Dynamic Analysis of Viscoelastic Sandwich Beam

K. Srinivas[1], B. Kiran Kumar[2], B. Dasharatha[3],

[1] Assistant Professor, Department of Mechanical Engineering, JBIET, Moinabad, Telangana.
[2,3] Research Scholar, Department of Mechanical Engineering, JBIET, Moinabad, Telangana.

ABSTRACT

In this research work vibration analysis of a viscoelastic sandwich beam is studied. A finite element model will be developed for the three-layer viscoelastic sandwich beam. Sandwich beam is modelled using linear displacement field at face layer and non-linear displacement field at core layer, under harmonic loading viscoelastic core exhibits complex modules. The equations of the motion for the viscoelastic sandwich beam is derived by using Euler–Bernoulli beam theory. Specimens are modelled by varying taking core material as viscoelastic and face layers are isotropic and studied under the fixed-fixed, cantilever, simply supported and free- free boundary conditions for model analysis. The natural frequencies are obtained for various models are estimated by multi layered core sandwich beam theory. The response of the beam under various boundary conditions are performed by FEA. The obtained theoretical results are compared with the results obtained from FEA.

Keyword - *Vibration Analysis, Sandwich Beam*

1. INTRODUCTION

Vibration mainly influences the life of engineering structures and their performance and invariably, damping in structures influences its behavior. Many types of damping mechanisms have been developed over time to control the undesired vibration of structures. Basically damping refers to the extraction of mechanical energy from a vibrating system, mainly by converting the mechanical energy into heat energy by means of some dissipation mechanism. Mostly all m aterials exhibit some amount of internal structural damping. Most of the time it is not substantially effective to minimize the vibration around resonant frequencies. Hence, by bringing these materials in contact with the highly damped and dynamically stiffed material it is possible to control the vibration. In the last years many studies have been presented concerning the structural vibration reduction making use of passive damping control techniques by means of surface treatments with viscoelastic materials. This kind of vibration control technique is largely used nowadays for several industrial applications, such as aeronautical and automotive components. The free layer damping (FLD) and constrained layer damping (CLD) technologies are two of these viscoelastic surface treatments, consisting of adding a damping viscoelastic layer to the structural system. Specifically, FLD consists of adding that viscoelastic layer on a vibrating metallic base, and the configuration can be analysed as the flexural behavior of a two-layer beam.

VISCOELASTIC MATERIALS

A purely elastic material is one in which all the energy stored in the sample during loading is returned when the load is removed. As a result, the stress and strain curves for elastic materials move completely in phase. For elastic materials, Hooke's Law applies, where the stress is proportional to the strain, and the modulus is defined at the ratio of stress to strain. A complete opposite to an elastic material is a purely viscous material, also shown in Figure 1. This type of material does not return any of the energy stored during loading. All the energy is lost as "pure damping" once the load is removed. In this case, the stress is proportional to the rate of the strain, and the ratio of stress to strain rate is known as viscosity, μ. These materials have no stiffness component, only damping. For all others that do not fall into one of the above extreme classifications, we call viscoelastic materials. Some of the energy stored in a viscoelastic system is recovered upon removal of the load, and the remainder is dissipated in the form of heat. The cyclic stress at a loading frequency of ω is out-of-phase with the strain by some angle δ, (where 0 <δ< ¹/2). The angle f is a measure of the materials damping level; the larger the angle the greater the damping

