# EVALUATION OF MICROWAVE PARAMETERS OF MESFET BY OPTIMIZATION

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**ABSTRACT**

*This article helps to understand the behavior and extraction of GaAs MESFET transistor model. The extraction of the model is carried out by the Simplex optimization Method, based on the S - parameters.*

***Keywords:*** *GaAs MESFET, S-Parameters, Modeling, Simplex Method, Equivalent Circuit.*

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## 1. INTRODUCTION:

The development of electronic technology marked the end of the 20th Century, thanks to the discovery of transistors around 1949. The mastery of the elements of groups III and V of the Mendeleïeva table made it possible to develop semiconductor compounds like gallium arsenide. These materials are used in GaAs MESFET technology. This article proposes to evaluate the parameters of a model with weak signals in the microwaves, from a modeling. The optimization method is the "Simplex Method".

## 2. CHARACTERISTIC OF GAAS:

### 2.1. CRYSTAL STRUCTURE:

In GaAs, the crystal is formed by a Ga atom surrounded by 4 As atoms forming a "tetrahedral" configuration [2] (Fig 01). This structure is called the "zinc-blende structure", and crystallizes in the face-centered cubic system.

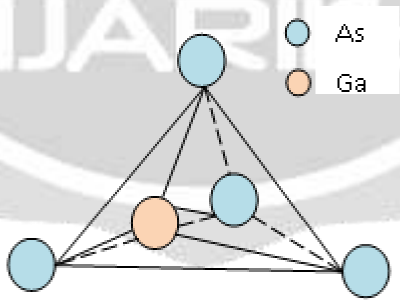



Figure 01: Tetrahedral bond in a GaAs crystal.

### 2.2. MOBILITY:

GaAs has greater mobility than Si, and under application of an electric field, more amplification is obtained for GaAs. This affects high frequency operation, as the current is modulated. Table 1 shows comparative values of electron-hole mobility in Ge, Si and GaAs semiconductors.

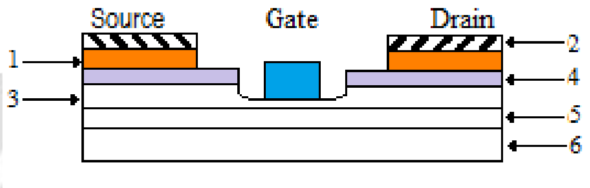
Table 1: Electron mobility / holes of Si, Ge and GaAs. [2]

| | $\mu_n$ [$m^2$/Vs] | $\mu_p$ [$m^2$/Vs] |
|---|---|---|
| Ge | 0,39 | 0,09 |
| Si | 0,15 | 0,06 |
| GaAs | 0,85 | 0,04 |

In this table, electrons have a greater mobility than holes and that those of GaAs are more mobile than those of Si or Ge.

### 3. i-v CHARACTERISTIC OF GAAS MESFET:

In the MESFET, the charge carriers come from the source to the drain through a channel. It is a doped region in the semiconductor. The flux of the carrier is controlled by the voltage applied to the gate which is a reverse biased Schottky diode. Figure 02 shows a cross section of a low gate MESFET.




1 Ohmic contact 2 Metal layer
3 Channel n 4 Layer n +
5 Buffer 6 GaAs Substrate

Figure 02: Section of GaAs MESFET [3].

Two models that describe the i-v characteristic are presented in this section. First, Schockley's description [4], using a graduated channel approximation to simplify the two-dimensional Poisson equation. Then the Liechti model [5] which predicts the formation of a dipole in the channel when the channel is saturated.

