EFFECT OF CONTACT RATIO ON SPUR GEARS USING MODEL-BASED ANALYSIS AND SIMULATION

Pradeep Kumar Mehta [1] & Vijay Karma [2]

[1] *Student, Department of Mechanical Engineering (Design & Thermal Engineering), Institute of Engineering & Technology, DAVV University, Indore, Madhya Pradesh, India.*

[2] *Assistant Professor, Department of Mechanical Engineering, Institute of Engineering & Technology, DAVV University, Indore, Madhya Pradesh, India.*

ABSTRACT

Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio. This paper presents a computer modelling and simulation showing how the gear contact ratio affects a spur gear system. The analysis presented in this paper is performed by using a Six Degree of Freedom (6DOF) dynamic model of a one-stage spur gear system. In the analysis, the contact ratio was varied over the range 1.6456 to 2.1256. In order to simplify the analysis, other parameters related to contact ratio were held constant. The simulation was performed using MATLAB and the model was tested at two frequencies of 570 and 833 Hz. The contact ratio was found to have a significant influence on gear dynamics. A contact ratio close to 2 provided maximum values for Minimum and Maximum tooth stiffness at both the tested frequencies. The paper derives the conclusion that contact ratio 2 is best suited to the model under consideration to obtain highest tooth stiffness.

Keyword: - *Spur gears, Contact ratio, MATLAB, Tooth stiffness, Vibration response simulation.*

---

1. INTRODUCTION

Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio. The vibration of a gear train are significantly affected by the gear contact ratio [1]. The contact ratio can be influenced by parameters such as the pressure angle, the tooth size (diametral pitch), and the centre distance. Of these parameters, varying the length of the tooth addendum is the most desirable way to control the contact ratio without increasing the tooth stress. The contact ratio can be increased by reducing the pressure angle but this also increases the tooth bending moment and hence the stress. Finer pitch gears have higher contact ratios, but their smaller teeth are subjected to higher stress [2]. The gear tooth addendum was varied to create contact ratios in the range 1.6456 to 2.1256. In order to simplify the analysis, the torque and other parameters were held constant. For this analysis we will adopt the mathematical model of a one-stage spur gearbox system with torsional and lateral vibration reported by Bartelmus [3] and Tian et al. [4]. Computer simulation is used to study the effects of contact ratio on a one-stage spur gearbox. Mesh stiffness and its time-variations are recognized as key parameters controlling gear dynamics.

1.1 A Dynamic Model of One-Stage Spur Gear

The schematic diagram of the dynamic model of a one-stage spur gearbox system is shown in Fig. 1, is a model with torsional and lateral vibration reported by Bartelmus [3] and Tian [4]. This is a two-parameter model, involving stiffness and damping with torsional and lateral vibration and has six degrees of freedom. The main parameters that are considered in the model are given in Table 1. It is assumed that all gears are perfectly mounted rigid bodies with ideal geometries. Inter-tooth friction is ignored here for simplicity.

The following notation is used in this study [4].

F_k stiffness inter-tooth force
F_c damping inter-tooth force
F_u internal stiffness force of input bearing
F_{uc} internal damping force of input bearing
F_l internal stiffness force of output bearing
F_{lc} internal damping force of output bearing
M_{pk} stiffness moment of input couplings
M_{pc} damping moment of input couplings
M_{gk} stiffness moment of output couplings
M_{gc} damping moment of output couplings
K_t total mesh stiffness
c_t mesh damping coefficient
I_m mass moment of inertia of motor
I_b mass moment of inertia of load
I_1/I_2 mass moment of inertia of pinion/gear
M_1 input motor torque $M2$: output torque from load
m_1/m_2 mass of the pinion/gear
k_p torsional stiffness of input flexible coupling
k_g torsional stiffness of output flexible coupling
c_p damping coefficient of input flexible coupling
c_g damping coefficient of output flexible coupling
k_1 vertical radial stiffness of input bearings
k_2 vertical radial stiffness of output bearings
c_1 vertical radial viscous damping coefficient of input bearings
c_2 vertical radial viscous damping coefficient of output bearings
θ_1/θ_2 angular displacement of pinion/gear
θ_m angular displacement of motor
θ_b angular displacement of load
y_1/y_2 linear displacement of pinion/gear in the y direction
R_{b1}/R_{b2} base circle radius of pinion/gear
R_{O1}/R_{O2} outside circle radius of pinion/gear