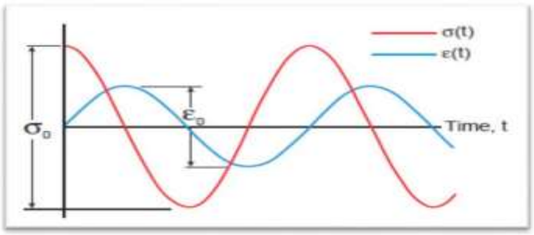


Fig.1.2 viscous material

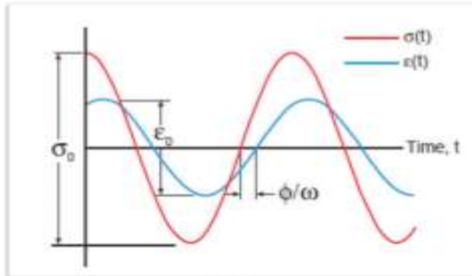


Fig.1.2.1 viscoelastic material

For a viscoelastic material, the modulus is represented by a complex quantity. The real part of this complex term (storage modulus, E') relates to the elastic behavior of the material, and defines the stiffness. The imaginary component (loss modulus, E'') relates to the material's viscous behavior, and defines the energy dissipative ability of the material. Using Hooke's Law to define the modulus for complex values, we can define the complex modulus, E*. Viscoelastic materials are one such that they are capable of storing strain energy when they are deformed; these types of materials exhibit the material characteristics of both viscous fluid and elastic solid. Viscoelastic damping property was exhibited by the large variety of polymeric materials ranging from synthetic/natural rubbers to various thermoset/thermostat materials used in different industries. Here polymers display rheological behavior intermediate between a simple fluid and crystalline solids, due to having tangled molecules and large molecular order. This type of viscoelastic materials offers a wide range of possibilities for developing a desire damping level provided by the designer to completely comprehend their mechanical behavior. In viscoelastic material the mechanical energy is released through normal deformation and cyclic shear.

There are mainly three methods of treatment of viscoelastic material viz., unconstrained layer or free layer treatment, constrained layer and partially constrained layer treatment. Depending upon the functional requirements in obtaining efficient properties of all layers sandwich structures utilizes the constrained layer treatment. In this constrained layer damping treatment, the viscoelastic material was sandwiched between the surface of structure and thin facings of elastic metallic materials. Normally Sandwich construction includes a relative thick core of low density material, sandwiched between the bottom and top face sheets (face layers) of relatively thin in size.

BEHAVIOUR OF VISCOELASTIC MATERIALS

One of the unique characteristics of viscoelastic materials is that their properties are influenced by many parameters. They can include: frequency, temperature, dynamic strain rate, static preload, time effects such as creep and relaxation, aging, and other irreversible effects. In working with this class of materials, we strive to define the materials complex modulus (stiffness and damping properties) as a function of these parameters. Most important of these include temperature and frequency effects. Viscoelastic materials are typically characterized as having the type of behavior

Viscoelastic materials are characterized by possessing infinite memory, i.e., their actual mechanical response is modulated by the past, based on superposition principle.

Loss factor = $E' \; E'' = \eta = tan \; \delta$

Where E'= storage modulus (elastic modulus)

E''=loss modulus (damping modulus)

δ= phase lag between stress and strain

σo = Stress;

ϵo = Strain Then complex (or dynamic) modulus of elasticity can be expressed as

$E* = E' + E''$

Where

$E*$= complex modulus= $\sigma \; \varepsilon$ = sinusoidal modulus

E' is the real part and E'' is the imaginary part.

The complex modulus is composed of the storage modulus E' (real part) and the loss modulus E'' (imaginary part)

2. LITERATURE

So many researchers contribute lot of work to predict ageing of viscoelastic materials under various parameters individually.Among those

Dr. Ferry et.al, Dr. Monsia et.al, V.S. Wani et.al did wast of work in this area. V.S. Wani et.al, study on dynamic mechanical analyser (DMA) to evaluate the dynamic mechanical properties and quantify the storage life of four different propellants based on hydroxyl terminated polybutadiene, aluminium and ammonium perchlorate of 3 years, 6 years and 10 years of time having different burning rates ranging from 5 mm/s to 25 mm/s. Each sample was given a multi-frequency strain of 0.01 per cent at three discrete frequencies (3.5 Hz, 11 Hz, 35 Hz) in the temperature range - 80 °C to + 80 °C. The storage modulus, loss modulus, tan delta and glass transition temperature (Tg) for each propellant samples have been evaluated and it is observed that all the propellants have shown time (freq uency) and temperature dependent behaviour on deformation conduct a series of experiments like strain and temperature ramp / frequency sweeps, creep, stress relaxation, etc. using high burning rate composite propellant (burn rate ~20 mm/s at 7,000 kPa), in order to determine the precise effects of such parameters on the results obtained. The evaluated data revealed that as the temperature increases the storage modulus, loss modulus, and tan delta curves with respect to the frequency shift towards the lower side. Moreover, there is equivalency between the increase in the temperature and the decrease in the frequency, which can be used for the time-temperature superposition principles. Further, in transient tests, the relaxation modulus has been found to decre ase when increasing strain levels in the given time range. Also, relaxation modulus versus time curves were found to shift towards the lower side with increasing temperature while creep compliance decreases with the increase in stress and decrease in temperature. The glass transition value of the composite propellant increases when there is an increase in the heating rate.

