GENERALIZED FUZZY METRIC SPACE WITH APPLICATIONS & FUZZY MAPPINGS

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Abstract

In this paper, Based on the above definitions, some interesting coincidence points, common fixed points, and fixed point results are obtained that generalize not only the applications and fuzzy mapping and several important results of generalized fuzzy metric space with multiplayer mapping in recent literature. We extend the application of generalized fuzzy metric space and generalized locations with fuzzy mapping such as quasi-pseudo-metric spaces and cone metric spaces. Some assumptions are also acceptable for α-commuting, α-weakly consistent mapping, L-fuzzy mapping for L-fuzzy sets, and a pair of βFL - L-fuzzy mappings. They do, but also decrease. Some survival theory for the solution of a generalized class of nonlinear integral equations. Some practical examples have also been presented to increase the validity of this work.

Keywords: *Fuzzy Set, Fuzzy Theory. Metric Spaces and Cone Metric Spaces*

1.1 Introduction

Fuzzy set theory / Fuzzy logic comes under Artificial intelligence or soft computing. The other useful tools of soft computing or Artificial Intelligence (A.I) are Artificial Neural Network(A.N.N), Genetic Algorithm (G.A) , Support Vector Machine (S.V.M) , Probabilistic Reasoning etc.

The theory of fuzzy logic of soft computing is based on the notion of relative graded membership, as inspired by the processes of human perception and cognition. Lotfi A. Zadeh published his first famous research paper on fuzzy sets in 1965[1]. Fuzzy logic can deal with information arising from computational perception and cognition, that is, uncertain, imprecise, vague, partially true, or without sharp boundaries. Fuzzy logic allows for the inclusion of vague human assessments in computing problems. Also, it provides an effective means for conflict resolution of multiple criteria and better assessment of options. New computing methods based on fuzzy logic can be used in the development of intelligent systems for decision making, identification, pattern recognition, optimization, and control. While utilizing the topological methods in the theory of differential equations, French mathematician Poincar´e initiated the idea of fixed point theory. In 1904, Bohl [34] proved a result about the non-retraction which was preceded by Brouwer and Hadamard in the form of a very famous Brouwer fixed point theorems in 1910 [36]. Although this theorem attained a lot of importance and recognition among the existence principles in mathematics it never provided any practical approach towards the calculation of a fixed point. Banach resolved this issue in 1922 by presenting a revolutionary contraction principle (namely called Banach contraction principle) in which the Picard iteration process was used for the evaluation of a fixed point. Since the theorem and its many equivalent formulations or extensions are powerful tools in showing the existence of solutions for many problems in pure and applied mathematics.

As an important consequence, some real problems can be solved most effectively by using hybrid systems what is increasing the interest on them. The rest and probably the most successful hybrid approach till now are the so-called neurofuzzy systems, although some other hybridations are being developed with great success as, for instance, the genetic fuzzy systems. Soft computing is an emerging collection of methodologies, which aim to exploit tolerance for imprecision, uncertainty, and partial truth to achieve robustness, tractability and total low cost.

Fixed point theory itself is a magnificent combination of analysis (pure and applied), topology, and geometry. Over the last few decades, the theory of fixed points has become a very influential and important tool in the study of

nonlinear analysis. In particular the use of fixed-point techniques has been increased enormously in such diverse fields as biology, chemistry, economics, engineering, dynamics, optimal control, game theory, and physics.
From the commencement of modern science until the start of twentieth century, uncertainty was mostly regarded as objectionable in science and the indication was to escape it. This approach progressively changed with the development of probability theory in the field of statistical mechanics. Probability theory effectively defined and categorized the phenomenon of uncertainty and was thought to be appropriate for dealing with all types of ambiguities. With the advent of fuzzy set theory in 1965, a tremendous modification was observed in the classical ideas of probability theory. Irrespective of the expectation of future events, fuzzy set theory is basically concerned with the concepts arising in the linguistic terms of natural languages, such as hot, very hot, warm, cold, educated, highly educated and so on.