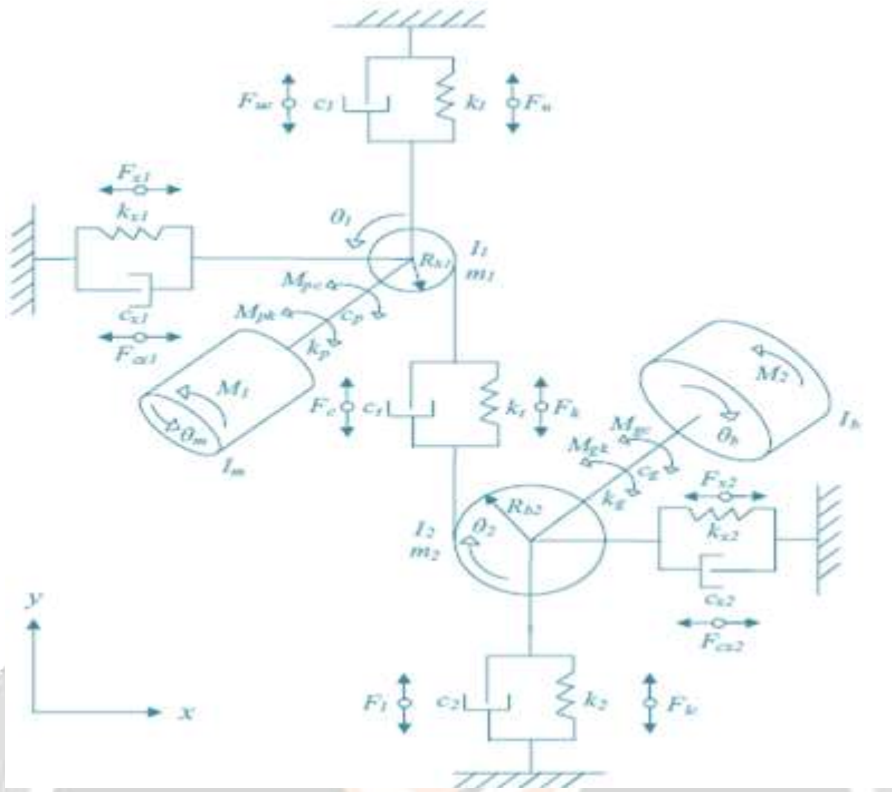

Fig. 1. A one stage gearbox system [4].

The main parameters of the gearbox system in this study are listed in Table 1.

Table 1. Major parameters of the spur gears used in this model [4].

Mass moment of inertia of the motor (I_m)	0.0021 kgm^2
Mass moment of inertia of the load (I_b)	0.0105 kgm^2
Mass moment of inertia of the pinion (I_1)	4.3659×10^{-4} kgm^2
Mass moment of inertia of the gear (I_2)	8.3602×10^{-3} kgm^2
Contact ratio (C_r)	1.6456
Mass of the pinion (m_1)	0.96 kg
Mass of the gear (m_2)	2.88 kg
Input shaft frequency (f_1)	30 Hz
Mesh frequency (f_m)	570 Hz
Input motor torque (M_1)	11.9 Nm
Output torque from load (M_2)	48.8 Nm
Torsional stiffness of the coupling (k_c)	4.4×10^4 Nm/rad

Damping coefficient of the coupling (c_c)	5.0×10^5 Nm/rad
Radial stiffness of the bearing (k_r)	6.56×10^7 N/m
Damping coefficient of the bearing (c_r)	1.8×10^5 Ns/m
Young's modulus (E)	2.068×10^{11} Pa
Pressure angle	20°
Diametral pitch (P)	0.2032 m^{-1}
Width of teeth (L)	0.016 m
Poisson's ratio (v)	0.3
Number of teeth on pinion and gear	$N_1 = 19$; $N_2 = 48$

