULTS

4-1. Human population dynamics

4.1.1. Markov chain of life and death and population growth rate:

Let be (H_n) the random variable that models the size of the human population where n is the rank of the year. We denote by h_n the value taken by the random variable (H_n) at rank n.
We denote by E the state space of all possible values of the random variable (H_n).
Let n_1 and n_2 such that $\Delta n = n_2 - n_1$($n_1 < n_2$) where Δn is called the duration of the period between $[n_1, n_2[$.
For all $n \in [n_1, n_2[$, and for all $j \in E$, let $\propto_j$ be the birth rate and μ_j the death rate during the period Δn so that the size of the population is equal to j in year $n + \Delta n$. Then

$$h_{n+\Delta n} = (1 + \sigma_{\Delta n})h_n = j \in E.$$

Where, for all $n \in [n_1, n_2[$, $\sigma_{\Delta n} =\propto_j - \mu_j$ and this is the rate of population during the period Δn.
Let $i, j \in E$ and be $p_{ij}(n) = Pr\{ H_{n+\Delta n} = j / H_n = i\}$ the probability of having a population size j in year $n + \Delta n$ given that i the size in year n.
As the size of the population at time $n + \Delta n$ only depends on its initial state at time n, thus the value taken by the random variable $(H_{n+\Delta n})$ only depends on the value taken by the random variable (H_n) ($h_{n+\Delta n}$ only depends on h_n). Then, (H_n) is a Markov chain who's the state space is $E = \{(1 + \sigma_{\Delta n})i = j, i \in IN^* \; et \; j \geq 0\}$ and the transition matrix is $P = \left(p_{ij}(n)\right)$.
In particular, for human population estimation, it is more realistic to work in a finite state space.
It is difficult to imagine the extinction of the human population, thus, it is assumed that there exists a smallest strictly positive integer g such that $E = \{g, g + 1, g + 2, \dots, N\}$ where N is a finite natural number.
According to the processes of birth and death (homogeneous in time) in a finite state space, given the state i ($i > g, i \neq N$):

- $p_{i,i+1}(\Delta n) =\propto_i \Delta n + o(\Delta n)$
- $p_{i,i-1}(\Delta n) = \mu_i \Delta n + o(\Delta n)$
- $p_{i,i}(\Delta n) = 1 - (\propto_i \Delta n + \mu_i \Delta n) + o(\Delta n)$
- $\lim_{n\to+\infty} p_{ij}(n) = p_j$ (it is the stationary probability for the state to be in a state j whatever the future years) and $V = (p_j)$(the stationary law).

That is to say, to the state i one can pass in a very short time Δn only to the state $i - 1$ or $i + 1$, in particular, from N one can only pass to $N - 1$.
From the Chapman- Kolmogorov equation, $p_{ij}(n + \Delta n) = \sum_{k=0}^{\infty} p_{ik}(n)p_{kj}(\Delta n)$. This sum can be written as

$$p_{ij-1}(n)p_{j-1j}(\Delta n) + p_{ij}(n)p_{jj}(\Delta n) + p_{ij+1}(n)p_{j+1j}(\Delta n) + \sum_{\substack{k<j-1\\ k>j+1}} p_{ik}(n)p_{kj}(\Delta n)$$

As $\sum_{\substack{k<j-1\\ k>j+1}} p_{ik}(n)p_{kj}(\Delta n) \leq \sum_{\substack{k<j-1\\ k>j+1}} p_{kj}(\Delta n)$

But $\sum_{\substack{k<j-1\\ k>j+1}} p_{kj}(\Delta n) = 1 - \left(p_{jj}(\Delta n) + p_{j-1j}(\Delta n) + p_{j+1j}(\Delta n)\right)$

$$= 1 - \left(1 - \left(\propto_j \Delta n + \mu_j \Delta n\right) - o(\Delta n) + \mu_j \Delta n + o(\Delta n) +\propto_j \Delta n + o(\Delta n)\right) = 1 - 1 - o(\Delta n) = o(\Delta n)$$

Thereby, $p_{ij}(n + \Delta n) = p_{ij-1}(n)p_{j-1j}(\Delta n) + p_{ij}(n)p_{jj}(\Delta n) + p_{ij+1}(n)p_{j+1j}(\Delta n) + o(\Delta n)$

$$p_{ij}(n + \Delta n) = p_{ij-1}(n) \propto_{j-1} \Delta n + p_{ij}(n)(1 - (\propto_j \Delta n + \mu_j \Delta n)) + p_{ij+1}(n)\mu_{j+1}\Delta n + o(\Delta n)$$

$$p_{ij}(n + \Delta n) - p_{ij}(n) = p_{ij-1}(n) \propto_{j-1} \Delta n - \left(\propto_j \Delta n + \mu_j \Delta n\right)p_{ij}(n) + p_{ij+1}(n)\mu_{j+1}\Delta n + o(\Delta n)$$

$$\frac{p_{ij}(n + \Delta n) - p_{ij}(n)}{\Delta n} = p_{ij-1}(t)\alpha_{j-1} - \left(\alpha_j + \mu_j\right)p_{ij}(n) + p_{ij+1}(n)\mu_{j+1} + \frac{o(\Delta n)}{\Delta n}$$

If $\Delta n \to 0$ ($n_1 \to n_2$), we have $p_{ij-1}(t)\alpha_{j-1} - \left(\alpha_j + \mu_j\right)p_{ij}(n) + p_{ij+1}(n)\mu_{j+1} = 0$
Since N is the upper bound of E and all elements of E are greater than or equal to g, then

- $\mu_g = 0$ (no person died if the size of the population is equal to the lower bound g of E)
- for $j = g$, we have $-\alpha_g p_{ig}(n) + p_{ig+1}(n)\mu_{g+1} = 0$
- for $j = N$, we have $-\alpha_{N-1}p_{iN-1}(n) + p_{iN+1}(n)\mu_{N+1} = 0$
- for any j, we have $p_{ij-1}(n)\alpha_{j-1} - \left(\alpha_j + \mu_j\right)p_{ij}(n) + p_{ij+1}(n)\mu_{j+1} = 0$

These equations are called equations of the future and they provide the equations of states.
As $\lim_{n\to+\infty} p_{ij}(n) = p_j$ and $g - 1 \notin E$ then:
$-\alpha_g p_g + \mu_{g+1}p_{g+1} = 0$ (1-1)
$\propto_{N-1} p_{N-1} - \mu_N p_N = 0$ (1-2)

$\propto_{j-1} p_{j-1} - (\propto_j + \mu_j)p_j + \mu_{j+1}p_{j+1} = 0$ (1-3)
Equation (1-1) implies $p_{g+1} = \frac{\propto_g}{\mu_{g+1}} p_g$ (1-4)
From the relations (1-4) and (1-3), we get
$\mu_{g+2}p_{g+2} = (\propto_{g+1} + \mu_{g+1}) \frac{\propto_{g+1}}{\mu_{g+1}} p_g - \propto_g p_g = \propto_g p_g \left(\frac{\propto_{g+1}}{\mu_{g+1}}\right) \Leftrightarrow p_{g+2} = \frac{\propto_g \propto_{g+1}}{\mu_{g+1}\mu_{g+2}} p_g$ (1-5)
According to relations (1-5) and (1-2) and by induction, we have
$p_{N-1} = \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} p_g \Rightarrow p_N = \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N} p_g$ (1-6)
According to the recurrence relation (1-6), we have

$$\sum_{j=g}^{N} p_j = p_g + \frac{\propto_g}{\mu_{g+1}} p_g + \cdots + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} p_g + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N} p_g$$
$$= p_g \left(1 + \frac{\propto_g}{\mu_{g+1}} + \cdots + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N}\right)$$
$$\Longrightarrow p_g = \frac{\sum_{j=0}^{N} p_j}{1 + \frac{\propto_g}{\mu_{g+1}} + \cdots + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N}}$$