#### 3.1. ANALYSIS OF SCHOCKLEY'S THEORY [4]:

The gate is reverse biased and below it a depletion region forms. The gradual channel approximation reduces the Poisson equation to one dimension:

$$\frac{dE_y}{dy} = -\frac{d^2\psi}{dy^2} = \frac{\rho}{\varepsilon}$$

In Figure 03, the depletion thickness can be determined on the drain $x_2$ and on the source $x_1$:

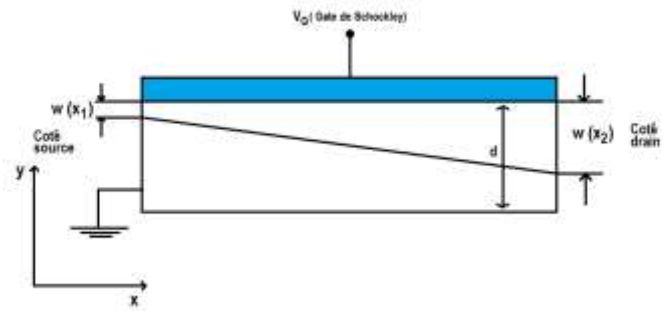



Figure 03: MESFET depletion region.

For a MESFET, the channel is pinched when w ($x_2$) = d and the pinch voltage is reached:

$V_p = V_D + |V_G| + V_{bi} = \frac{q\, d^2\, N_D}{2\varepsilon_s}$

with

$\varepsilon_s$ the permittivity of the semiconductor
q the charge of the electron
$N_D$ semiconductor doping
$V_D$ the voltage applied to the drain
$| V_G |$ that of the gate, and
$V_{bi}$ the internal voltage.

The length L of the MESFET is defined as the direction of current flow. When the transistor is polarized, we have the pinch current $I_p$:

$$I_p = \frac{z\, q^2 N_D^2 \mu_n d^3}{6\, L\varepsilon_s}$$

$\mu_n$ the mobility of the electron
z the width of the gate

The MESFET is in the saturation region when the depletion region is pinched. The saturation current is determined by evaluating $I_D$ at point

$V_p = V_D + | V_G | + V_{bi}$

And we get:

$$I_{Dsat} = I_P \left[ \frac{1}{3} - \left( \frac{|V_G| + V_{bi}}{V_P} \right) + \frac{2}{3} \left( \frac{|V_G| + V_{bi}}{V_P} \right)^{\frac{3}{2}} \right]$$

$I_{Dsat}$ remains constant until the onset of the "avalanche phenomenon", created by the increase in the voltage of the drain, causing an increase in the energy of the electron. These electrons collide with the atoms of the semiconductor lattice and in turn will create pairs (electron-hole). This results in a sharp increase in the drain current.

### 3.2 LIECHTI'S MODEL [5]:

For this model, the expression of the drain ID current begins in the channel as:

$$I_D = z\, q\, n(x)\, v(x)\, y(x)$$

n (x) doping in the channel
v (x) the speed of carriers (electrons), and
y (x) the thickness of the conductive channel (i.enon depopulated).

The gate is biased and the drain voltage is gradually increased. The depletion thickness is greatest near the drain since the potential is highest on this side. When the electric field E increases, v(x) increases and therefore $I_D$ increases.

When E reaches the critical value, the speed being saturated, no longer increases and the unpopulated channel becomes very thin. According to this model, a constant current is maintained by introducing more negative charge into this region of the channel. Negative and positive charges form a dipole layer (Fig. 04).

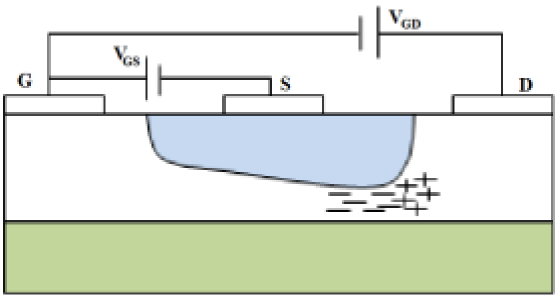


Figure 04: Liechti model.

### 4 SMALL SIGNAL OPERATION MODEL:

An equivalent circuit of a device is useful to be able to understand the behavior of this device in a circuit, then to "predict" the performance of this circuit before reaching "the drawing table".