Because friction is ignored, the vibration in the x direction is free response and will disappear due to inherent damping. In this paper, we focus only on the motion in the y direction. Based on the work of Bartelmus [3], to simulate the gearbox vibration, the vertical motion (in the y direction) equations of the pinion and gear are

$$m_1\ddot{y}_1 = F_k + F_c - F_u - F_{uc} \quad (1)$$

$$m_2\ddot{y}_2 = F_k + F_c - F_l - F_{lc} \quad (2)$$

$$I_1\ddot{\theta}_1 = M_{pk} + M_{pc} - R_{b1}(F_k + F_c) \quad (3)$$

$$I_2\ddot{\theta}_2 = R_{b2}(F_k + F_c) - M_{gk} - M_{gc} \quad (4)$$

$$I_m\ddot{\theta}_m = M_1 - M_{pk} - M_{pc} \quad (5)$$

$$I_b\ddot{\theta}_b = -M_2 + M_{gk} + M_{gc} \quad (6)$$

$$F_k = k_t(R_{b1}\theta_1 - R_{b2}\theta_2 - y_1 + y_2) \quad (7)$$

$$F_c = c_t(R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2 - \dot{y}_1 + \dot{y}_2) \quad (8)$$

$$F_u = k_1 y_1 \quad (9)$$

$$F_{uc} = c_1\dot{y}_1 \quad (10)$$

$$F_l = k_2 y_2 \quad (11)$$

$$F_{lc} = c_2\dot{y}_2 \quad (12)$$

$$M_{pk} = k_p(\theta_m - \theta_1) \quad (13)$$

$$M_{pc} = c_p(\dot{\theta}_m - \dot{\theta}_1) \tag{14}$$

$$M_{gk} = k_p(\theta_2 - \theta_b) \tag{15}$$

$$M_{gc} = c_g(\dot{\theta}_2 - \dot{\theta}_b) \tag{16}$$

Equations 1–2 are the vertical motion equations of the pinion and the gear. Equations 3–4 are the rotary motion equations of the pinion and the gear. Equations 5 and 6 are the rotary motion equations of the motor and load, respectively. Equations 7–12 represent the values of the forces, and Equations 13–16 represent the values of the moments.

The following further assumptions are made [4], the vertical radial stiffness of the input bearings and that of the output bearings are constant and equal, i.e., $k_1 = k_2 = k_r$; the damping coefficients of the input and the output bearings are constant and equal, i.e., $c_1 = c_2 = c_r$; the torsional stiffness values of the input and the output flexible coupling are constant and equal, i.e., $k_p = k_g = k_c$; the damping coefficients of the input and the output flexible coupling are constant and equal, i.e., $c_p = c_g = c_c$; the mesh damping coefficient, c_t , is assumed to be proportional to the total mesh stiffness, k_t : $c_t = \mu k_t$, where μ is set to be constant 3.99 × 10–6 (s).

2. CONTACT RATIO

When two gears are working together the number of gear teeth which are in contact varies during the meshing cycle. In order for the gear drive to work properly there must be at least one pair of teeth in contact at all times. The average number of gear teeth in contact when the gears are operating is called the contact ratio, CR. In practice the contact ratio varies between two discrete values and a contact ratio of e.g. CR = 1:3 describes that some of the time there is one gear tooth pair in contact and for the rest of the time there are two pairs in contact. Acceptable values for the contact ratio is usually CR > 1:2 with a absolute minimum of CR = 1:1. If a contact ratio below this value occurs, correct motion transfer cannot be assured [5].

3. GEAR MESH STIFFNESS EVOLUTION

Mesh stiffness and its time-variations are recognized as key parameters controlling gear dynamics and tooth loading to a large extent. The teeth in a healthy gear in good running condition will deflect under load. The meshing process is always varying from one and two pairs of teeth in contact. Harris [6] defined mesh stiffness fluctuation as a periodic excitation which would cause gear pair vibrations even if the gears were free from any manufacturing errors. The duration of contact depends on the contact ratio ε .

Thus, gear mesh stiffness is periodic with the period $T_{eng} = 60/N_1 . Z_1$ and can be approximated as [7]

$$k_t(t) = \begin{cases} k_{max}, nT_{eng} \le t \le (n+\varepsilon-1)T_{eng'} \\ k_{min}, (n+\varepsilon-1)T_{eng} \le t \le (n+1)T_{eng'} \end{cases} \tag{17}$$

where N_1 is the pinion rotational speed in rpm, Z_1 is the tooth numbers, ε represents the contact ratio and n is an integer representing the nth gear mesh period.

