Double negative medium modeling using Drude model in FDTD method

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ABSTRACT

In electromagnetism, a material with a negative refractive index is a dispersive material called a double negative material (DNG). In order to numerically modeling this type of material, the FDTD method is used. To do this, the DB-FDTD formulation is used, and the dispersive medium is modeled by the Drude model. The numerical values used for the models are those of the plasmas of four noble metals. The frequencies of sources made up of sinusoids are defined so that the permittivity and permeability are equal to -1, so the reflection coefficient of the DNG medium is $n_r = -1$*. These electromagnetic parameters can be obtained with the Drude model by acting on the frequency. 1D simulations based on noble metals allow us to see the effect of the negativity of the refractive index of a material.*

Keyword: *DNG material, Numerical Modeling, DB-FDTD, Drude Model, Noble Metal, Negative refractive index*

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1. INTRODUCTION

The double negative material (DNG: Double-Negative Material) or simply metamaterial (MTM: Metamaterial)) is a dispersive material which was first introduced in theory by Veselago in 1968. An interface between vacuum and a medium made of DNG can produce negative refraction of light incident at the interface. Negative index materials do not exist in nature. However, artificially designed materials can act as DNG material. However, it should be noted that obtaining a DNG material requires both negative values of permittivity and permeability ($\varepsilon < 0$ and $\mu < 0$) over the same frequency range [1].

The goal of this paper is to model a DNG material with the FDTD method. To do this, the Drude model will be used for modeling the electrical and magnetic susceptibilities of a dispersive medium. The purpose being to be able to obtain negative values of the permittivity as well as of the permeability from the Drude model. The impact of the conductivity of DNG materials will be studied at the end of this paper.

2. IMPLEMENTATION OF A DNG MATERIAL

For DNG materials, the refractive index is $n_r < 0$. In order to model this type of material, in this work, the desired refractive index is $n_r = -1$ (Eq.1). In special cases of incident wave frequency range, dispersive materials may exhibit negative refractive indices. In order to simulate DNG materials, the ideal case is considered, i.e. the susceptibility functions are equal (Eq.2). The susceptibility functions are modeled using Drude's model for frequency-dependent materials [2].

$$n_r = \sqrt{\varepsilon_r \mu_r} = -1, \text{ then: } \varepsilon_r = \mu_r = -1 = j^2 \quad (1)$$

$$\hat{\chi}_e(\omega) = \hat{\chi}_m(\omega) = \frac{\omega_p^2}{\omega(j\nu - \omega)} \quad (2)$$

2.1. Construction of a DNG material with Drude model

For a DNG material consisting of Drude material, the permittivity as well as the permeability are defined in Eq.3, with $\varepsilon_\infty = 1$ and $\mu_\infty = 1$.

$$\hat{\varepsilon}_{r,D}(\omega) = \hat{\mu}_{r,D}(\omega) = 1 + \frac{\omega_p^2}{\omega(j\nu - \omega)} \quad (3)$$

Since making a DNG material requires that both permittivity and conductivity have the same variations in the frequency domain, the following will only discuss the treatment of relative permittivity. The real part of the relative permittivity is given in Eq. 4. For $\omega < \sqrt{\omega_p^2 - \nu^2}$, the relative permittivity is negative. And for $\omega_{DNG} = \sqrt{\frac{\omega_p^2 - 2\nu^2}{2}}$, the relative permittivity is $\varepsilon_r = -1$.

$$\Re\{\hat{\varepsilon}_{r,D}(\omega)\} = 1 - \frac{\omega_p^2}{\nu^2 + \omega^2} \quad (4)$$

For a medium constituted by a Drude material, there exists a frequency f_0 for which the permittivity is negative provided that $\omega_p^2 - \nu^2 > 0$. So the medium can have a refractive index $n_r = -1$ at the frequency f_{DNG}, it is necessary that $\omega_p^2 - 2\nu^2 > 0$.

2.2. DNG materials consisting of noble metal plasma

For noble metal plasma, the numerical parameters for a Drude model are given in Tab. 1 and the relative permittivity functions are illustrated in Fig. 1 [3][4]. Fig. 1 highlights the frequencies for the negative relative permittivity of each of the noble metals.