Fernando Corte´s et.al, suggested a relaxation function characterising viscoelastic materials whose storage modulus is constant with frequency, and whose loss factor shows the representative peak of damping materials. The new model gives a way to provide comparative data for different materials in a form which can easily be incorporated into simulations. The physical meaning of the model parameters is defined from the analysis of the complex modulus in frequency domain. The presented relaxation function is validated by curve fitting to experimental measurements carried out on polymer concrete specimens, made of epoxy resin matrix with min eral aggregates.

M. D. Monsia et.al proposed a one-dimensional nonlinear mathematical model consisting of a modified and extended Voigt model for the prediction of time dependent deformation response of a variety of materials exhibiting elastic, viscous and inertial nonlinearities simultaneously. Numerical examples are presented to investigate the effects of material parameters action on the model.

V.M Kulik et.al, introduced and improved method to measure the dynamic viscoelastic properties of elastome rs. The method is based on the analysis of forced oscillation of a cylindrical sample loaded with an inertial mass. No special equipment or instrumentation other than the ordinary vibration measurement apparatus is required. Upper and lower surfaces of the viscoelastic material sample were bonded to a load disc and a rigid base plate, respectively. The rigid base plate was subject to forced oscillations driven by a vibration exciter. Two accelerometers were attached to monitor the displacement of the base plate and the load disc. The recorded magnitude ratio and the phase difference between the load disc and the base plate vibrations represent the axial, dynamic deformation of the sample. The data are sufficient to obtain the dynamic properties of the sample, oscillation properties of vibration exciter, whereas the sensitivity of gauges having no effect on the calculation results. For accurate calculation of the properties, a two-dimensional numerical model of cylindrical sample deformation was used. It was shown that the modulus of elasticity and the loss tangent fall on a single curve, but the loss tangent curves showed some degree of scatter. Studied temperature dependence and nonlinear behavior of viscoelastic properties is found not to be associated with this effect. As the extension to **V.M Kulik et.al, Andrey V Boiko et.**al, invistigate complex poisons ratio as a function of frequency. In his work Standard composition (90% PDMS polymer + 10% catalyst) of silicone RTV rubber (Silastic S2) were used for prep aring three samples for axial stress deformation and three samples for shear deformation. Comprehensive measurements of modulus of elasticity, shear modulus, loss factor, and both real and imaginary parts of Poisson's ratio were determined for frequencies from 50 to 320 Hz in the linear deformation regime (at relative deformations 106 to 104) at temperature 250C. In order to improve measurement accuracy, an extrapolation of the obtained results to zero load mass was suggested. For this purpose measurements with several masses need to be done. Different combinations of the samples with different sizes for the shear and stress measurements exhibited similar results. The proposed method allows one to measure imaginary part of the Poisson's ratio, which appeared to be about 0.04–0.06 for the material of the present study.

M.Suceska et.al, evaluate the mechanical changes of rocket propellants – sustainers, built in in-service antitank guided missiles systems, induced by natural ageing at ambient conditions during up to 35 years of storage. The mechanical and viscoelastic properties were tested using a dynamic mechanical analyser, an uniaxial tensile and compression tester, and a notch toughness tester. The results have shown that the changes of the studied mechanical and viscoelastic properties are evident, although the results of the tests are rather scattered (as a consequence of measuring uncertainty, different ageing histories of propellants, etc.) or changes of some properties are not too pronounced. For example, after 15 years of storage at ambient conditions the glass transition temperature increases for about 5 °C, the tan δ in the glass transition region decreases for about 5%, the storage and loss modulus at 25 °C increase for about 15%, Young modulus at 23 °C increases up to 30%, the notch toughness at -30 °C decreases up to 15%, etc. Muhammad

Majduddin et.al, Presents a novel technique to measure the stress strain response at the exposed surface of propellant grain using a miniature testing device. This specially designed device is able to measure the stress response while the propellant surface is compressed at a constant rate which cannot possible accurately by traditional techniques. This measured stress strain behavior is then correlated with the physical properties measured by routine tensile tests of the similar type of propellant which is aged artificially. It is observed that there exists an excellent correlation between the measured stress values by the sensor and physical properties measured by uniaxial tensile test. This nondestructive technique provides properties of propellant grains of all the motors in the batch comprehensively. The technique is safe as well as economical as compared to the traditional methods.