As $\sum_{j=g}^{N} p_j = 1$, then,

$$p_g = \frac{1}{1 + \frac{\propto_g}{\mu_{g+1}} + \cdots + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N}}$$

And for everything $g < j \leq N$,

$$p_j = \frac{\propto_g \propto_{g+1} \cdots \propto_{j-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_j} p_g = \frac{\frac{\propto_g \propto_{g+1} \cdots \propto_{j-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_j}}{1 + \frac{\propto_g}{\mu_{g+1}} + \cdots + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-2}}{\mu_{g+1}\mu_{g+2} \cdots \mu_{N-1}} + \frac{\propto_g \propto_{g+1} \cdots \propto_{N-1}}{\mu_{g+1}\mu_{g+2} \cdots \mu_N}}$$

They are the limit values independent of the initial state, and therefore they are the components of the vector $V = (p_j)$ (the stationary law of the Markov chain (H_n)).
For all $n \geq n_1$, let be σ_j the population growth rate so that the state of the human population in years $n + \Delta n$ be equal to $j \in E$, and let $\sigma_{n+\Delta n}$ be the estimated value of the population growth rate. Then, this rate follows the stationary probability distribution V such that:

$$\sigma_{n+\Delta n} = \sum_{j \in E} \sigma_j . p_j$$

4.1.2. Error calculation for the estimated values of (H_n) :

It is known that for all $n \geq n_1$, the estimated value of the size of the human population $v(h_{n+\Delta n})$ is determined by the relationship:

$$v(h_{n+\Delta n}) = (\sigma_{n+\Delta n} + 1)\, h_n$$

In this section, it is assumed that the rank of the initial year is 0 and for any rank n, we denote by $v(h_n)$ the estimated size and by h_n the actual size of the human population, such as $v(h_n), h_n \in E$.
Given h_0, let be σ the estimated value of the population growth rate per year from rank 0 to rank n $(n_1 = 0)$.
By definition, for all $n \geq 0$, $\sigma = \frac{v(h_n) - v(h_{n-1})}{v(h_{n-1})} \Rightarrow v(h_n) = (1 + \sigma)v(h_{n-1})$, then $(v(h_n))$ is a geometric sequence with common ratio $(1 + \sigma)$ whose first term is $v(h_0) = h_0$. Thus, $v(h_n) = (\sigma + 1)^n\, h_0$

As $\sigma_{n+\Delta n} = \frac{v(h_n) - h_0}{h_0} = \frac{(\sigma+1)^n h_0 - h_0}{h_0} = (\sigma + 1)^n - 1 \Longrightarrow \sigma = (\sigma_{n+\Delta n} + 1)^{1/n} - 1$
Therefore, for all $n > 0$, $v(h_{n+1}) = \left((\sigma_{n+\Delta n} + 1)^{1/n} - 1\right) v(h_n)$
Let $\varepsilon(h_n) = |v(h_n) - h_n|$ therefore $\varepsilon(h_n)$ characterize the errors of calculations on the estimated size of the human population for the year n.
Let p the rank of the current year, let's put $\varepsilon_h = \max_{0<n\leq p}\left\{\frac{\varepsilon(h_n)}{h_n}\right\}$. Suppose if $0 \leq \varepsilon_h \leq 0.025$ (maximum error of 2.5% of the true population size), then, $\varepsilon(h_n)$ is negligible compared to h_n.

In the sequel, we assume that for all $n > 0$, the value of $\varepsilon(h_n)$ is negligible compared to h_n. Thus, for any rank $n > 0$, $h_n = v(h_n)$ (we will use the estimated values of the size of the human population up to rank $2p$ such that h_0 is the initial size).

4-2. Area model :

Let F_0 the initial size of the forest area.
The zone model is a relationship that models the dynamics of the forest surface in interaction with the size of the human population:

$$F_{n+1} = F_n + R(h_n)$$

Where F_n is the size of the forest area for the n-th year and $R(h_n)$ the residual area between the years n and $n+1$.

4.2.1. Residual area:

The residual area in $n-th$ years is characterized by parameters $(r_n, \varphi_n, e(h_n), \tau_n)$ such as:
$R(h_n) = \tau_n r_n - \varphi_n - e(h_n)$ where

- r_n is the surface of the afforestation or reforestation carried out
- τ_n is the success rate of the afforestation or reforestation carried out
- φ_n is the forest area destroyed by accidental or intentional fire
- $e(h_n)$ is the forest area exploited for daily needs

According to this relationship, area $e(h_n)$is the only parameter that depends on the size dynamics of the human population.

4.2.2. Area $e(h_n)$:

Forest consumption by daily needs plays a big role in the study of forest dynamics. Thus, it is obvious that the consumption of wood in the rural area is quite different compared to the consumption of wood in the urban area.
Let $\theta(r)$ and be $\theta(u)$ the average rates of the respective distribution of demography according to the rural zone and the urban zone. So $h_n = \theta(r)h_n + \theta(u)h_n$ where $\theta(r) + \theta(u) = 1$.
Let $c_n(r)$ and be $c_n(u)$ $t$he surface consumption of the forests of an individual in the $n-th$ year. Then, the area $e(h_n)$ is determined by the following relation:

$$e(h_n) = [\theta(r)c_n(r) + \theta(u)c_n(u)]h_n$$

In the event of a massive migration, the values of $\theta(u)$ and $\theta(r)$ can be modified according to the years.
Therefore, the residual area is:

$$R(h_n) = \tau_n r_n - \varphi_n - [\theta(r)c_n(r) + \theta(u)c_n(u)]h_n$$

4.2.3. Estimation of F_n by zone model

Considering all these parameters, the zone model is given by the following recurrent sequence:

$$F_1 = F_0 + R(h_0), n = 0$$
$$F_{n+1} = F_n + R(h_n), \ \ n \geq 1$$

There fore,

$$\begin{cases} F_1 = F_0 + \tau_0 r_0 - \varphi_0 - [\theta(r)c_0(r) + \theta(u)c_0(u)]h_0 \\ F_{n+1} = F_n + \tau_n r_n - \varphi_n - [\theta(r)c_n(r) + \theta(u)c_n(u)]h_n, \ \ n \geq 1 \end{cases}$$

4.2.4. Error calculation for the estimated values of F_n :

We denote by $v(F_n)$ is the estimated value and by F_n the actual size of the forest area if we start from the row 0 up to the row n.
Let $\varepsilon(F_n) = |v(F_n) - F_n|$ therefore $\varepsilon(F_n)$ characterizes the calculation errors on the estimated value of the size of the forest area for the year n.
Let p the rank of the current year, let's put $\varepsilon_F = \max_{0<n\leq p}\left\{\frac{\varepsilon(F_n)}{F_n}\right\}$. Suppose if $0 \leq \varepsilon_F \leq 0.01$ (maximum error of 1% of the actual size of the forest area), then, $\varepsilon(F_n)$ is negligible compared to F_n.
The zone model is not longer reliable as long as it $\varepsilon(F_n)$ exceeds the reasonable value ($\varepsilon_F > 0.01$), so predictions into the future are limited.