Tab.1: Frequency parameters of noble metal plasmas

	Plasma frequency	**Electron collision velocity**	**Frequency for $\Re\{\hat{\varepsilon}_r(\omega)\} = 0$**	**Frequency for $\Re\{\hat{\varepsilon}_r(\omega)\} = -1$**
	f_p (THz)	ν (THz)	f_0 (THz)	f_{DNG} (THz)
Silver	2 224.55	5.07	2 224.5	1 573
Gold	2 200.37	17.41	2 200.3	1 555.8
Copper	2 127.83	22.25	2 127.7	1 504.4
Aluminum	3 651.16	146.29	3 6482	2 576.7

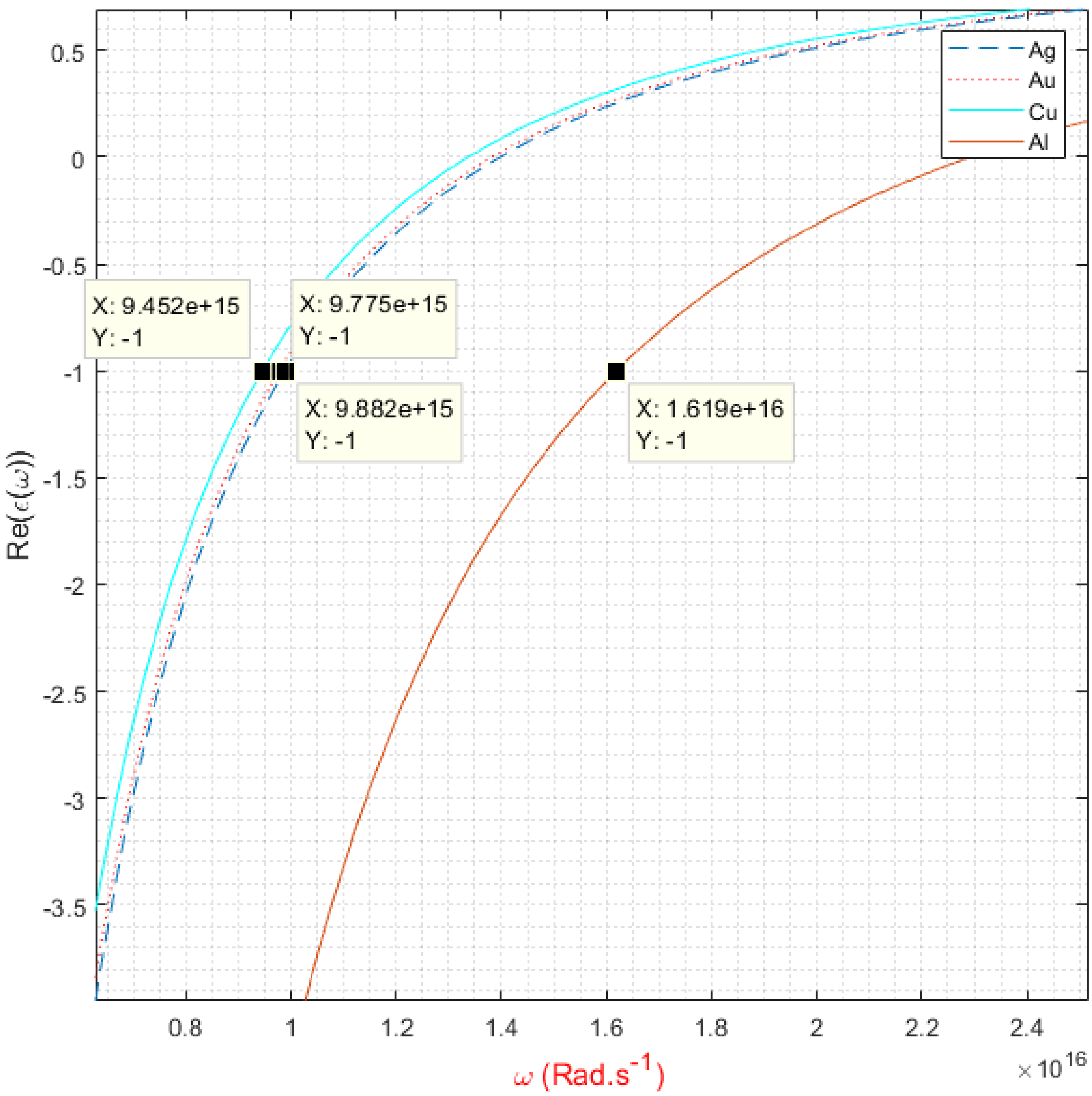

Fig.1: Relative permittivity of noble metal plasma as function of angular frequency