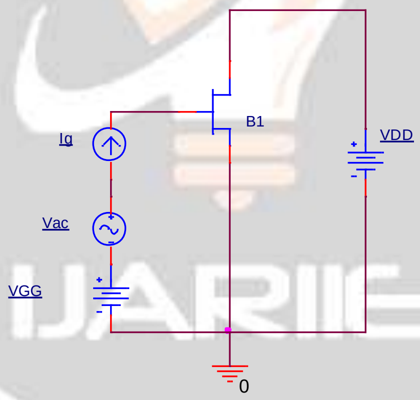
Modeling an equivalent circuit does not require knowledge of the doping of the transistor. The models are classified into two groups: the weak signal model and the strong signal model.

The small signal models describe the operation of the transistor in the saturation region. A small change in the gate-source voltage produces a linear change in the drain-source current. The application limit point is the point where the variation in the drain current is no longer linear, for a given gate-source voltage value.

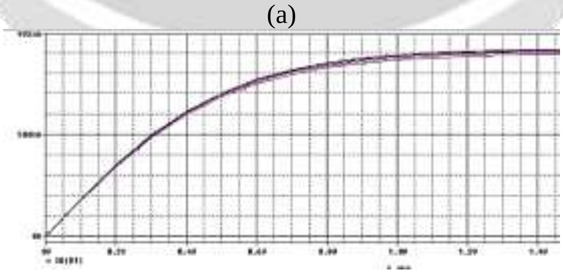
In this model, the elements of the circuit no longer vary either with the voltage or with the working frequency. An equivalent circuit of the microwave MESFET, a general model, simply consists of intrinsic devices with parasitic inductors, capacitances and resistances.

### 4.1 MESFET POLARIZATION:

In this report, the common source equivalent circuit model will be examined. The transistor is in the saturation region and the idle operating point (point Q) is far from the pinch region.
A sinusoidal signal is applied such that the transistor does not go out of the saturation region when the signal changes alternations. Figure 05 (a) illustrates the MESFET mounted as a common source and shows the existence of an AC signal at the gate, and in Fig. 05 (b), the iv characteristic of this device and the sinusoidal voltage around the point Q which is in the saturation region.




(a)



(b)

Figure 05: (a) common source bias, (b) voltage in small signals applied to the saturated MESFET, at the gate.

### 4.2 GENERAL MODEL:

The general model, in Fig. 06, is the most frequent model [6], [7], [8], and illustrates the physical significance of each parameter.

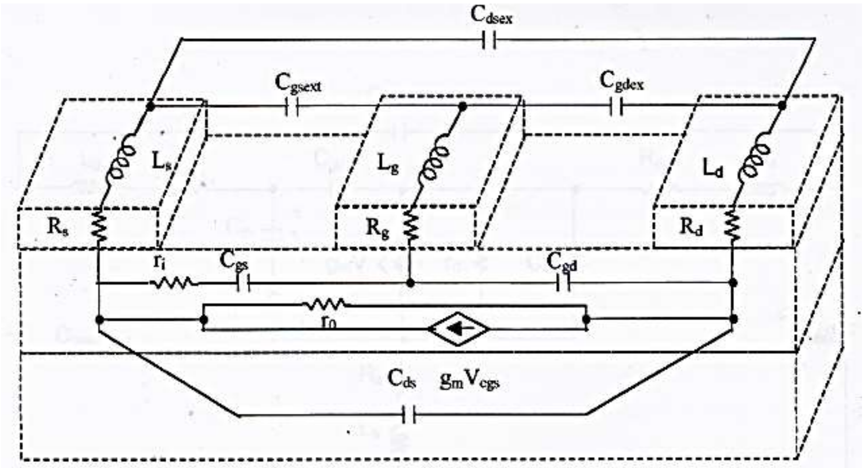



Figure 06: Physical elements of the equivalent circuit [6].

For the proposed analyzes, the MESFET is divided into two sections: intrinsic and extrinsic systems. The intrinsic model is made up of a Schottky diode and a doped semiconductor which forms the channel. The extrinsic model adds the parasitic elements associated with the conductors and contacts of the metal.