Fourier development of $k_t(t)$ yields:

$$k_t(t) = k_m + \frac{\Delta k}{\pi}\sum_{i=1}^{\infty}\frac{1}{i}\left[\sin(2i\pi(\varepsilon-1))\cos\frac{2i\pi t}{T_{eng}} + (1-\cos(2i\pi(\varepsilon-1)))\sin\frac{2i\pi t}{T_{eng}}\right] \tag{18}$$

with: $k_m = k_{max}$ (ε - 1) = (2 - ε)k_{min} and Δk = $k_{max} - k_{min}$.

By introducing the gear stiffness ratio and using some geometrical and material properties, the maximum and the minimum value of the gear stiffness can be calculated

$$k_{max} = 14\times10^9 \frac{E}{2.1\times10^{11}} b\times s, \ \ k_{min} = rk_{max}$$

where E = $2.068\text{x}10^{11}$ N/m^2 is the mean value of Young’s modulus of the gear bodies, b = 0.16 m is the effective width of meshing gears, s = 0.47 is the shape factor and r = 0.5476 is the stiffness ratio.

4. MATHEMATICAL SIMULATION

After gear modelling MATLAB simulation is carried out to predict the natural frequencies of a system. MATLAB's ODE15s is a standard solver in MATLAB for ordinary differential equations. The Runge-Kutta method is implemented in this function. Thus, in this study, the function of ODE15s is used to obtain the numerical results of the motion equations. This method employs a linearized iterative procedure that involves dividing the mesh period into many equal intervals. Initial angular displacements are obtained by preloading the input shaft with the nominal torque carried by the system. Initial angular speeds are taken from the nominal system operating speed. For steady state operation the dynamic motions of the system can be found from this iterative procedure. The method is described in detail in Lin et al. [9].

The displacement plots for the perfect gear tooth can be obtained by computer simulations; and the results are presented in Figure 2.

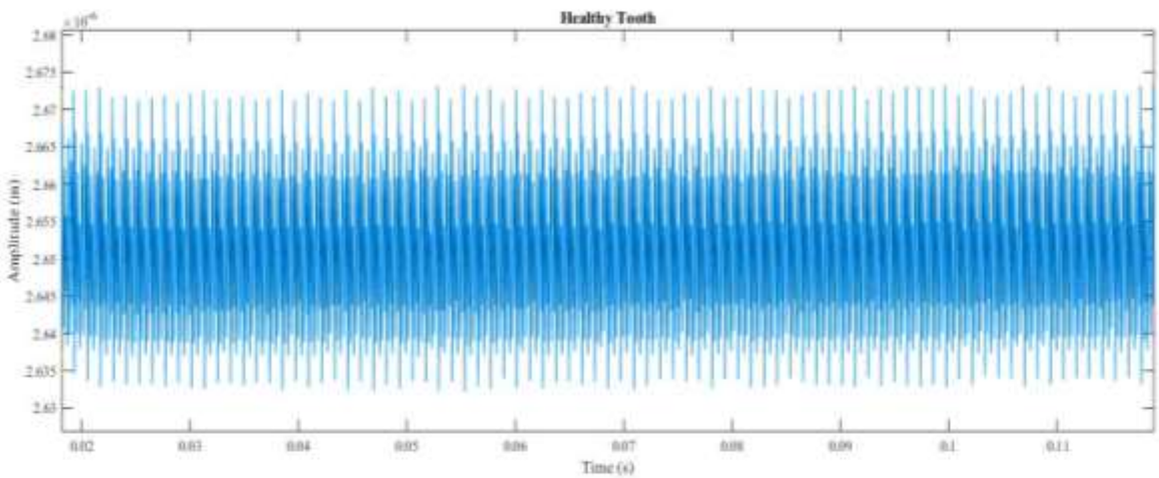

Fig. 2. The pinion's vibration displacement response in y direction for a perfect gear tooth

In order to observe more details, the Fourier Transform is applied to the time domain signal, and the power spectral density is presented in Figure 3.