3. THEORY OF VIBRATION

The equations of motion of a beam are derived according to the Euler–Bernoulli, Rayleigh, and Timoshenko theories. The Euler–Bernoulli theory neglects the effects of rotary inertia and shear deformation and is applicable to an analysis of thin beams. The Rayleigh theory considers the effect of rotary inertia, and the Timoshenko theory considers the effects of both rotary inertia and shear deformation. The Timoshenko theory can be used for thick beams. The equations of motion for the transverse vibration of beams are in the form of fourth-order partial differential equations with two boundary conditions at each end. The different possible boundary conditions of the beam can involve spatial derivatives up to third order. The free vibration solution, including the determination of natural frequencies and mode shapes, is considered according to Euler–Bernoulli theory.

3.1 NATURAL FREQUENCY AND MODE SHAPE OF A BEAM

Consider the free-body diagram of an element of a beam shown in fig. 3.1. The bending vibrations of a beam are described by Euler-Bernoulli Beam theory the fundamental equation is

$$EI\,\frac{d^4y}{dx^4} + \rho A\,\frac{d^2y}{dt^2} = 0 \qquad \text{....(Eq 3.1)}$$

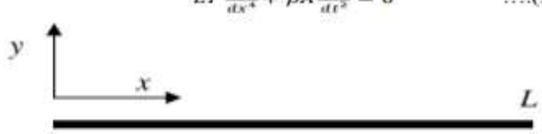


Figure. 3.1 Beam of Uniform Thickness

Where E, I, ρ, A are respectively the Young Modulus, second moment of area of the cross section, density and cross section area of the beam. L is the length of the beam. The solution of Eq. 3.1 can be written as a standing wave* y(x, t) = v(x) T(t) , separating the spatial and temporal component. This leads to the following characteristic equation that relates the circular frequency ω to the wavenumber kn : then equation (1) becomes

$$\frac{EI}{\rho A}\frac{d^4v}{dx^4}T(t) - v(x)\frac{d^2T}{dt^2} = 0 \qquad \text{....(Eq 3.2)}$$

3.2 MATERIAL PROPERTIES AND NATURAL FREQUENCIES OF A SANDWICH BEAM

Material properties, length, width and thickness of all layers are same for a symmetric sandwich structure as shown in

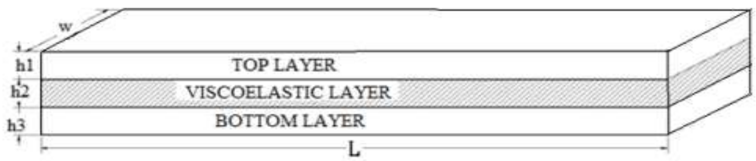


Figure: 3.2 Viscoelastic sandwich beam

The sandwich beam model described here based on the following assumptions

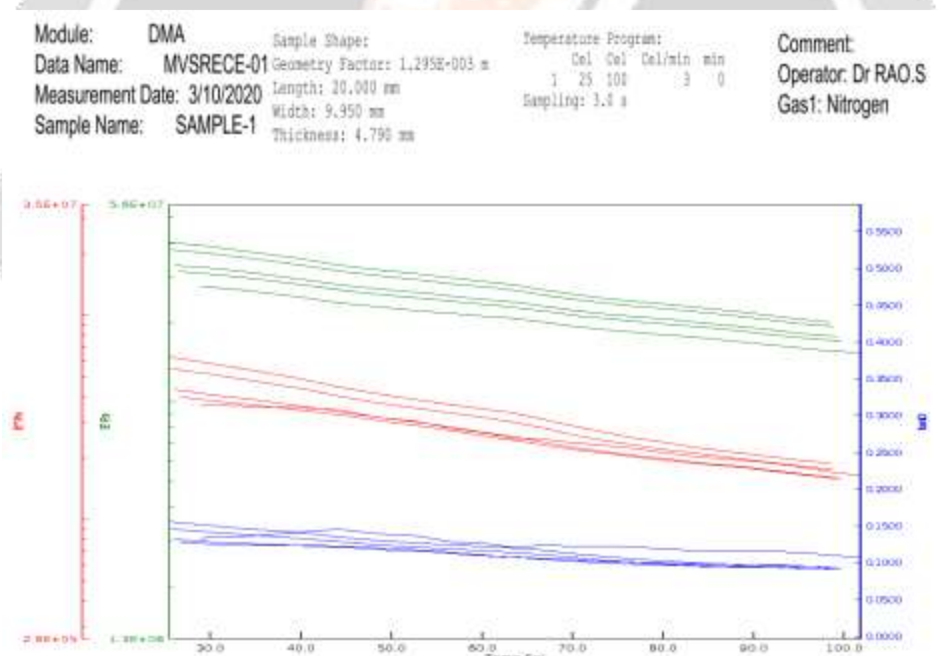
1. Top and bottom layers are considered as ordinary beams with axial and bending resistance.
2. The core layer carries negligible longitudinal stress, but takes the nonlinear displacement fields in x and z directions.
3. All the three layers are assumed to be perfectly bonded and there is no slippage between the layers.
4. Transverse displacements of top and bottom layers equal transverse displacement of core at interfaces. The sandwich beam considered here consists of three layers with viscoelastic material as a core layer, the top and bottom layers are isotropic and linear elastic material with thickness hf. The viscoelastic core layer has a thickness of hc under harmonic loading exhibits complex modulus in the form of $Ec = E'$ (1 + iη) where η is the loss factor. For static analysis, static elastic modulus of viscoelastic material is static modulus which can be calculated form