In the MESFET, we have two feedback loops, via the drain-gate capacitance and via the parasitic impedance of the source. The feedback inductance increases stability below 8GHz, but decreases it above this value.

### 4.3 SIMPLIFICATION OF THE GENERAL MODEL:

For this model, the impedance between the drain and the gate is assumed to be infinite and the parasitic resistances and inductances are neglected.

The first assumption is due to the fact that Cgd is very small compared to Cgs, and in low frequencies this impedance is considered to be infinite. Then, for the sake of simplification, the parasitic (weak) resistances have been omitted (Fig. 07).

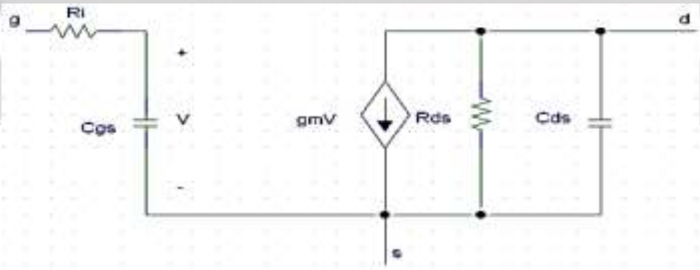



Figure 07: General model simplified in small signals [9].

## 5 METHOD OF EVALUATION OF THE ELEMENTS OF THE EQUIVALENT CIRCUIT:

This work describes a technique for extracting equivalent circuit parameters from measured S parameters. First, the intrinsic Y parameters are determined, then a Z matrix transformation to obtain the parasitic resistances, followed by a re-transformation into Y parameters to include the parasitic capacitances [6]. [10], [1]. Finally, the Y parameters are converted into S parameters and they are compared with those measured. The method used in this draft is the “Simplex Method” [1].

### 5.1 EXTRACTION OF THE INTRINSIC CIRCUIT:

The intrinsic circuit (Fig. 08) is that of the MESFET without the parasitic elements and containing the following elements: the gate-source capacitance $C_{gs}$, the load resistance $r_i$, the inter-electrode capacitance between the gate and the drain $C_{gd}$, the parameter $g_m$ associated with the generator voltage controlled current, the delay time τ associated with the change in the thickness of the depletion region, the output resistance of the channel $r_0$ and the drain-source capacitance $C_{ds}$.

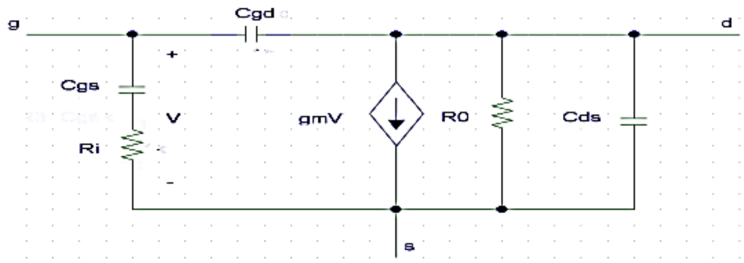



Figure 08: Intrinsic MESFET [6]

The elements of the equivalent circuit can be expressed in terms of parameters Y according to the representation of a quadrupole in Fig. 09.

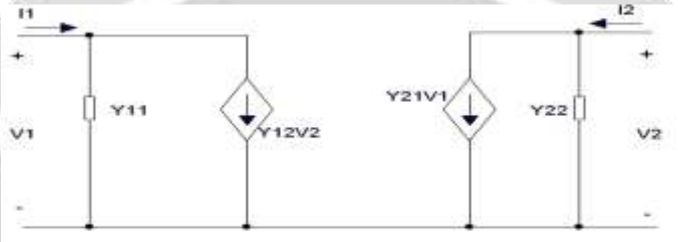



Figure 09: The Y parameters for a quadrupole.