Fuzzy Set

Fuzzy set theory is generalization of the classical or crisp set. Let X be Universal set.The crisp set is defined in such a way as an element will be either a member or non member of a set and its characteristic function will be expressed as $\chi_{\tilde{A}} : X \to \{0, 1\}$
The function can be generalized such that the values assigned to the elements of the universal set X fall within a specified range [0,1] and indicate the membership grade of the elements in the set . Larger values denote higher degrees of element membership. Such a function is called a membership function and is defined as

$$\mu_{\tilde{A}} : X \to [0, 1]$$

Thus a Fuzzy set Ã is defined $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$
Where $\mu_{\tilde{A}}(x)$ is the grade of membership of the element of X in A i.e $x \in A$ where $A \subseteq X$.
There are various notations to denote fuzzy set of a set & its membership function. Here we used Ã as fuzzy set, $\mu_{\tilde{A}}$ as membership function & $\mu_{\tilde{A}}(x)$ as degree of membership for element of X.
The significance of Zadeh’s contribution was that it challenged not only probability theory as a sole agent for uncertainty; but the very foundations upon which the probability theory is based, Aristotelian two – valued logic. For when A is a fuzzy set and x is a relevant object the proposition x is a member of A is not necessarily either true or false as required by two valued logic, but it may be true only to some degree the degree to which x is actually a member of A.
Fuzzy Theory not only helps in the representation of the measurement of uncertainties but also gives a meaningful representation of vague concepts in a simple natural language.

Objective

1. To evaluate the Fixed Points and Coincidence Points of fuzzy metric space with Applications & Fuzzy mappings
2. The 2nd kind class of Volterra integral equations is to study the existence and uniqueness of solutions of a common class of integral equations, which arise from differential equations of the form $dx/dt\, f(x(t)) = K(t, x(t))$,under different assumptions on the tasks involved.

Literature Review

Heilpern [58] introduced the concept of fuzzy mapping and proved a fixed point theorem for fuzzy contraction mappings in metric linear space, which is a fuzzy extension of the Banach contraction principle. Subsequently several other authors [59- 61] have studied existence of fixed points of fuzzy mappings. Many authors have proved fixed and common fixed point theorems in metric and fuzzy metric spaces.
Mishra et al. [6] extended the notion of compatible maps under the name of asymptotically commuting maps in fuzzy metric spaces and prove common fixed point theorems using the continuity of one map and completeness of the involved maps. Singh and Jain [7] introduced the notion of weak and semicompatible maps in fuzzy metric spaces and showed that every pair of compatible maps is weakly compatible but the converse is not true in general. Pant [8] initiated the study of common fixed points of non-compatible maps in metric spaces. For a non-compatible maps, Aamri and El Moutawakil[9] introduced a new property named as (E.A) property,Pant [10] studied the common fixed points for non-compatible maps using (E.A) property in fuzzy metric spaces.
The concept of fuzzy metric spaces has been studied by many authors in several ways. Kramosil and Michalek [2] introduced the concept of KM-fuzzy metric space as a generalization of probabilistic metric space given by

Menger [3] and Schweizer and Sklar [4]. George and Veeramani [5] modified this concept to GV-fuzzy metric space and obtained a hausdorff topology for this kind of fuzzy metric spaces. Fuzzy set theory has applications in applied sciences such as mathematical programming, modeling theory, engineering sciences, image processing, control theory, and communication.

Sintunavarat and Kumam [11] introduced the notion of common limit range property (or (CLR) property) for a pair of maps as a generalization of (E.A) property and prove common fixed point theorems in fuzzy metric spaces. The concept of (CLRg) property for hybrid maps is an extending of single maps. There are some similar results in deferent ways such as [12–14].

Result Analysis

A fuzzy coincidence theorem for a pair of fuzzy mappings satisfying a generalized contraction in a metric space is established in chapter three, which also generalizes the Heilpern contraction theorem for fuzzy mappings. A coincidence theorem for multivalued contractions is obtained as an improvement of the Nadler fixed point theorem. For the existence of the solution of nonlinear integral equations an application of the above-mentioned fuzzy coincidence theorem is achieved, which involves the completeness property of the function space (C[a, b] ,R).

In the same chapter we have also proved some fuzzy coincidence theorems by using MT-function. The last section of this chapter deals with the newly defined concepts of α-commuting and α-weakly compatible mapping. A fuzzy fixed point result for two α-weakly compatible mappings in connection with MT-function is also obtained. Another very fascinating aspect of metric space is quasi-pseudospace. The concepts of K-Cauchy sequences and K-sequentially complete quasi-pseudo-metric spaces are the two forceful inspirations behind chapter four. In this chapter, some local versions of fixed point theorems satisfying Banach, Kannan, and Chatterjea type fuzzy contractive conditions in a left(respectively right) K-sequentially complete quasi- pseudo-metric space are obtained. Our analysis is based on the fact that fuzzy fixed point results can be obtained from the fixed point theorems of multivalued mappings with closed values. An interesting example is also generated for the clarification of results.