$$\frac{Storage\ modulus}{\cos\delta} \qquad \text{....(Eq 3.26)}$$

Where δ = phase shift. For dynamic analysis storage modulus can be used as elastic modulus. The sandwich concept is based on two main ideas: increasing the stiffness in bending of a beam or a panel and doing so without adding excessive weight. Here, sandwich beam structures may refer to multi layered structures wit symmetrical cross-sections. Suppose the considered sandwich structure is generated by a periodic distribution of unit cells, each of which consists of upper face, soft core and lower face which is consider to be the upper face of the second cell and so on as presented in fig. 3.2. To figure out the problem, consider firstly the bending of a multi layered beam with (nc) number of cores. The general term for bending stiffness is the flexural rigidity (D), which is the product of the material(s) elastic modulus(E), and the crosssection moments of inertia (I).

4. RESULTS AND DISCUSSIONS

Module	DMA			
Channel	1			
Data Name	MVSRECE-01			
Measurement Time	3/10/2020 3:07:40 PM			
Sample Name	SAMPLE-1			
Sample Shape	Geometry Factor	0.001295	m	
	Length	20	mm	
	Width	9.95	mm	
	Thickness	4.79	mm	
Temperature Program		Cel	Cel	Cel/min
	1	25	100	3
	Sampling	3	s	
	Temperature Program Mode	Ramp		
Measurement Mode	Bend (Dual Cantilever)			
Meas. Frequency Information				
	Meas. Frequency	5	Frequency	
		0.1	Hz	
		0.5	Hz	
		1	Hz	
		5	Hz	
		10	Hz	



THEORETICAL CALCULATIONS:

Free vibration analysis was carried out on various viscoelastic sandwich beams of rectangular cross section using different edge conditions like clamped-free, clampedclamped, simply supported and free - free boundary conditions. The materials properties are considered for face and core layers are listed below. Material properties of Steel and NBR (Nitrile Butadiene Rubber) as follows:

Young's Modulus of steel (f) = 2 × 105 N/mm3

Young's Modulus of NBR (c) = 4 N/mm3

Density of steel (f) = 8.05 × 10−6 Kg/mm3

Density of NBR (c) = 9.3 × 10−7 Kg/mm3

Thickness of steel (hf) = 0.45 mm

Thickness of NBR (hc) = 5 mm

Width (b) = 35 mm

Length (L) = 300 mm

Equivalent mass :-

$$m_e = 2\,[bh_f\rho_f] + n_c bh_c\rho_c = b(2h_f\rho_f + h_c\rho_c)$$
$$= 35 \times [(2 \times 0.45 \times 8.05 \times 10^{-6}) + (5 \times 9.3 \times 10^{-7})]$$
$$= 35 \times [\,0.000007245 + 0.0000465]$$
$$= 18.810 \times 10^{-4}\ \mathrm{Kg}/mm$$

flexural rigidity (D) :-

$$D = 2\frac{E_f bh_f^3}{12} + \frac{E_c bh_c^3}{12} + \frac{E_f bh_f}{2}\,(h_f + h_c)^2$$
$$= \left[\frac{2 \times 2 \times 10^5 \times 35 \times 0.45^3}{12}\right] + \left[\frac{4 \times 35 \times 5^3}{12}\right] + \left[\frac{2 \times 10^5 \times 35 \times 0.45 \times (0.45 \times 5)^2}{2}\right]$$
$$= 1{,}06{,}312.5 + 1458.33 + 46781437.5 = 468.892 \times 10^5$$