The equations from this figure are:

$$\begin{cases} i_1 = Y_{11} + V_1 + Y_{12} + V_2 \\ i_2 = Y_{21} + V_1 + Y_{22} + V_2 \end{cases}$$

Short-circuit measurements are adopted to determine the parameters:

$$Y_{11} = \frac{i_1}{V_1}\bigg| V_2 = 0$$

$$Y_{11} = \left(\frac{r_i^2 w^2 c_{gs}^2}{1+w^2 c_{gs}^2 r_i^2}\right) + jw\left(\frac{c_{gs}}{1+w^2 c_{gs}^2 r_i^2} + c_{gd}\right)$$

$$Y_{12} = \frac{i_1}{V_2}\bigg| V_1 = 0$$

$$Y_{12} = jwc_{gd}$$

$$Y_{21} = \frac{i_2}{V_1}\bigg| V_2 = 0$$

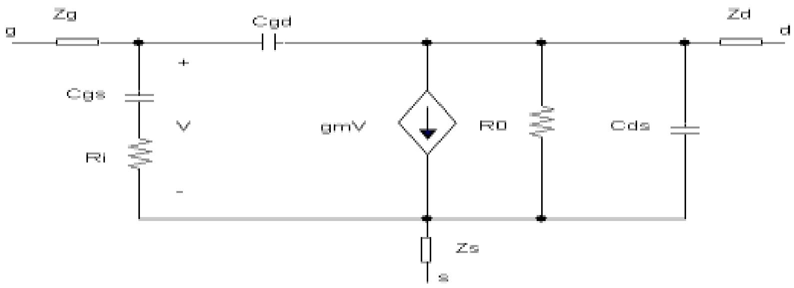
$$Y_{21} = \frac{g_m e^{-j\omega t}}{1 + jwc_{gs} r_i} + jwc_{gd}$$

$$Y_{22} = \frac{i_2}{V_2}\bigg| V_1 = 0$$

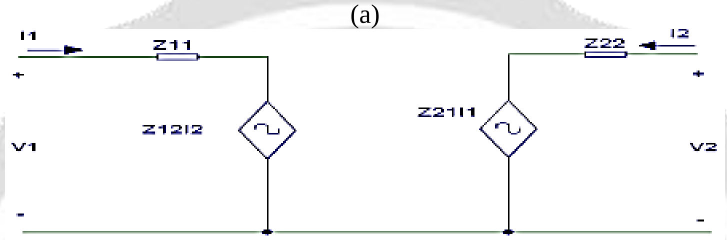
$$Y_{22} = \frac{1}{r_0} + jw\left(c_{ds} + c_{gd}\right)$$

**5.2 EXTRACTION OF PARASITES:**

The representation in Z parameters is necessary to determine the parasitic resistances and inductances [6], [10]. Fig. 10 (a) shows the parasitic elements in series of the MESFET and that of 10 (b) the equivalent network in parameter Z:




(a)




(b)

Figure 10: (a) Intrinsic MESFET with serial parasites, (b) representation in Z parameters.

The Y parameters can be converted into Z parameters using the following transformation:

$$[Z] = \begin{bmatrix} \frac{Y_{11}}{|Y|} & \frac{Y_{12}}{|Y|} \\ \frac{Y_{21}}{|Y|} & \frac{Y_{22}}{|Y|} \end{bmatrix}$$

Whith $| Y | = Y_{11} Y_{22} - Y_{12} Y_{21}$

The next step is the equation of the parasitic capacitances according to Fig. 11.