3. 1D FDTD SIMULATION OF A DNG MATERIAL USING THE DRUDE MODEL

3.1. 1D simulation parameters

The simulation uses the 1D FDTD method using the DB-FDTD formulation [5]. The grid is terminated on both sides by absorbent layers with progressive absorption coefficients [6]. In order to simulate a wave propagating only in the positive x direction, the TFSF formulation is used [7].

The parameters of the FDTD 1D grid, for simulations of a DNG material consisting of plasma are given in Tab. 2. The time steps and the space steps differ depending on the material chosen. The 400-cell grid is completed with absorbent layers of 15 cells. The source is a sinusoidal of frequency f_{DNG}, introduced by limit TFSF at node 30 of the grid. The layer of DNG material starts at node 160 and ends at node 240. The spatial resolution is defined by $n_\lambda = 40$, two wavelengths of the source are therefore contained in the DNG material medium.

Tab.2: 1D FDTD Grid Parameters for DNG Materials Made of Noble Metal Plasma

	Silver	**Gold**	**Copper**	**Aluminum**
f_{DNG} (THz)	1 573	1 555.8	1 504.4	2 577.6
Δt (fs)	1.5 10^{-2}	1.52 10^{-2}	1.57 10^{-2}	9.21 10^{-3}
Δx (nm)	4.76	4.82	4.98	0.9

3.2. Numerical results

Fig. 1 gives the snapshots of the electric field traveling through a vacuum and encountering a DNG material made up of silver plasma. Traveling through the DNG material, the wave propagates in the opposite direction. On leaving the medium made of DNG material, the wave is in a vacuum and propagates again in the normal direction of propagation that is to say in the direction of x positive.