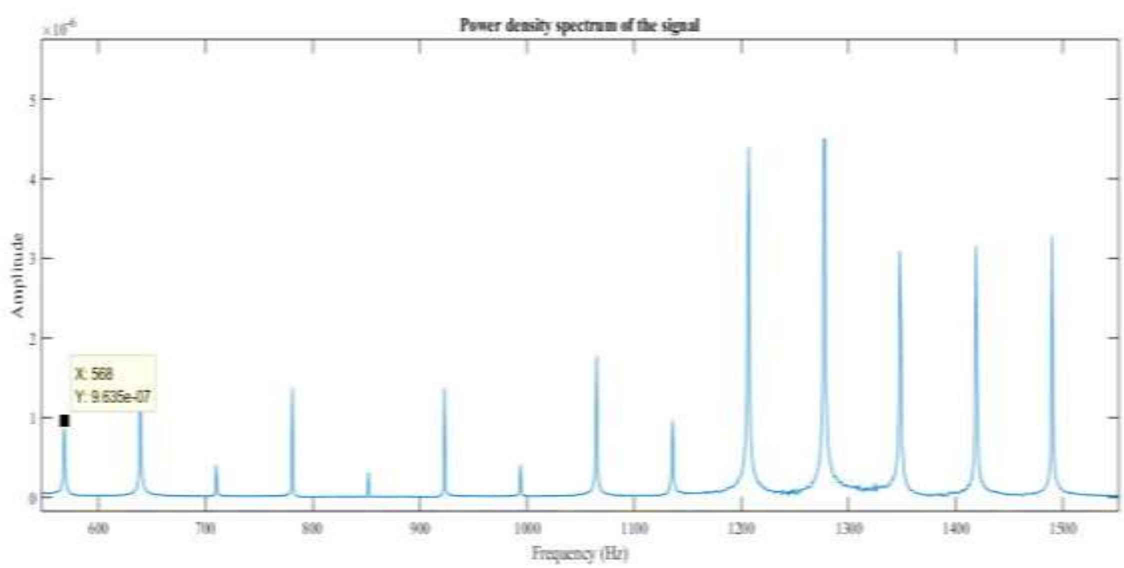

Fig. 3. Spectra of the pinion's vibration displacement response in y direction for a perfect gear tooth.

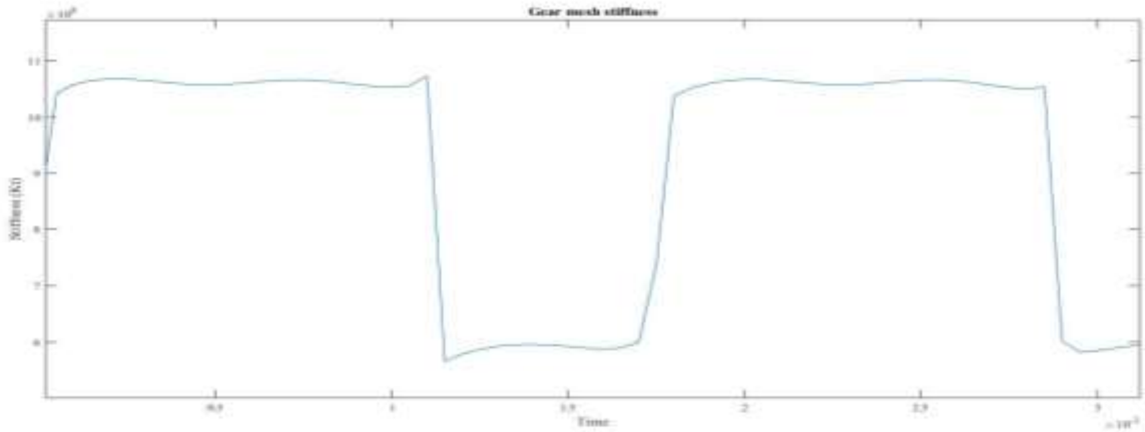

Fig. 4. Time variation of stiffness gear mesh k (t)

The teeth in a healthy gear in good running condition will deflect under load. The meshing process is always varying from one and two pairs of teeth in contact. The duration of depends on the contact ratios. The gear mesh stiffness will fluctuate around a mean value K_m as shown in Figure 4.