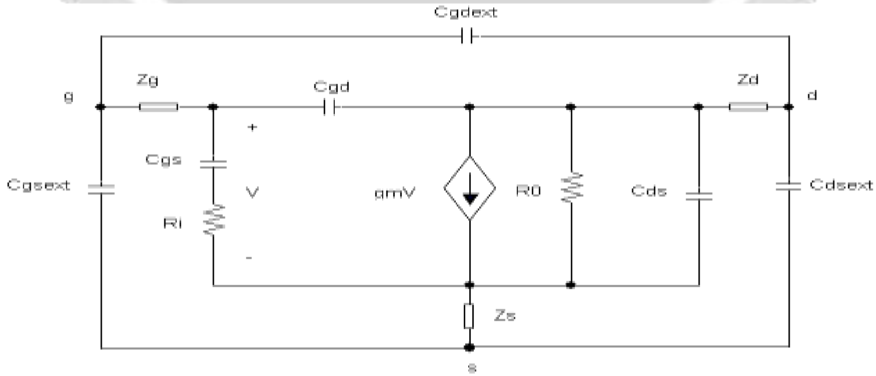



Figure 11: MESFET with extrinsic elements.

The Y parameters are used and a transformation can be done between [Y] and [Z]:

$$[Y] = \begin{bmatrix} \frac{Z_{11}}{|Z|} & \frac{Z_{12}}{|Z|} \\ \frac{Z_{21}}{|Z|} & \frac{Z_{22}}{|Z|} \end{bmatrix}$$

Whith $|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$

### 5.3 DISTRIBUTION PARAMETERS:

The Y and Z parameters are experimental parameters, which have their drawbacks. For example, open circuit measurements are necessary to find the Z parameters, but in the microwave (microwave) band, parasitic capacitors can "short-circuit" open circuits, and can distort the results. Other parameters are therefore necessary: these are the distribution parameters or S parameters. They measure the incident and reflected waves at the entry and exit of the MESFET (Fig. 12). These parameters S are evaluated experimentally.

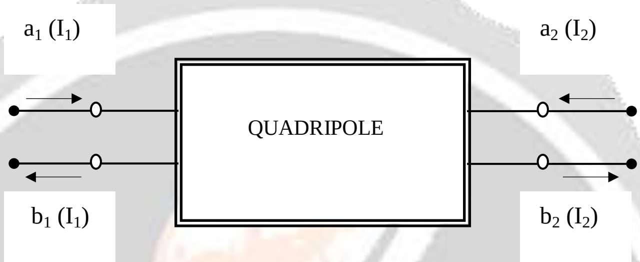



Figure 12: Reflected and incident waveform in microwave frequencies.

From Figure 12, we have:

$$\begin{cases} b_1(I_1) = S_{11}a_1(I_1) + S_{12}a_2(I_2) \\ b_2(I_2) = S_{12}a_1(I_1) + S_{22}a_2(I_2) \end{cases}$$

with
$a_1(I_1)$ the incident wave at input 1 (side 1),
$a_2(I_2)$ that of side 2.
$b_1(I_2)$ the wave reflected at input 1 (side 1),
$b_2(I_2)$ that of side 2.

$S_{11}$ the reflection coefficient at input 1 (side 1),

$$S_{11} = \frac{b_1(I_1)}{a_1(I_1)}\bigg| a_2(I_2) = 0$$

$S_{12}$ the inverse transmission coefficient on side 2

$$S_{12} = \frac{b_1(I_1)}{a_2(I_2)}\bigg| a_1(I_1) = 0$$

$S_{21}$ the direct transmission coefficient at input 1 (side 1),

$$S_{21} = \frac{b_2(I_2)}{a_1(I_1)}\bigg| a_2(I_2) = 0$$

$S_{22}$ the reflection coefficient of output 2

$$S_{11} = \frac{b_1(I_1)}{a_1(I_1)}\bigg| a_2(I_2) = 0$$

The measurement of the S parameters requires that the device is terminated with an impedance adapted to the characteristic impedance of the transmission line.

**5.4 MESFET PARAMETER EVALUATION TOOL:**

The relations between the parameters S and the elements of the equivalent circuit have been determined via transformation matrices. If the matrices include the real and imaginary parts of the parameters S, we have 8 equations and 16 unknowns. The problem is said to be "poorly conditioned". This type of problem is usually solved using an optimization method such as the "Simplex Method". [11]

The evaluation begins with the introduction of the measured S parameters and a starting point for the initial value of the equivalent circuit. Then 16 others are generated for the start of the simplex. Then, the cost function is calculated on each point. The M.S module makes it possible to find the values of the cost function, and to minimize the error. Once the error reaches a minimum tolerance, the process stops. The best point of the 17 is used to generate the S parameters (Fig. 13) as a function of frequency.