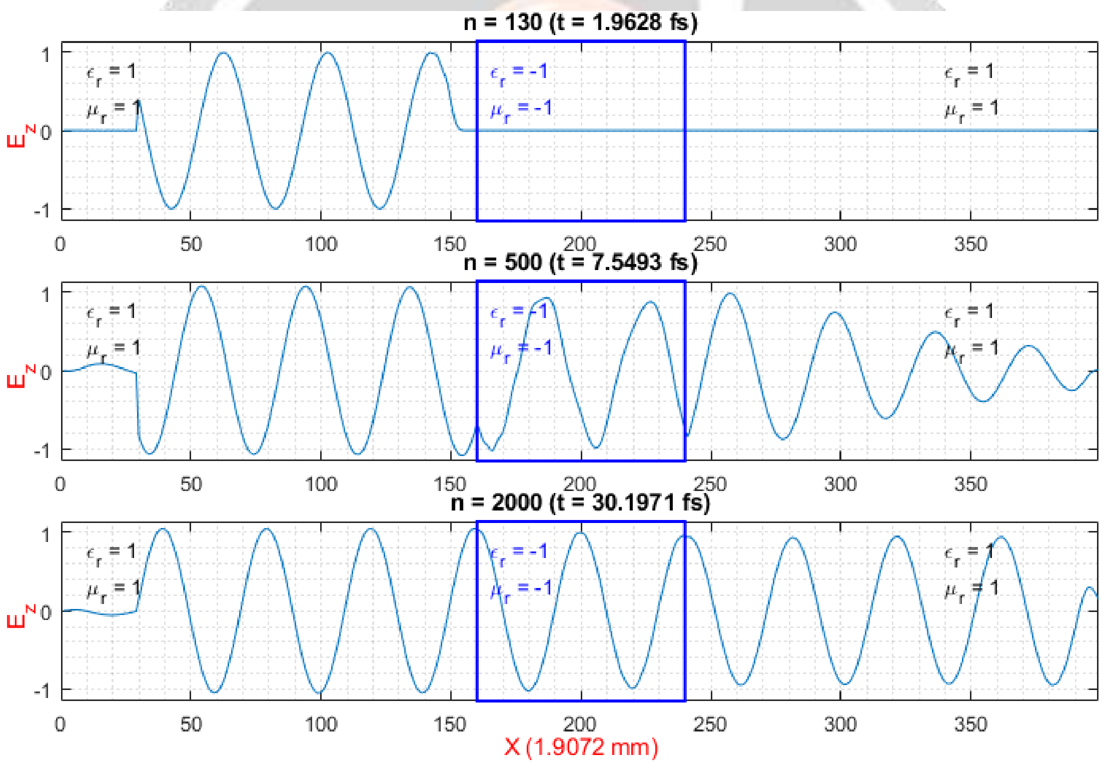

Fig.2: Snapshots of the electric field for a DNG Material with silver Plasma

Fig. 3, Fig. 4 and Fig. 5 respectively illustrate the instantaneous electric field for DNG materials consisting of gold plasma, copper plasma and aluminum plasma. Propagation in the opposite direction of the wave within DNG materials is also observed.