In the sequel, we assume that for all $n \geq p$, the value of $\varepsilon(F_n)$ is negligible compared to F_n. Thus, for any rank $n \geq p$, $F_n = v(F_n)$ (we will use the estimated values of of the forest area up to rank $2p$ such that F_0 is the initial size).

4-3. Reforestation trend and confidence interval:

Let be p the rank of the year that characterizes the present, in this section, we take $n \geq p$. Thus, $\varepsilon(F_n)$ taking the values of $v\left(\varepsilon(F_n)\right)$ such that $F_n = v(F_n)$ at $\varepsilon(F_n) - close$.

4-3.1. Forest condition trend:

Definitions: between the interval $[n, n+1]$, we say that the trend of the forest condition is:

- Stable when $F_{n+1} = F_n$
- Red when $F_{n+1} < F_n$
- Green when $F_{n+1} > F_n$

Properties: between the interval $[n, n+1]$

- The trend is stable if and only if $R(h_n) = 0$
- The trend is red if and only if $R(h_n) < 0$
- The trend is green if and only if $R(h_n) > 0$

4-3.2. Confidence interval of reforestation

To avoid the red tendency in the $(n+1)-th$ year, it is necessary that $R(h_n) > 0$, which is equivalent to $\tau_n r_n - \delta_n > 0$ where $\delta_n = \varphi_n + [\theta(r)c_n(r) + \theta(u)c_n(u)]h_n$

From where, it is necessary that $r_n > \frac{\delta_n}{\tau_n}$ because $\tau_n > 0$.

To determine the limit of the reforestation area, it is necessary to look for a framework for r_n.

Consider S_T the total area of the area concerned and $S(h_n) = S_c + sh_n$ where S_c is the area of cultivable areas (to be kept for food crops), s the usual area to be kept for an individual's infrastructure and dwellings (this value is chosen arbitrarily). If F_n the area of forest cover at row n, then the area available for afforestation or reforestation actions between rows n and $n+1$ is the area $S_R(n+1) = S_T - S(h_n) - F_n$

However, so that the surface of the forest cover F_{n+1} does not exceed the surface $S_R(n+1)$, then, it is necessary that

$$F_{n+1} = F_n + R(h_n) < S_R(n+1)$$
$$\Leftrightarrow F_n + \tau_n r_n - \delta_n < S_T - S(h_n) - F_n$$
$$\Rightarrow \tau_n r_n - \delta_n < S_T - S(h_n) - 2F_n$$
$$\Rightarrow \tau_n r_n < S_T - S(h_n) + \delta_n - 2F_n, \tau_n > 0$$
$$\Rightarrow r_n < \frac{S_T - S(h_n) - 2F_n + \delta_n}{\tau_n}$$

Let $\omega(n) = \frac{S_T - S(h_n) - 2F_n}{\tau_n}$ and $\omega_{inf}(n) = \frac{\delta_n}{\tau_n}$, therefore

$$\omega_{inf}(n) < r_n < \omega(n) + \omega_{inf}(n)$$

It should be known that if $\omega(n) > 0$, the actions of reforestation continue $(S_R(n+1) > F_n)$ and if $\omega(n) = 0$ then, the actions stop $(S_R(n+1) = F_n)$. So $\omega(n)$ is always positive or zero.

On the other hand, if $\omega(n) < \omega_{inf}(n)$, by hypothesis $r_n > \omega_{inf}(n) > \omega(n) \Rightarrow \tau_n r_n > S_R(n+1) - F_n$ $\Rightarrow F_n + \tau_n r_n > S_R(n+1) \Rightarrow F_{n+1} > S_R(n+1) - \delta_n \Rightarrow F_{n+1} = S_R(n+1)$ to $\delta_n - close$. This result shows that if $\omega(n) < \omega_{inf}(n)$, then the area available for reforestation $S_R(n+1)$ is almost everywhere filled at $\delta_n - close$. In this case, to control the reforestation activities by protecting the surface $S(h_n)$, then it is preferable to reforest an area strictly less than δ_n or to interrupt the reforestation activities.

Normally, in our objective, when we reforest, then its surface must be higher $\omega_{inf}(n)$. Therefore, in the sequel, we consider this case as a negligible case. That is to say that throughout the sequel, we will take $\omega(n) \geq \omega_{inf}(n)$.

As p is the rank that characterizes the present, let $d > p$ (d is the last rank for forecasting into the future) and let

$n \in \{p, p+1, \dots, d\}$.

Since the value of the area S_T is finite, the sequence (F_n) is bounded and $\tau_n \neq 0$, then the sequence $(\omega(n))$ is bounded.

We pose $\omega_{sup}(n) = \frac{\min_{k\in\{p,p+1,\dots,d\}}\{\omega(k)\} + \omega_{inf}(n)}{2(d-p+1)}$.

We know that whatever the values taken by r_n, so that the surface $S_R(n+1)$ to be always greater than or equal to the surface F_{n+1}, then:

$$\omega_{inf}(n) < r_n \leq \omega_{sup}(n) < \omega(n) + \omega_{inf}(n)$$

1st case: We assume that $(\omega(n))$ is monotonous

- If the sequence $(\omega(n))$ is increasing, then $\min_{n\in\{p,p+1,\dots,d\}}\{\omega(n)\} = \omega(p)$

$$\Rightarrow \forall n \in \{p, p+1, \dots, d\}, \sum_{n=p}^{d} r_n \leq \sum_{n=p}^{d} \omega_{sup}(n) = \frac{1}{2(d-p+1)} \sum_{n=p}^{d} \left(\omega(p) + \omega_{inf}(n)\right)$$

$$\Rightarrow \sum_{n=p}^{d} r_n \leq \frac{1}{2}\omega(p) + \frac{1}{2(d-p+1)}\sum_{n=p}^{d}\omega_{inf}(n)$$

But $\omega(p)$ is the greatest lower bound of $(\omega(n))$ and $\forall n \in \{p, p+1, \dots, d\}, \omega(n) \geq \omega_{inf}(n)$, then $\omega_{inf}(n) \leq \omega(p)$

$$\Rightarrow \sum_{n=p}^{d} r_n \leq \frac{1}{2}\omega(p) + \frac{1}{2}\omega(p) = \omega(p) < \omega(d) + \omega_{inf}(d)$$

- If the sequence $(\omega(n))$ is decreasing, then $\min_{n\in\{p,p+1,\dots,d\}}\{\omega(n)\} = \omega(d)$

$$\Rightarrow \ \forall n \in \{p, p+1, \dots, d\}, \sum_{n=p}^{d} r_n \leq \sum_{n=p}^{d}\omega_{sup}(n) = \frac{1}{2(d-p+1)}\sum_{n=p}^{d}\left(\omega(d) + \omega_{inf}(n)\right)$$

$$\Rightarrow \sum_{n=p}^{d} r_n \leq \frac{1}{2}\omega(d) + \frac{1}{2(d-p+1)}\sum_{n=p}^{d}\omega_{inf}(n)$$