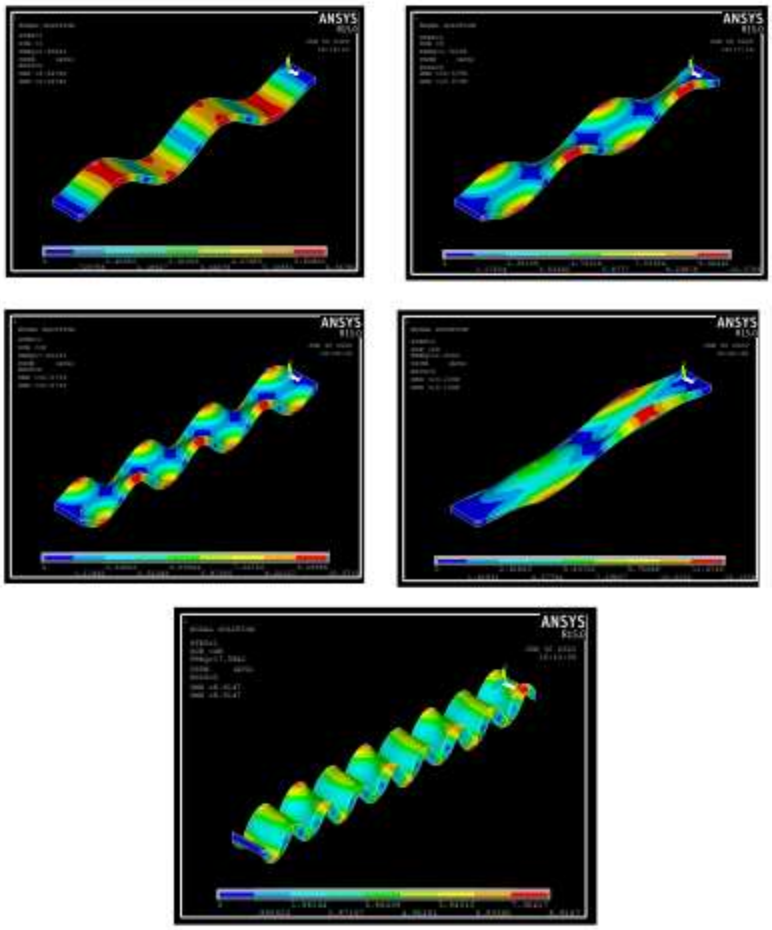
Equivalent stiffness :-

$$K_e = \frac{3D}{L^3}$$
$$= \left[\frac{468.892 \times 10^5}{300^3}\right] = 5.21$$

THEORETICAL VALUES:

BEAMS	NATURAL FREQUENCY				
	1st freq (Hz)	2nd freq (Hz)	3rd freq (Hz)	4th freq (Hz)	5th freq (Hz)
Free – Free	1.3348	3.675	7.213	11.92	17.8125
Clamped – Clamped	1.3348	3.675	3.675	11.92	17.8125
Clamped – Free	0.21	1.3146	3.6809	7.213	11.92
Simply - Supported	0.589	2.356	5.299	9.4216	14.721

CLAMPED - CLAMPED



RESULTS OF COMPARISON:

BEAM/FREQUENCY		1st(HZ)	2nd(HZ)	3rd(HZ)	4th(HZ)	5th(HZ)
Free – Free	THEORY	1.334	3.675	7.213	11.924	17.812
	EXP	1.518	3.671	7.514	11.971	17.846
Clamped – Clamped	THEORY	1.334	3.675	7.213	11.924	17.812
	EXP	1.558	3.722	7.06	12.00	17.584
Clamped – Free	THEORY	0.21	1.315	3.680	7.213	11.92
	EXP	0.172	1.312	3.552	7.268	11.971
Simply - Supported	THEORY	0.589	2.356	5.299	9.421	14.721
	EXP	0.679	2.623	5.525	9.573	14.947