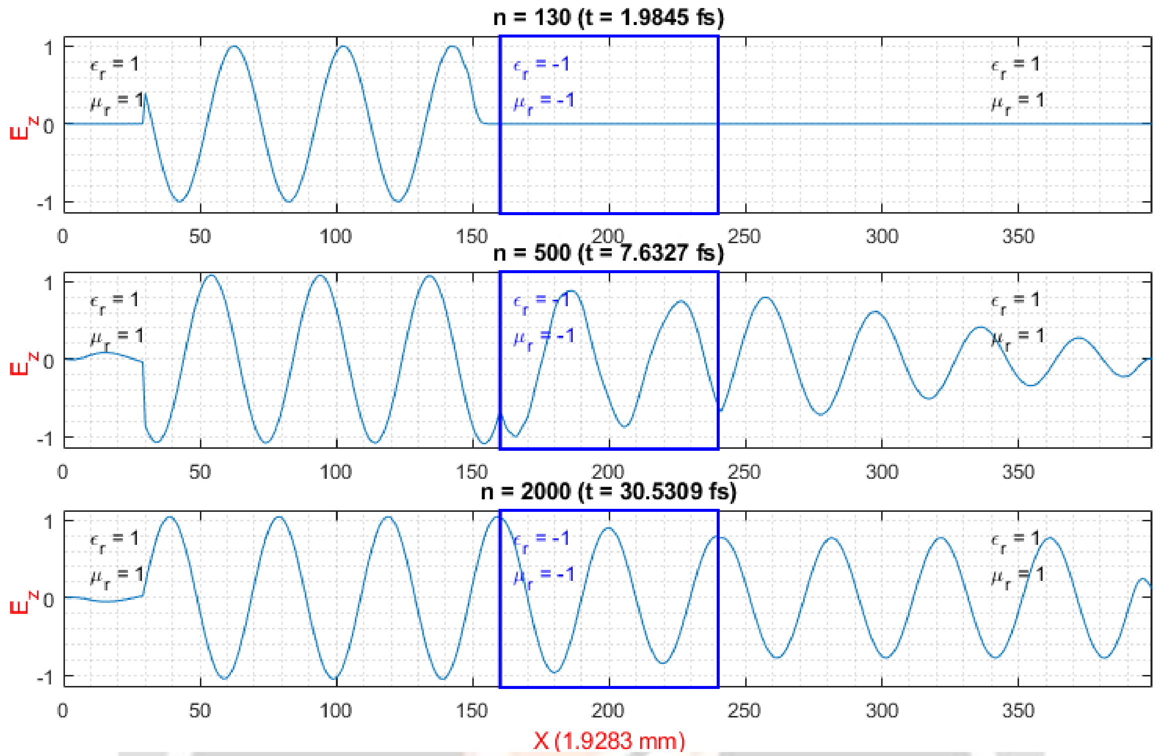

Fig.3: Snapshots of the electric field for a DNG Material with gold Plasma

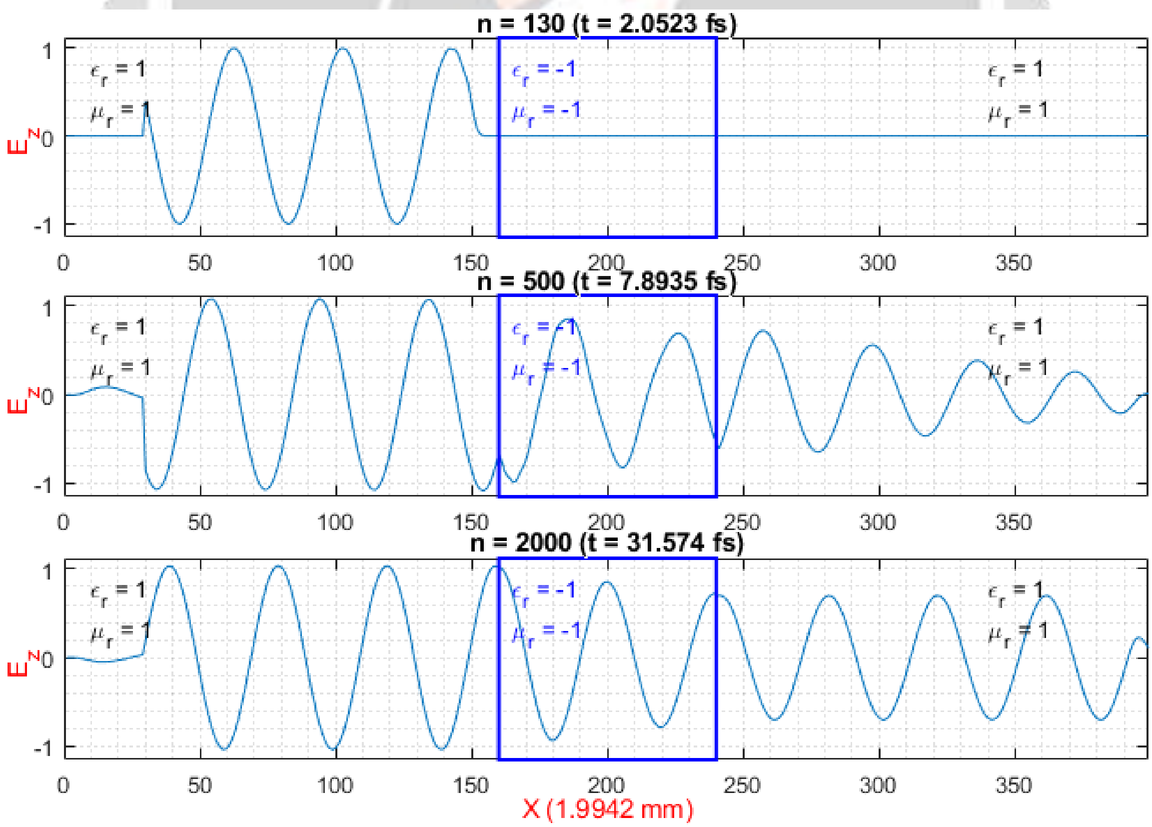

Fig.4: Snapshots of the electric field for a DNG Material with copper Plasma

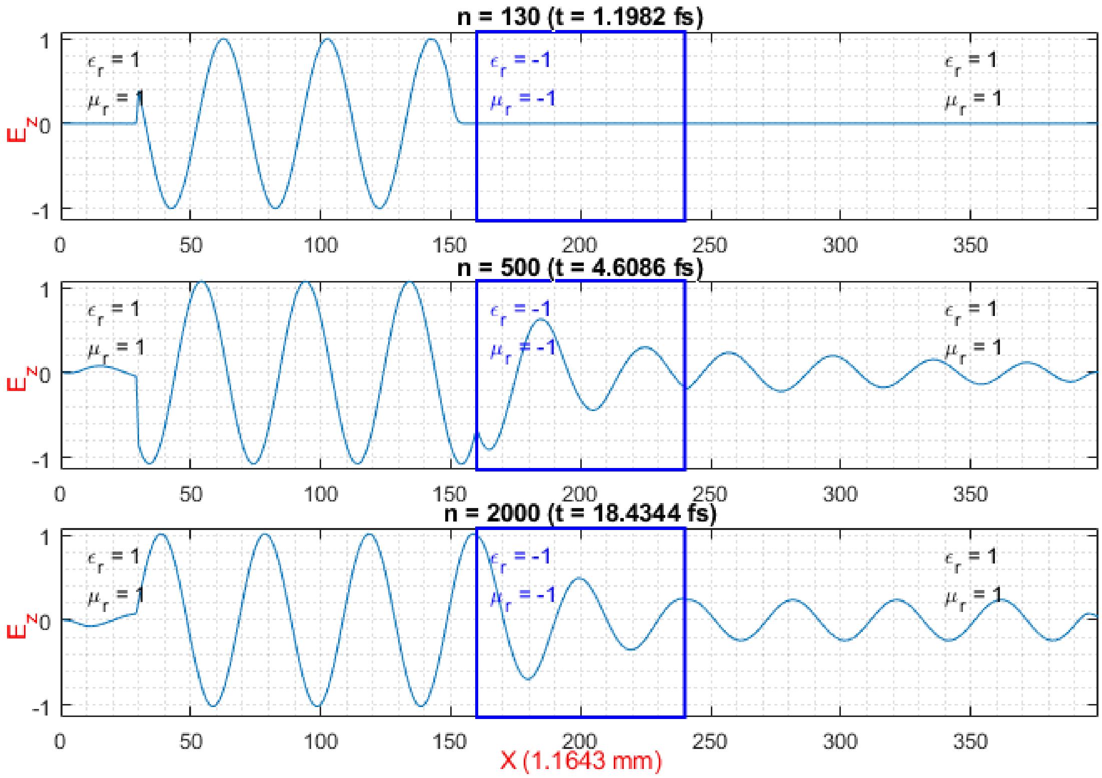

Fig.5: Snapshots of the electric field for a DNG Material with aluminum Plasma

4. IMPACT OF THE CONDUCTIVITY OF THE MATERIAL

In Fig.2, Fig.3, Fig.4 and Fig.5, the attenuation of the wave is observed at the outlet of the medium made of DNG materials. The DNG material has been designed so that ε_r and μ_r vary at the same time with the same values. According to the theory of perfectly matched absorbent layers, the medium behaves like an absorbent layer with a reflection coefficient close to zero (equal to zero for the continuous world, Eq. 5). The attenuation of the wave is proportional to the conductivity σ (Eq.6) of the material.

$$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1 - \sqrt{\frac{\mu_r}{\varepsilon_r}}\right)}{\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1 + \sqrt{\frac{\mu_r}{\varepsilon_r}}\right)} = 0 \tag{5}$$

$$\hat{\sigma}_D = \hat{\sigma}_e = \hat{\sigma}_m = \Re\{\hat{\sigma}(\omega)\} = \frac{\varepsilon_0 \omega_p^2 \nu}{\nu^2 + \omega^2} \tag{6}$$