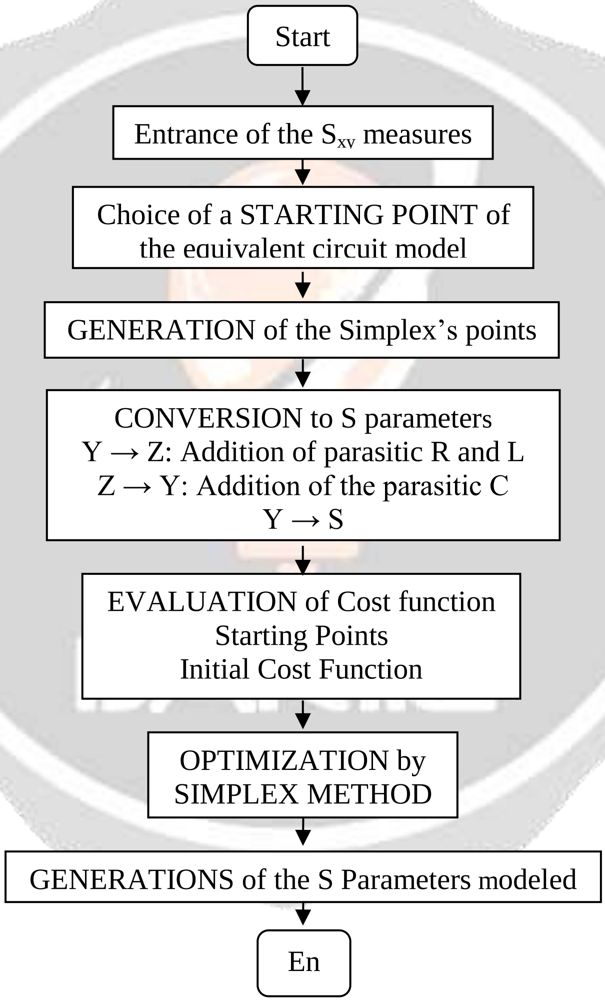



Figure 13: Flowchart for extracting the S parameters of a MESFET.

**6 MEASUREMENTS AND DATA ANALYSIS:**

A wideband equivalent circuit from 0.05 GHz to 6.05 GHz is appropriate for each S parameter. A key assumption is that capacitors, resistances, inductors and current generator should not vary with voltage.

### 6.1 DATA PROCESSING:

The general model of a MESFET used for data processing is that of Fig. 06. The processing process is done using the M.S, but in matrix form. Table 2 shows the starting values for the M.S. process.

Two different types of treatments are compared:

- no limits have been imposed on the placement of the points of the simplex

- limits have been introduced.

Table 2: Limits of circuit elements and starting values.

| Circuit element | Minimum | Departure | Maximum |
|---|---|---|---|
| $L_g$ (nH) | 0,05 | 0,13 | 1,30 |
| $L_d$ (nH) | 0,02 | 0,08 | 1,08 |
| $L_s$ (nH) | 0,01 | 0,01 | 1,07 |
| $R_g$ (Ω) | 0,11 | 1,44 | 10,0 |
| $R_d$ (Ω) | 1,02 | 2,10 | 50,5 |
| $R_s$ (Ω) | 0,01 | 0,01 | 0,55 |
| $c_{ds}$ (pF) | 0,01 | 0,02 | 0,03 |
| $r_i$ (Ω) | 0,05 | 1,72 | 6,01 |
| $g_m$ (mS) | 20,03 | 2987,41 | 3500 |
| $c_{gs}$ (pF) | 0,12 | 0,70 | 30,31 |
| $c_{gd}$ (pF) | 0,04 | 7,01 | 30,06 |
| $r_0$ (Ω) | 1,00 | 6,22 | 54,75 |
| $c_{gsext}$ (pF) | 0,00 | 1,11 | 10,03 |
| $c_{gdext}$ (pF) | 0,00 | 0,80 | 10,02 |
| $c_{dsext}$ (pF) | 0,00 | 0,41 | 10,30 |

The simplex has a total of 17 points. The starting point generates the other 16 points, the 16 points with the 16 coordinates are considered as a 16x16 matrix. Then, limits are placed on the positions of the points of the simplex (Min and Max values in Table 2).