5. CONCLUSIONS

The study of the project is based on development of the three layer viscoelastic sandwich beam with face layer as steel and core layer as NBR with pvc. By testing the viscoelastic material under Dynamic mechanical analyser, storage and loss modulus values are captured. The equation of the motion of vibration sandwich beam is derived by using Euler- Bernoulli theory. The Natural frequency is obtained for various models by using different Boundary condition i.e.., Free-free, Fixed -Fixed , Simply supported and cantilever Beams. The Experimental Results of the viscoelastic sandwich beam with different Boundary Conditions are performed by using FEA (Ansys APDL). The Experimental values from FEA by using different boundary conditions are compared to the theoretic al values and the results are approximately same.

6. ACKNOWLEDGMENT

We express our grateful thanks to our project guide Mr. K. Srinivas, Assistant Professor ME, JBIET for guiding us throughout the project by giving continuous feedback and suggestions which helped for the progress of the project

We take the opportunity in expressing our heartfelt gratitude to Dr. Anoop Kumar Shukla, Head of the Department, Mechanical Engineering, JBIET for allowing us to take up this project.

We take immense pleasure in the thanking Dr. P.C. Krishnamachari, Principal, J.B. Institute of Engineering and Technology for permitting us to carry out this project at our college.

7. REFERENCES

[1] Cui Huiru, Tang Guojin, Shen Zhibin, "A three-dimensional viscoelastic constitutive model of solid propellant considering viscoelastic Poisson's ratio and its implementation", European Journal of Mechanics A/Solids, Vol. 61 (2017) 235-244.

[2] Vilas Wani, Mehilal, Sunil Jain, Praveen Prakash singh, Bikash Battacharya, "Prediction of storage of propellants having direct burning rates using dynamic mechanical analysis" Defense science journal, Vol. 62, No 5, September 2012, pp. 290-294.

[3] Vilas Wani, Mehilal, Sunil Jain, Praveen Prakash singh, Bikash Battacharya, "Studieson the Influence of Testing Parameters on Dynamic and Transient Properties of Composite Solid Rocket Propellants Using a Dynamic Mechanical Analyzer" Journal of Aerospace Technology and Management, Vol.4, No 4, pp. 443-452, Oct.-Dec., 2012.

[4] Fernando Corte´s, Maria Jesus, Elejabarrieta "Modelling viscoelastic materials whose storage modulus is constant with frequency" International Journal of Solids and Structures 43 (2006) 7721–7726.

[5] M. D. Monsia "A Mathematical Model for Predicting The Nonlinear Deformation Response of Viscoelastic Materials" journal of applied mathematics and mechanics – (8) 16 (2012): pp 22-30.

[6] S.M.R. Khalili Free vibration analysis of sandwich beam carrying sprung masses International Journal of Mechanical Sciences 52 (2010) 1620–1633.

[7] J.R. Banerjee and A.J. Sobey Dynamic stiffness formulation and free vibration analysis of a three-layered sandwich beam International Journal of Solids and Structures 42(2005) 2181– 2197.

[8] M. Chehel Amirani Free vibration analysis of sandwich beam with FG core using the element free Galerkin method Composite Structures 90 (2009) 373–379.

[9] Fei Lin and Mohan D. Rao Vibroacoustical Analysis of Multiple-Layered Structures with Viscoelastic Damping Cores: Hindawi Publishing Corporation ISRN Mechanical Engineering Volume 2013, Article ID 645232, 13 pages.

[10] Farough Mohammadi and Ramin Sedaghati. Vibration analysis and design optimization of sandwich cylindrical panels fully and partially treated with electrorheological fluid materials. Journal of Intelligent Material Systemsand Structures 23(15) 1679–1697.

[11] Binod P. Yadav, Vibration damping using four-layer sandwich. Journal of Sound and Vibration. 317 (2008) 576–590.

[12] E.M. Daya, L. Azrar, M. Potier-Ferry. An amplitude equation for the non-linear vibration of visco elastically damped sandwich beams. Journal of Sound and Vibration 271 (2004) 789–813.

[13] F.S. Barbosa_, M.C.R. Farage. A finite element model for sandwich viscoelastic beams: Experimental and numerical assessment. Journal of Sound and Vibration 317 (2008) 91– 111.