Tab.3 shows the conductivity values of four noble metals. Silver plasma has the smallest conduction and therefore less wave attenuation. Aluminum plasma has the highest conduction, which is the highest absorption rate. This leads to the fact that a wave passing through the DNG material consisting of aluminum plasma is strongly attenuated.

Tab.3: Conductivities of plasmas formed by noble metals

	Silver	**Gold**	**Copper**	**Aluminum**
$\boldsymbol{\sigma\ (S.m^{-1})}$	564	1937	2476	16288

5. CONCLUSION

This paper concerns the modeling of DNG materials consisting of noble metal plasma. The numerical results confirm the robustness of the FDTD method with high precision results. The behavior of DNG materials conforms to Veselago's theory, because the materials reverse the direction of propagation of the incident waves and at the outputs the waves resume their normal propagations. At the limits of the medium made of DNG material, the wave behaves as if it encountered an absorbent layer perfectly matched to the vacuum. At the left limit, a tiny part of the wave is reflected at the interface. At the left limit, the wave is attenuated due to the crossing of the medium of DNG materials.

This work is a first draft highlighting the efficiency of the FDTD method for modeling DNG materials. However, the medium considered here is only plasma. For the modeling of a DNG material consisting of a dielectric, other models of dispersive material should be considered.

6. REFERENCES

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