But $\omega(d)$ is the greatest lower bound of $(\omega(n))$ and $\forall n \in \{p, p+1, \dots, d\}, \omega(n) \geq \omega_{inf}(n)$, then $\omega_{inf}(n) \leq \omega(d)$

$$\Rightarrow \sum_{n=p}^{d} r_n \leq \frac{1}{2}\omega(d) + \frac{1}{2}\omega(d) = \omega(d) \leq \omega(d) + \omega_{inf}(d)$$

2nd case: Assuming that $(\omega(n))$ is not monotonous, as it is bounded, then there is always at least one rank n_0 between the ranks $p$and d such that $\min_{n\in\{p,p+1,\dots,d\}}\{\omega(n)\} = \omega(n_0)$.
As in the previous cases, $\omega_{inf}(d) \leq \omega(n_0) < \omega(d)$

$$\Rightarrow \sum_{n=p}^{d} r_n \leq \frac{1}{2}\omega(n_0) + \frac{1}{2}\omega(n_0) = \omega(n_0) < \omega(d) < \omega(d) + \omega_{inf}(d)$$

For this reason, in any case, if $\omega_{sup}(n) = \frac{\min_{n\in\{p,p+1,\dots,d\}}\{\omega(n)\} + \omega_{inf}(n)}{2(d-p+1)}$, then

$$\sum_{n=p}^{d} r_n \leq \omega(d) + \omega_{inf}(d) \Leftrightarrow \tau_d \sum_{n=p}^{d} r_n + F_d - \delta_d \leq S_R(d+1)$$

As $F_{d+1} = \sum_{n=p}^{d}\tau_n r_n + F_d - \delta_d$, then $F_{d+1} \leq \tau_d \sum_{n=p}^{d} r_n + F_d - \delta_d$ if and only if, for all $n \in \{p, p+1, \dots, d\}$, $\tau_n \leq \tau_d$.
So, to keep the surface $S(h_n)$, we must take $\tau_d = \max_{n\in\{p,p+1,\dots,d-1\}}\{\tau_n\}$. In this case, we have :

$$F_{d+1} \leq S_R(d+1)$$

This last relation means that if we take $\tau_d = \max_{n\in\{p,p+1,\dots,d-1\}}\{\tau_n\}$ such that $\omega_{inf}(n) < r_n \leq \omega_{sup}(n)$, then the area of reforested areas never exceeds the area of available areas as long as $r_n \in]\omega_{inf}(n); \omega_{sup}(n)[$.
To conclude, the confidence interval for reforestation is:

$$\mathcal{I}(n) =]\omega_{inf}(n); \omega_{sup}(n)[$$

In other words, for all $n \in \{p, p+1, \dots, d\}$, $r_n \in \mathcal{I}(n)$ means that up to rank d, the trend of the forest area is always stable or green such that the area $S(h_n)$ will be kept at $\varepsilon(F_n) - close$.

4-3.3. Area Pattern Green Trend Range

For all $n \in \{p, p+1, \dots, d\}$, either $\varepsilon = \max_{n\in[p,d]}\{\varepsilon(F_n)\}$.
We suppose $\frac{\omega_{min}(n) - \omega_{inf}(n)}{2} > \varepsilon$ (which is always possible because ε takes a small value)

Let $\omega_{min}(n) = \omega_{inf}(n) + \varepsilon$ and $\omega_{max}(n) = \omega_{sup}(n) - \varepsilon$.
Thus, the confidence interval of the green trend of the zone model is given by:

$$\mathcal{I}_\varepsilon(n) = [\omega_{min}(n); \omega_{max}(n)]$$

In other words, for any $n \in \{p, p+1, \dots, d\}$ such that

- If the reforestation area $r_n = \omega_{min}(n)$ then, the trend of the forest area is stable up to the rank d.
- If the reforestation area $r_n > \omega_{min}(n)$ then, the trend of the forest area is green up to rank d.

In general, $r_n \in \mathcal{I}_\varepsilon(n)$ means that up to rank d, the trend of the forest surface is certainly green or at least stable (the surface F_p will be at least preserved at rank d) of which the surfaces of food crops, dwellings and infrastructures are absolutely preserved.

4-3.4. Confidence interval of an ideal reforestation area per year

In order to have a reasonable green tendency (so that the tendency is green and the surfaces S are $S(h_{n,})$ preserved), it is natural to think of the financial constraint on the plan of reforestation. Then it is necessary to find the interval of the ideal area of reforestation per year to set the annual budget for reforestation during the period $d-p$.

Either $n \in \{p, p+1, \dots, d\}$ and either $\psi(n) = \frac{\omega_{min}(n)+\omega_{max}(n)}{2}$. For all $n \in \{p, p+1, \dots, d\}$, if $r_n = \psi(n)$, then the green trend is certain and the forest cover never exceeds the limit $S_R(n+1)$. Indeed, $\psi(n)$ characterizes the ideal areas of reforestation per year.

For $n \in \{p, p+1, \dots, d\}$, the arithmetic mean $\overline{\psi} = \frac{\sum_{n=p}^{d} \psi(n)}{d-p+1}$ is the ideal average area of reforestation per year. On a practical level, from this value we can set the annual budget necessary for the reasonable green trend to be certain.

In reality, the budgetary means of the regions are different (depending on their country), therefore, it is necessary to find the means so that the trend is green.

Given $\overline{\omega}_{min} = \frac{\sum_{n=p}^{d} \omega_{min}(n)}{d-p+1}$ and $\overline{\omega}_{max} = \frac{\sum_{n=p}^{d} \omega_{max}(n)}{d-p+1}$, then there are two possible cases (depending on the budgetary means) for setting the annual budget in order to guarantee the green trend:

- **Case of the region which does not manage to reach the annual budget for $\overline{\psi}$:**

In this case, it is necessary to take a value of area to be reforested (during the period $d-p$) in the interval $[\overline{\omega}_{min}, \overline{\psi}[$, that is to say that it is necessary to at least preserve the area F_p. Then the ideal reforestation interval for this region is $[\overline{\omega}_{min}, \overline{\psi}[$.

When the surface $\overline{\omega}_{min}$ is quite small compared to the surface $\overline{\psi}$, it means that the surface of the areas to be reforested is vast. So, for all $k \geq 2$, such that $k\overline{\omega}_{min} \leq \overline{\psi}$, we can take the interval $[\overline{\omega}_{min}, k\overline{\omega}_{min}]$ as the ideal reforestation interval for the gradual green trend (virtually certain green trend).

- **Case of the region that manages to reach or exceed the annual budget for $\overline{\psi}$:**

In this case, preferably, it is recommended to take a value of area to be reforested (during the period $d-p$) in the interval $[\overline{\psi}, \overline{\omega}_{max}]$ and this is the ideal reforestation interval for this region.