### 6.2 RESULTS AND ANALYSIS:

Table 3 presents a comparison of these two treatments. The cost functions and the error terms are given there for the four S parameters. The cost function is equal to the sum of the error terms.

Table 3: Comparison of cost functions for the two types of processing.

| Treatments | Fº cost | $S_{11}$ | $S_{12}$ | $S_{21}$ | $S_{22}$ |
|---|---|---|---|---|---|
| No limit | 20,2198 | 3,24457 | 3,24457 | 7,94400 | 6,73666 |
| Limited | 22,5724 | 5,28662 | 2,85660 | 7,88850 | 6,52073 |

The $S_{21}$ parameter has the best approximation; this is because $S_{21}$ typically has the largest modulus among the four S parameters, especially at low frequencies. In fact, $|S_{21}|$ gives an idea on the voltage gain of the device.

The limitless case has the lowest value of the cost function and the value of parameter $S_{11}$ has the greatest dispersion.

Table 4 shows the values of the equivalent circuits for the different approaches discussed.

Table 4: Equivalent circuit for the different extractions.

| Circuit element | No limit | With limit |
|---|---|---|
| $L_g$ (pH) | 0,157480 | 0,183875 |
| $L_d$ (pH) | 0,186913 | 0,187753 |
| $L_s$ (pH) | 0,0110833 | 0,010139 |
| $R_g$ (Ω) | 1,10155 | 1,58602 |
| $R_d$ (Ω) | 0,101228 | 1,00104 |
| $R_s$ (Ω) | 0,0114797 | 0,0172276 |
| $c_{ds}$ (pF) | 0,0311370 | 0,0201548 |
| $r_i$ (Ω) | 0,588542 | 1,99895 |
| $g_m$ (mS) | 1317,12 | 1223,81 |
| $c_{gs}$ (pF) | 0,558419 | 0,315232 |
| $c_{gd}$ (pF) | 7,11441 | 6,30023 |
| $r_0$ (Ω) | 1,00143 | 6,30122 |
| $c_{gsext}$ (pF) | 1,69517 | 1,86660 |
| $c_{gdext}$ (pF) | -0,899193 | 0,0793905 |
| $c_{dsext}$ (pF) | 0,818402 | 0,00829952 |

With unrestricted values, they can turn negative like $c_{gdext}$ does. A negative capacity is physically impossible. This is due to the fact that the problem contains a lot of variables, and that the Simplex Method is very sensitive to the initial conditions.

## 7 CONCLUSIONS:

An evaluation of the parameters S of the equivalent power MESFET circuit from the parameters S measured experimentally and from the data in the “data book” was made. The measurements were performed on devices in the frequency band 50 MHz to 6.05 GHz.

An analytical model of the device was presented, resulting in an equivalent circuit that many designers use. This model called "general model" consists of 16 circuit elements, the latter constitute the starting point of the optimization method the "Simplex Method" for solving the problem.

Beginning by extracting the Y parameters from an intrinsic model, transforming them into Z parameters for parasitic resistances and inductances. Then transforming them back into Y parameters in order to complete the model with parasitic capacitances. Finally, to resolve the problems linked to parameters in the “High Frequencies”, the distribution parameters or S parameters are necessary.

Two types of operations have been developed for the M.S departure points: the limitless case and the imposed limit case. The results will be obtained in good agreement with the experimental results, however negative results of the elements are observed. The sensitivity of the “Simplex Method” is the cause.

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