5. CONTACT RATIO ANALYSIS

The periodic change of tooth stiffness, gear errors and friction force impulse at the pitch point are the principal causes of vibration and noise in gears. In high precision and heavily loaded gears, the effect of gear errors is insignificant, so the periodic variation of tooth stiffness and friction force impulse are the most significant causes of noise and vibration.

High contact ratio spur gears can be used to exclude or reduce the variation of tooth stiffness. Kasuba, [10]; established experimentally that the dynamic loads decrease with increasing contact ratio in spur gearing. Sato et al. [11]; demonstrated experimentally that the minimum dynamic factor corresponds to gears with a contact ratio slightly less than 2.00 (1.95). The same result was found experimentally by Kahraman and Blankenship, [12]. Data for the gear set used in this study are listed in Table 2.

Table 2. Impact of contact ratio on Tooth Stiffness at 570 Hz

S.No.	Contact ratio	Min. Stiffness	Max. Stiffness
1	1.6456	5.52×10^8	1.11×10^9
2	1.7056	5.24×10^8	1.08×10^9
3	1.7656	4.92×10^8	1.04×10^9
4	1.8256	4.67×10^8	1.02×10^9
5	1.8856	4.43×10^8	9.95×10^9
6	1.9456	3.97×10^8	9.56×10^9
7	2.0056	8.52×10^8	1.09×10^9
8	2.0656	8.15×10^8	1.36×10^9
9	2.1256	7.97×10^8	1.35×10^9

Table 3. Impact of contact ratio on Tooth Stiffness at 833 Hz

S.No.	Contact ratio	Min. Stiffness	Max. Stiffness
1	1.6456	5.52×10^{8}	1.11×10^{9}
2	1.7056	5.52×10^{8}	1.08×10^{9}
3	1.7656	5.52×10^{8}	1.04×10^{9}
4	1.8256	5.52×10^{8}	1.02×10^{9}
5	1.8856	5.52×10^{8}	9.95×10^{8}
6	1.9456	5.52×10^{8}	9.56×10^{8}
7	2.0056	5.52×10^{8}	1.09×10^{9}
8	2.0656	5.52×10^{8}	1.36×10^{9}
9	2.1256	5.52×10^{8}	1.35×10^{9}