If the area of the areas to be reforested is large ($\overline{\omega}_{min} \ll \overline{\psi}$), then for any $\chi > 0$ such that $\overline{\psi} - \chi > 2\overline{\omega}_{min}$, we can take the interval $\left[\overline{\psi} - \chi, \overline{\psi} + \chi\right]$ as the ideal reforestation interval to guarantee the absolute green trend.

V. CALCULATIONS, RESULTS AND INTERPRETATIONS: Case of the Haute Matsiatra region

5-1. Estimated population growth rate

In the Haute Matsiatra region, the birth rate varies from 24 to 37 per thousand, and the death rate varies from 9 to 13 per thousand (source: MDG/ INSTAT-RGPH 2018). That is there are 14 birth rate possibilities between the interval of years $[n, n+1]$. For the mortality rate of 9 to 13 per thousand, there are 5 possibilities between the year interval $[n, n+1]$ $(\Delta n = 1)$. Starting with the year 2000 (at rank 0) knowing that $h_0 = 677743$ (Source: MDG/ INSTAT-RGPH 2018), then there are 70 possible cases to obtain the value of σ_j such that $\sigma_j \in \{11,12,13,\dots,27,28\}$. We denote respectively by $x_j^{(k)}$ and $y_j^{(k)}$the possible values of the birth and death rates to have σ_j such that $\sigma_j = x_j^{(k)} - y_j^{(k)}$ where $k = 1, \dots, 71$. Since the birth rates are all greater than the maximum value of the death rate, then the smallest possible value of the population size (the smallest strictly positive integer in the state space E) is the initial size $h_0 = g = 677743$ (the trend is increasing).

Let X the random variable taking the values of σ_j and let $p(X = \sigma_j)$ the probability that the rate of population growth is equal to σ_j. So

$$p(X = \sigma_j) = \Pr\{h_1 = j/h_0 = 677743, \propto_j = x_j^{(1)}, \mu_j = y_j^{(1)} ou \dots ou \propto_j = x_j^{(k)}, \mu_j = y_j^{(k)}\}$$

We know that $p_j = \Pr\{h_1 = j/h_0 = 677743, \propto_j = x_j^{(k)}, \mu_j = y_j^{(k)}\}$

So,

$$\sum_{j=g}^{N} p_j = \sum_{j=g}^{N} [\sum_{k \in \{1,\dots,71\}} \Pr\{h_1 = j/h_0 = 677743, \propto_j = x_j^k, \mu_j = x_j^k\}]$$

Where N the upper bound of the possible value of the population size and N is finite.

Let A the set of all possible values of h_1, then, for all $j \in E$,

$$p_j = \begin{cases} p(X = \sigma_j) \neq 0 \; if \; j \in A \\ 0 \; if \; j \notin A \end{cases}$$

We know that

$$\sum_{j=g}^{N} p_j = \sum_{j=g}^{N} [\sum_{k \in \{1,\dots,70\}} \Pr\{h_1 = j/h_0 = 677743, \propto_j = x_j^k, \mu_j = x_j^k\}]$$

According to calculations on Matlab, we have

$$\sum_{j=677743}^{N} p_j = \sum_{j=677743}^{696720} p_j + \sum_{j=696721}^{N} p_j$$

The table 5-1-1 shows that the maximum possible value of the population growth rate is equal to 28 per thousand. So $\Pr\{h_1 > 696720/h_n = 677743\} = 0$

$$\sum_{j=677743}^{N} p_j = \sum_{j \in A} p_j + \sum_{j \notin A} p_j = p_{677743} + \sum_{\sigma_j=11}^{28} p(X = \sigma_j)$$

According to the values obtained in this table, we have:

$$\sum_{j=677743}^{N} p_j = 1$$

So the estimated population growth rate per year is

$$\sigma_{n+\Delta n} = \sigma_{n+1} = \sigma = \sum_{\sigma_j=11}^{28} \sigma_j p(X = \sigma_j) = 0.027732859066869, n \geq 0$$

Figure 5-1-1 shows the curve of the actual size (blue curve) and the estimated size (green curve) of the population between 2000 and 2023 (source : **https://fr.zhujiworld.com/mg/2855790-haute-matsiatra-region/#details**):

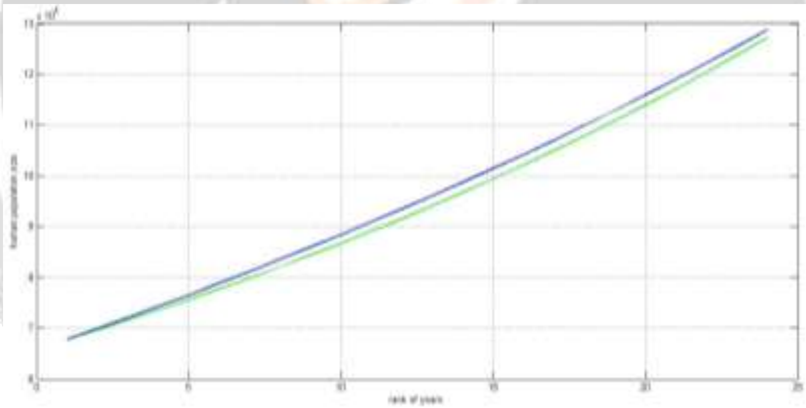

Figure 5-1-1: Curve of the h_n and the $v(h_n)$, (n=0,…,23) $h_0 = 677743$

According to this curve and the calculations on Matlab, we have the 5-1-2 curve which shows the ratio of errors compared to the real size of the population:

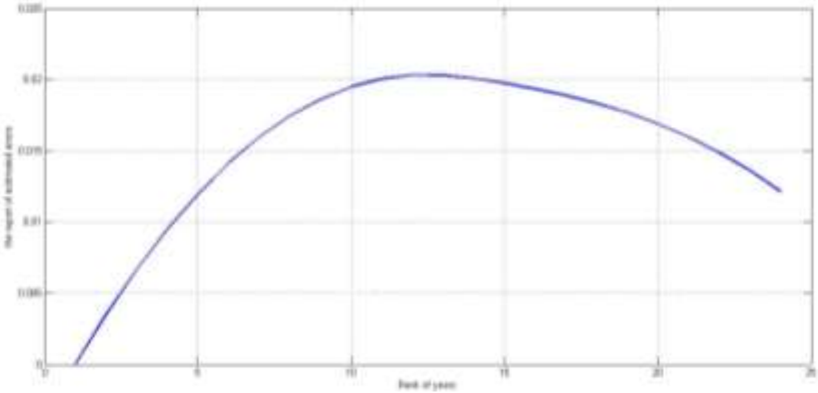

Figure 5-1-2 : Curve of the $\frac{\varepsilon(h_n)}{h_n}$, $0 \leq n \leq 23$

This curve shows that $0 < \varepsilon_h < 0.025$ (according to calculations on Matlab, $\varepsilon_h = \max_{0<n\leq p}\left\{\frac{\varepsilon(h_n)}{h_n}\right\} = 0.0203$) then the margin of error is acceptable.
Figure 5-1-3 shows the curve of the estimated size of h_n between 2000 and 2056:

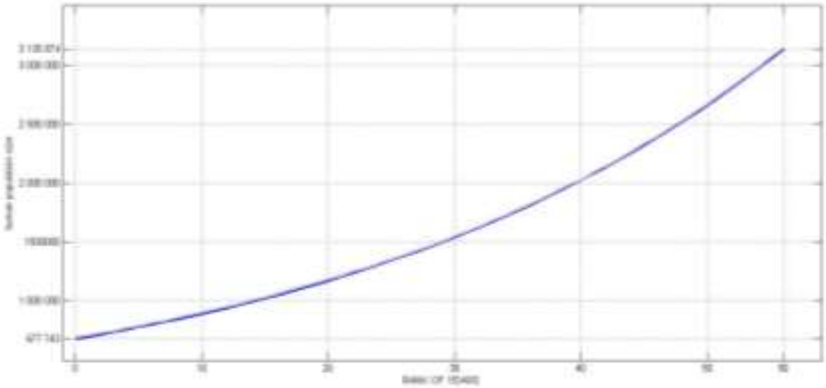

Figure 5-1-3: Curve of the dynamics of the size of the human population between the year 2000 (n=0) and the year 2056 (n=56),$h_0 = 677743$

5-2. Estimate of F_n based on h_n

According to the report on the evolution of the cover of natural forests 1990-2000-2005 by the Ministry of the Environment, Forests and Tourism and the JIRIALA_2008 project, the forest area of the Haute Matsiatra region in 2000 is 59675 ha while 59453 in 2005.
By exploiting the raw data of the monograph of the Haute Matsiatra region in 2014 (table 4, source BD/RSE/DREF-Haute Matsiatra, November 2011) and the report of the Director of Diversity Protection Raymond RAKOTONDRASOA of November 28, 2005 (Express de Madagascar), on average $r_n = 82.133 ha/year$, $\tau_n = 0.75$ et $\varphi_n = 15.45 ha/year$.
The following table 5-2-1 shows the results of surveys (in 2020) of wood consumers in the upper Matsiatra region:

Consumption/areas \ Wood requirements	Average consumption/month in rural area		Average consumption/month in the urban area	
	Per household (4.9 individuals)	Dry wood/Per individual	Per household (4.5 individuals)	Dry wood/Per individual
Charcoal for cooking	3 bags of 50 Kg=270Kg of dry wood	55,102Kg	2.5 bags of 50Kg = 225Kg of dry wood	50Kg
Dry wood for cooking (Kitay)	450Kg	91,837Kg	300Kg	66,667Kg
Papers	3Kg	0.613Kg	3.5Kg	0.778Kg
Others	1.5Kg	0.306	3Kg	0.668Kg

The following table 5-2-2 shows the distribution of individuals who use charcoal and dry wood:

Daily energy source	Charcoal	Dry wood (Kitay)
Rural population	80%	20%
Urban population	90%	10%

Knowing that the consumption of wood per person concerns charcoal, dry wood for cooking, paper and others, that is a_r the consumption of wood by one person per month in rural areas (respectively a_u in urban areas).
Using the data from Tables 5-2-3 and 5-2-3, we have:

$a_r = \left(\frac{80}{100}.91{,}837 + \frac{20}{100}.55{,}102 + 0{,}613 + 0{,}306\right)12 = 1024.9$Kg/individual/year
$a_u = \left(\frac{90}{100} \times 50 + \frac{10}{100} \times 66.667 + 0.778 + 0.668\right)12 = 637.3524$ Kg/individual/year
According to the report on data collection and analysis for sustainable forest management (reference: joining national and international efforts EC-FAO partnership program (1998-2002)/Budget line tropical forest B7-6201/97-15/ VIII/FOR PROJET GCP/INT/679/EC/Revue/ avril 1999), on average, a dry wood weighs 350Kg and a wood occupies on average 1m² of surface.
According to this standard, table 5-2-3 shows the results of calculations on the different types of wood consumption per person per year, according to their localities:

Areas	Consumption Kg/individual/year	Area (in m²) of forest /individual/year	Forest area (in ha) /individual/year
Rural	1024.9	2.928286	0.0002928286
Urban	777.3552	2.221015	0.0002221015

According to the PRD-HM-2016 version, on average, 19% of the population of the Haute Matsiatra region is in the urban area, of which 81% is in the rural area. Figure 5-2-1 shows the curve of the estimated size of F_n between 2000 and 2056:

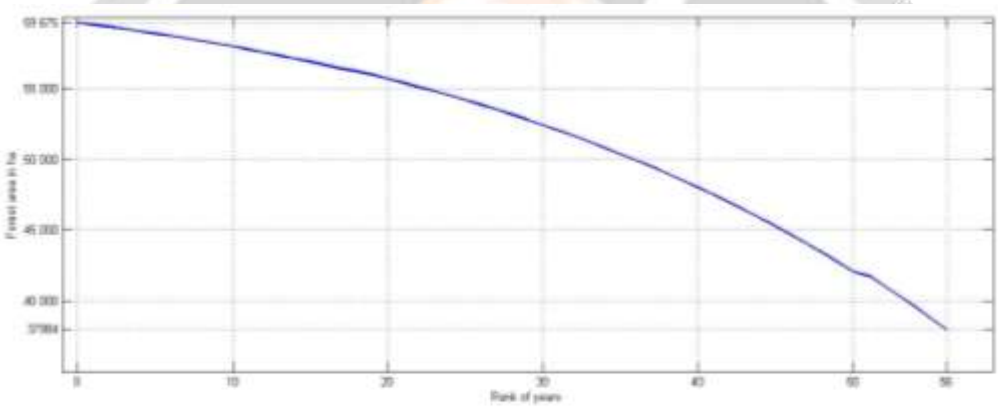

Figure 5-2-1: Curve of the dynamics of the forest area between the year 2000 (n=0) and the year 2056 (n=56), $F_0 = 59675\ ha$ *(Source: MDG/ INSTAT-RGPH 2018)*

The following figure 5-2-2 shows the curve of the dynamics of F_n population growth in the Haute Matsiatra region (according to the curve in Figure 5-1-3):

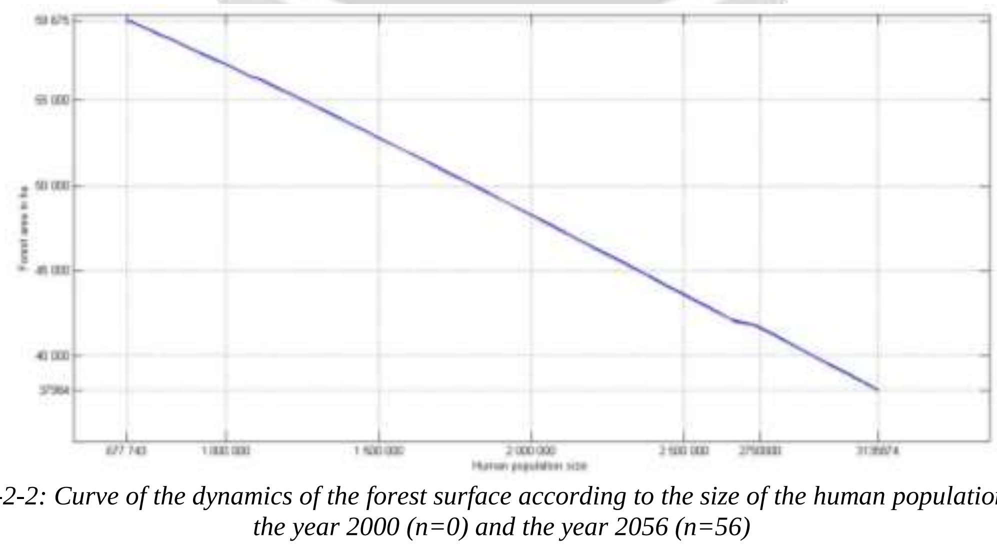

Figure 5-2-2: Curve of the dynamics of the forest surface according to the size of the human population between the year 2000 (n=0) and the year 2056 (n=56)

This curve shows that in 2056, the size of the human population is estimated at 3135874 and the forest reserve will be at 37984 ha.
Thus, it can be said that the growth in the size of the human population can lead to a rapid degradation of forest resources as long as the rate of reforestation is not yet rectified.