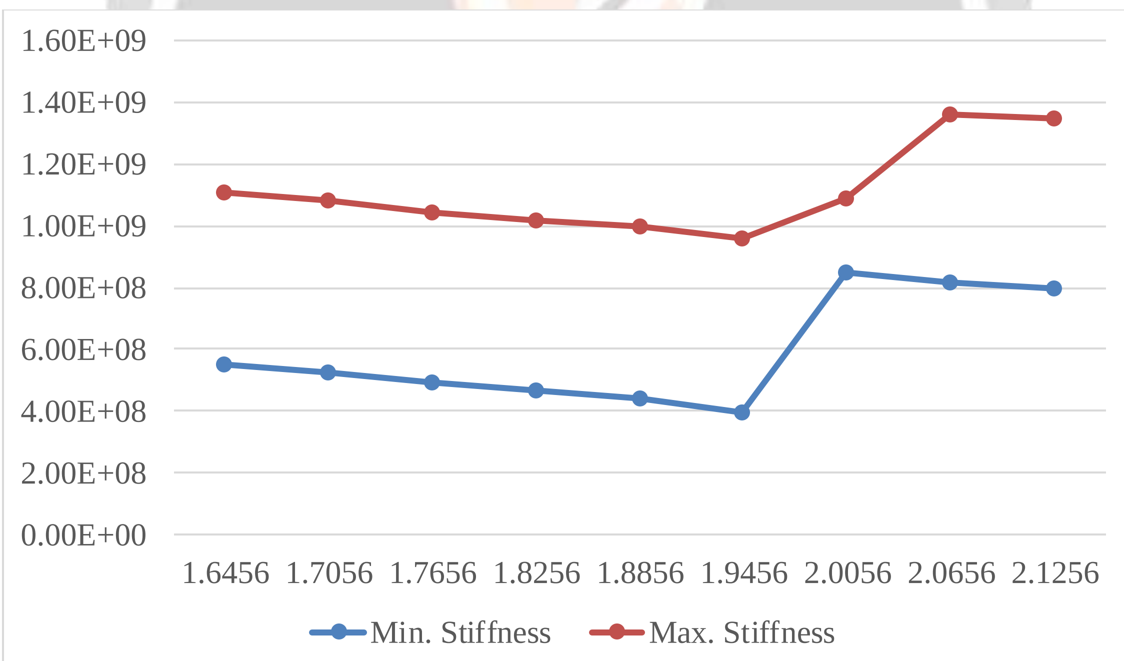

Fig. 5. Variation of Tooth Stiffness with Contact Ratio at 570 Hz

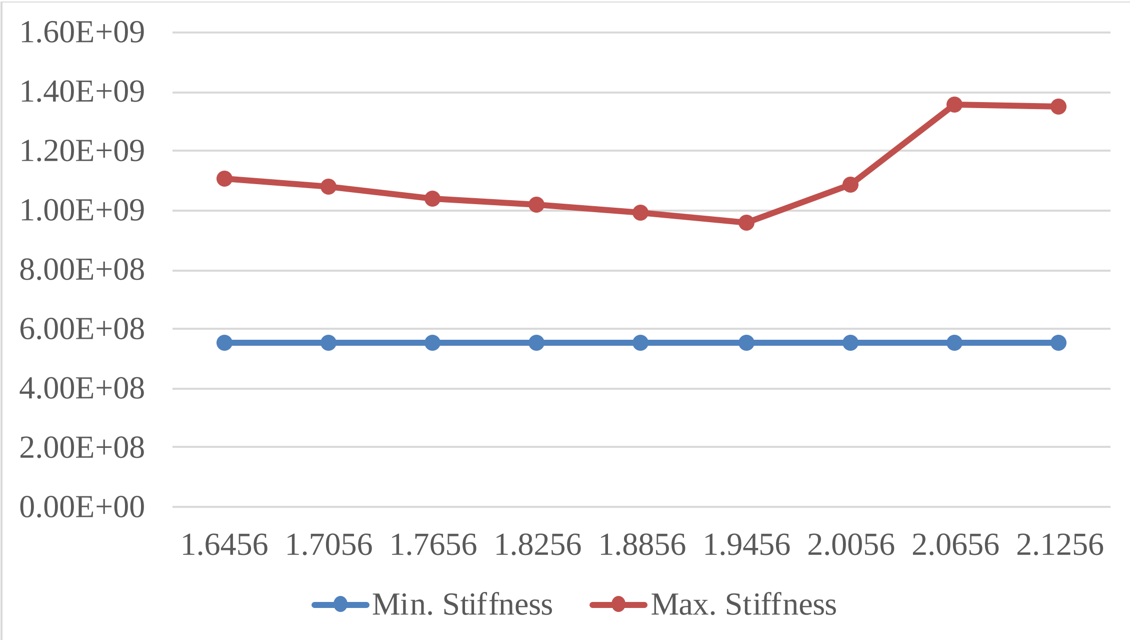

Fig. 6. Variation of Tooth Stiffness with Contact Ratio at 833 Hz

Dynamic modelling and simulation was applied on gear tooth to identify the effect of contact ratio on the tooth stiffness. The test was carried out at two frequencies i.e., 570 Hz and 833 Hz.

5.1 Results at 570 Hz

It is observed that at this frequency as the contact ratio is increased from 1.6456 to 1.9456 the minimum tooth stiffness decreases. As the contact ratio is increased to 2.0056 a sharp increase in tooth stiffness is observed. On further increasing the contact ratio beyond 2.0056 the Minimum tooth stiffness tends to decrease again. So it can be concluded that at contact ratio of 2.0056 ~ 2 the Minimum Tooth Stiffness has maximum value.

It is observed that at this frequency as the contact ratio is increased from 1.6456 to 1.9456 the Maximum tooth stiffness is observed to decrease constantly. As the contact ratio is further increased beyond 1.9456 till 2.0656 an increase in Maximum tooth stiffness is observed. On increasing the contact ratio beyond 2.0656 the Maximum tooth stiffness tends to decrease again. So it can be concluded that at contact ratio of 2.0656 ~ 2 the Maximum Tooth Stiffness has maximum value.