5-3. Confidence intervals:

The total area of the Haute Matsiatra region is $S_T = 2088000ha$and the cultivable area is $S_c = 503905ha$ (source: agricultural statistics service/DPEE/MIN AGRI, Madagascar). We assume that $s = 0.05\ ha/individu$, then $S(h_n) = 503905 + 0.05h_n$.
Because of the insufficiency of reliable data concerning the evolution of the size of the forest surface of the Haute Matsiatra region (since the year 2000), we are obliged to carry out the calculations of errors from the real size of the forest area in 2005, which is equal to 59453 ha (Source: JERIALA project/2008/mg_mef_monographie-region-haute-matsiatra_2014).

According to calculations on Matlab, we have $v(F_5) = 58904.984$. So $\varepsilon = 548.016ha$ and $\varepsilon_F = 0.00923 < 0.01$. Thus, we can say that the margin of error is acceptable.
We have $v(F_{22}) = F_{22} = 55126.554ha$ (estimated size of forest area in 2022).
Let $p = 22$ (year 2022) and $d = 50$ (year 2050). The following figure 5-3-1 shows the curves of the minimum reforestation areas:

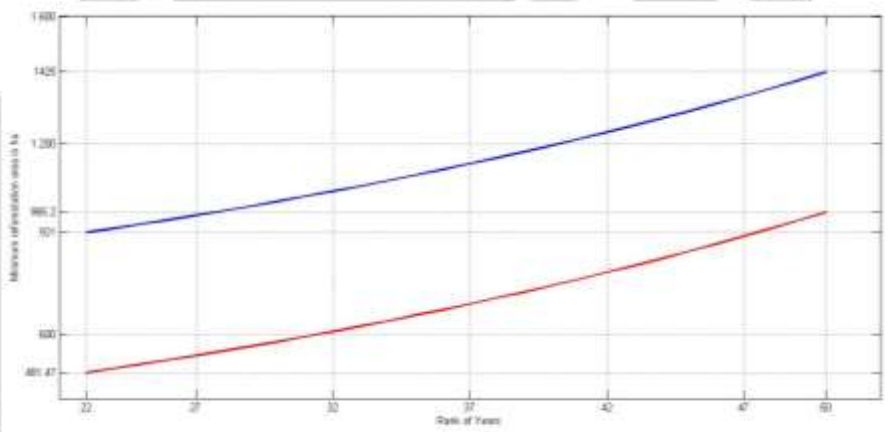

Figure5-3-1: curves of $\omega_{inf}(n)$ and $\omega_{min}(n)$, $s = 0.05ha$ and $\varepsilon = 548.016ha$
The red curve indicates the pace of $\omega_{inf}(n)$while the blue curve is that of $\omega_{min}(n)$.

This curve shows that in 2050 ($n = 50$), the minimum reforestation area for conservation is 1425ha. That is to say that population growth requires major reforestation efforts.
Thus, to preserve at least the forest area $F_{22} = 55126.554ha$ until 2050, then we must follow the pace of reforestation of the blue curve. In the event of financial difficulty (on the budgetary level), it is necessary to remain above the red curve to avoid the red trend.
The following figure 5-3-2 shows variation of the sequence $(\omega(n))$ and its minimum value:

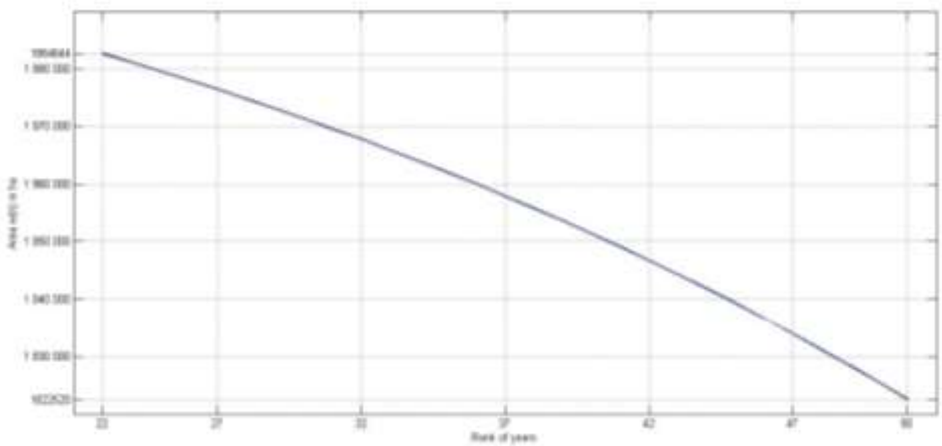

Figure 5-3-2: curve of $(\omega(n))$, $s = 0.05ha$ *and* $\varepsilon = 548.016ha$

This curve shows that the sequence $(\omega(n))$ is decreasing such that $\min_{n\in\{22,23,,\dots,50\}}\{\omega(n)\} = 1822519.815ha$ and $\max_{n\in\{22,24,\dots,50\}}\{\omega_{inf}(n)\} + \varepsilon = 985.2\ ha + 548.016\ ha = 1533.216ha.$

So for everything $n \in \{22,24,\dots,50\}, \omega_{inf}(n) < \omega(n)$. Thus, $\omega_{sup}(n) = \frac{1822519.815+\omega_{inf}(n)}{58}$.

The following figure 5-3-3 shows variation of the sequence $S_R(n+1)$:

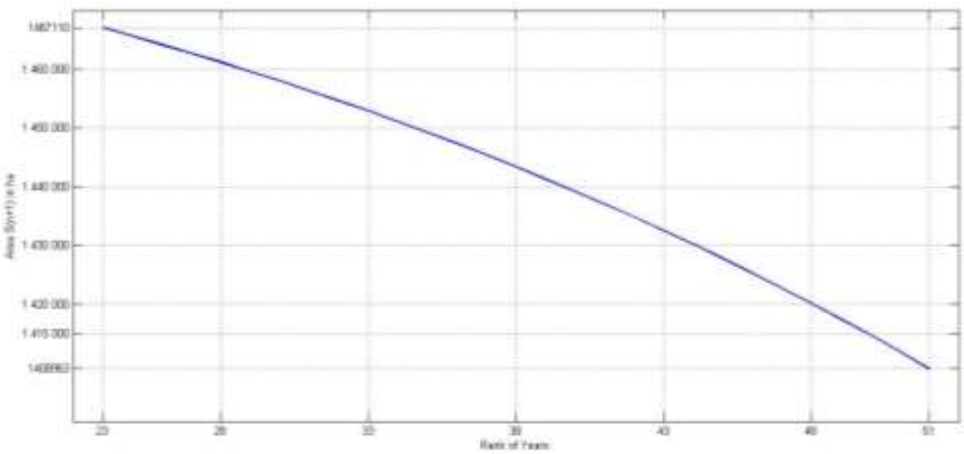

Figure 5-3-3: curves of $S_R(n+1)$, $s = 0.05ha and\ \varepsilon = 548.016ha$

This curve shows that the sequence $\left(S_R(n+1)\right)$ which characterizes the area of available reforestation is decreasing such that the area available in 2051 is $S_R(51) = 1409963ha$ while the area available in 2023 is $S_R(23) = 1467110\ ha$. That is to say that population growth constantly reduces the surface area of reforestation areas. Despite forest degradation, this result shows that if the size of the population increases, then the needs for housing and infrastructure also increase and that is why this phenomenon is quite natural.

The following figure 5-3-4 shows the curve of maximum reforestation areas $\omega_{max}(n)$:

Figure 5-3-4: curves $\omega_{max}(n)$, $s = 0.05ha$ *and* $\varepsilon = 548.016ha$

This curve shows that in 2050 ($n = 50$), the maximum reforestation area per year for the reasonable green trend increases to 31009.127ha while 31000.442ha in 2022. The latter shows that forest degradation due to population growth requires a gradual increase in efforts on reforestation activities.

According to the calculations on Matlab, if the reforestation plan takes the reforestation limit values $\omega_{sup}(n)$, then $\sum_{n=22}^{50} r_n = 880876ha$, as $S_R(51) = 1409963ha$. So, $\sum_{n=22}^{50} r_n < S_R(51)$and the latter shows that the area of reforested areas between 2022 and 2050 never exceeds the area available for reforestation between 2050 and 2051.

The following figure 5-3-5 shows shows the curve of ideal reforestation areas (for the certain green trend):

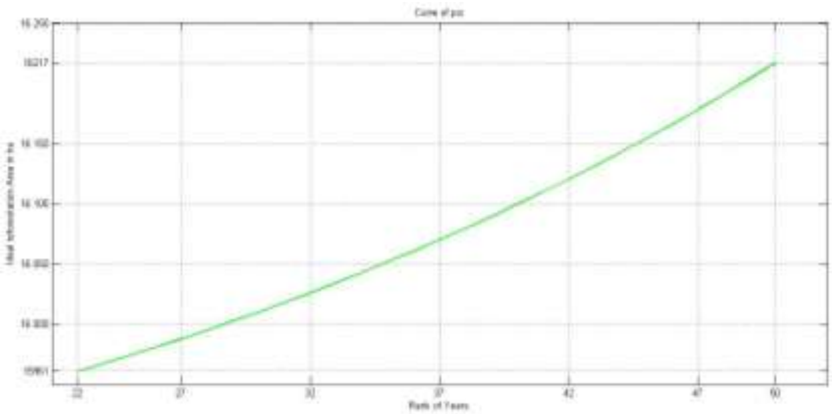

Figure 5-3-5: curve of $\psi(n)$, $s = 0.05ha$ *and* $\varepsilon = 548.016ha$

To guarantee the reasonable and certain green trend in 2050, then it is necessary to follow the neighborhoods of the shape of this curve and it represents the shape of the ideal reforestation surface.

5-4. Confidence interval of the ideal reforestation area for the Haute Matsiatra region

According to the calculations, $\overline{\omega}_{min} = 1104.178ha, \overline{\omega}_{max} = 29935.161ha$ and $\overline{\psi} = 15519.67ha.$

In Madagascar, it is difficult for the State to have a lot of funding for reforestation actions because of poverty. Moreover, the reforestation of $29935.161ha$ a year in a poor country like Madagascar is an almost impossible mission. On the other hand, this value shows that the region is vast and still exploitable despite population growth.

So, at the moment, the most logical option for the Haute Matsiatra region is to surpass or at least retain the forest cover of $55126.554ha$ (in 2022). Thus, to preserve the latter until 2050, it is necessary to reforest exactly $1104.178ha$ per year between 2022 and 2050.

As $\overline{\omega}_{min}$ is quite small in relation $\overline{\psi}$ (the area of the areas to be reforested is vast), and as the region is considered as regions in financial difficulty (because according to the PRD/HM_2014, only 170ha is the reforested area of the region between 2010 and 2011).

In this simulation, we choose $k = 2$ to have at least a progressive green trend and according to the calculations on Matlab, the confidence interval of the ideal reforestation for the Haute Matsiatra region is

$$[\overline{\omega}_{min}, 2\overline{\omega}_{min}] = [1104.178, 2208.356] \ (in \ ha)$$

For the choice $k = 2$, the following figure 5-4-1 shows the appearance of the reforestation confidence intervals, which suits the current situation of the Haute Matsiatra region:

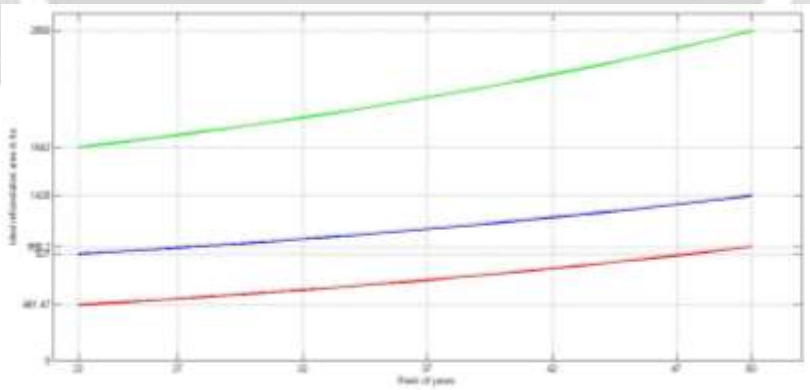

Figure5-4-1: the shapes of the reforestation confidence intervals, k=2 $\omega_{inf}(n)$ *the red curve,* $\omega_{min}(n)$ *la courbe bleuand* $2\omega_{min}(n)$*the green curve*

Thus, in the case of the Haute Matsiatra region, to avoid the red trend, in terms of reforestation, it is better to stay above the red curve. On the other hand, to have a progressive green trend, it is recommended to stay in the section between the blue curve and the green curve.

VI. CONCLUSION

To conclude, the results on the evolution of the forest area according to the dynamics of the human population show that population growth accelerates forest degradation and the conservation of forest resources has become a complex task. On the other hand, the results of the case of the Haute Matsiatra region can be used as decision-making tools for forest management in the face of population growth. Despite the region's financial problem and population growth, with this new approach, it is always possible to find the right decision to take thanks to the use of the confidence interval of the ideal reforestation area. In particular, for the Haute Matsiatra region, it is necessary to reforest at least $1104.178ha$ per year to guarantee the conservation of the forest cover $55126.554ha$ and the progressive green trend between the years 2022 and 2050. In general, given the population growth and whatever the economic problems and financial, it is still possible to gradually restore the parts ravaged by fire and by mining operations. To end this article, we propose as a perspective, the case study of Madagascar.

VII. REFERENCES

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