5.2 Results at 833 Hz

At this frequency it is observed that as the contact ratio is increased from 1.6456 to 2.1256 the minimum tooth stiffness remains unchanged at 5.52×10^8. So it can be concluded that contact ratio has no measurable impact on the Minimum Tooth Stiffness.

It is observed that at this frequency as the contact ratio is increased from 1.6456 to 1.9456 the Maximum tooth stiffness is observed to decrease constantly. As the contact ratio is further increased beyond 1.9456 till 2.0656 an increase in Maximum tooth stiffness is observed. On increasing the contact ratio beyond 2.0656 the Maximum tooth stiffness tends to decrease again. So it can be concluded that at contact ratio of 2.0656 ~ 2 the Maximum Tooth Stiffness has maximum value.

6. CONCLUSIONS

The research work was undertaken to identify impact of contact ration on the gear tooth stiffness. The simulation was performed using MATLAB and the model was tested at two frequencies of 570 and 833 Hz. The contact ratio was found to have a significant influence on gear tooth stiffness. A contact ratio close to 2 provided best values for Minimum and Maximum tooth stiffness at both the tested frequencies. So it can be concluded that contact ratio 2 is best suited to the model under consideration to obtain highest tooth stiffness. Hence while designing a spur gear a contact ratio close to 2 is recommended by the authors.

7. ACKNOWLEDGEMENT

The authors are thankful to the Director and Head of Mechanical Engineering Dept. IET DAVV, Indore, for their continuous guidance and support.

6. REFERENCES

[1]. Sato T., Urnezawa K., and Ishikawa, J. "Effects of Contact Ratio and Profile Correction on Gear Rotational Vibration," JSME Bulletin, Vol, 26, No, 221, pp. 2010-2016, 1983.

[2]. Liou C. H, and Lin H. H. “Effect of contact ratio on spur gear dynamic load with no tooth Profile modifications”, Journal of Mechanical Design, September 1996, Vol. 118/439.

[3]. Bartelmus W. “Mathematical modelling and computer simulations as an aid to gearbox diagnostics,” Mechanical Systems and Signal Processing, 15(5), 855–871, 2001.

[4]. Tian Z, Ming J.Z, Wu S. “Crack propagation assessment for spur gears using model-based analysis and simulation,” Journal of Intelligent Manufacturing, April 2012, Volume 23, Issue 2, pp 239-253.

[5]. Nilsson A. I. “Gear Whine Noise Excitation Model,” Master's Thesis in Mechanical Engineering, Sweden 2013.

[6]. Harris, S. L. "Dynamic Loads on the Teeth of Spur Gears," Proceedings of the Institution of Mechanical Engineers, Vol. 172, pp. 87-112, 1958.

[7]. Belalouache K., Benazzouz D, Rahmoune C. “A new method based on fast Kurtogram for the identification of pitting fault versus crack fault in gearbox systems,” JVE International Ltd. Journal of Vibroengineering, Vol 16, Issue 6, Issn 1392-8716, Sep 2014.

[8]. Math Works. MATLAB User's Guide. The Math Works, Inc., 2012.

[9]. Lin H. H., Huston R. L., and Coy J. J. "On Dynamic Loads in Parallel Shaft Transmissions: Part 1-Modeling and Analysis," ASME Journal of Mechanisms, Transmissions, And Automation in Design, Vol. 110, No, 2, pp. 221-225, 1988.

[10]. Kasuba R. "Dynamic Loads in Normal and High Contact Ratio Spur Gearing," International Symposium on Gearing and Power Transmissions, Tokyo, p. 49-55, 1981.

[11]. Sato T., Urnezawa K., and Ishikawa, J "Influence of Various Gear Errors on Rotational Vibration," Proc. of Int'l Symposium on Gearing and Power Transmissions, JSME, Tokyo, Japan, Paper C-10, pp, 55-60, 1981.

[12]. Kahruman A. and Blankenship G.W. "Effect of Involute Contact ratio on spur Gear Dynamics," Transactions of ASME, 121, Journal of Mechanical Design, p.112-118, March 1999.