Another remarkable feature of fuzzy set theory is associated with the concept of L-fuzzy sets. For the purpose of extension and modification of classical ideas related to fuzzy sets, an innovative notion of L-fuzzy mappings is introduced in chapter five. Motivated by the concept of admissible mappings, an interesting idea of βFL - admissible for a pair of L-fuzzy mappings is also established. On the basis of these definitions, a common L-fuzzy fixed point theorem is proved. The last section of this chapter establishes some new coincidence (and fixed-point) results in connection with a contractive relation (depending upon newly defined notions of DαL and d∞L distances) on a sequence of L-fuzzy mappings and a single valued crisp mapping in a complete metric space. This result not only generalizes several important results of fuzzy mappings and multivalued mappings in the current literature but also dicuss an existence theorem for the solution of a generalized class of nonlinear integral equations.

The most common application of fuzzy controller is Washing Machine which most of us uses in daily life which have fuzzy Controller. In Aero plane there is a fuzzy controller. With the inception of fuzzy theory it get importance due to his human behavior and in every field of mathematics e.g Analysis, topology, Matrix, algebra ,approximation and a lot of etc even in other field it get importance due to its practical human behavior. Many of the researchers are now working on Hybridization by using Fuzzy Theory with some other already established theory.

Due to its humanitarian behavior & result it get be better used in medical Science, Engineering etc.

Let X be any metric linear space and d be any metric in X & $\mu_{\tilde{A}}(x)$ is the grade of membership of the element of X in A i.e x € A where A ⊆ X.

The Collection of all fuzzy sets in X is denoted by Tau(X).

Let $\tilde{A} \in$ TAU(x) and $\alpha \in [0,1]$.The α-level set of Ã is defined by

$\tilde{A}_\alpha = \{x: \mu_{\tilde{A}}(x) \geq \alpha \}$ if $\alpha \in [0,1]$

& $\tilde{A}_0 = \{x: \mu_{\tilde{A}}(x) \geq 0 \}$

Now we distinguish from the collection Tau(X) a subcollection of approximate quantities denoted by W(X).

Defination 5.4.1. A fuzzy set of X is an approximate quantity iff its α-level set is a compact convex subset (non fuzzy) of X for each $\alpha \in [0,1]$, and sup $\mu_{\tilde{A}}(x) = 1$.

When $\tilde{A} \in W(x)$ and $\mu_{\tilde{A}}(x_0) = 1$ for some $x_0 \in X$, we will identify Ã with an approximation of x_0.

Theorem. For a non-normal solid cone P in a complete cone metric space (X, d) and an open subset U of X, let F : $[0, 1] \times \bar{U} \to EL(X)$ be an L−fuzzy mapping satisfying following conditions:

(a) $\zeta \notin [F(e, \zeta)]\alpha L$, for each $\zeta \in \partial U$, $\alpha L \in L$, $e \in [0, 1]$.

(b) $F(e, \bullet) : \bar{U} \to EL(X)$ is a fuzzy mapping satisfying

$ad(\zeta, \zeta') \in s([F(e, \zeta)]\alpha L, [F(e', \zeta')]\alpha L)$

and (1 − a)r ∈ s(˚ζ, [F (˚e, ˚ζ)]αL).
(c) the existence of a continuous increasing function h : (0, 1] → P is ensured for which; such that
h(i) − h(e) ∈ s([F (i, ζ)]αL, [F (e, ζ′))]αL,
h(i) ∈ h(e) + P,
for all e, i ∈ [0, 1], and each ζ ∈ U¯ .
Then F (0, •) has a fixed point if and only if F (1, •) has a fixed point.
Proof. Suppose that F (0, •) has a fixed point z, so that z ∈ [F (0, z)]αL. From (a), z ∈ U.
Define, for αL ∈L,
Clearly Q =
Q := {(e, ζ) ∈ [0, 1] × U : ζ ∈ [F (ζ, e)]αL }.
φ. We define a partial ordering in Q as;
(e, ζ) 4 (i, ζ′) ⇔ e ≤ i and d(ζ, ζ′) ≤ 2
(h(i) − h(e)).

which implies that $\{\zeta_n\}$ is a Cauchy sequence. Thus $\mathring{\zeta} \in X$ exists for which $\zeta_n \to \mathring{\zeta}$.

Choose $n_0 \in$ N, such that for $\theta \quad c$,we have $d(\mathring{\zeta},\zeta_n) \quad \frac{c}{2}$ and $d(\mathring{\zeta},\zeta_n) \quad \frac{c}{2}$ for all $n \geq n_0$. Then, in view of (a), we get

$$ad(\zeta_n,\mathring{\zeta}) \in s([F(e_n,\zeta_n)]_{\alpha_L},[F(\mathring{e},\mathring{\zeta})]_{\alpha_L}),$$

$$ad(\zeta_n,\mathring{\zeta}) \in s(\zeta_n,[F(\mathring{e},\mathring{\zeta})]_{\alpha_L}), \text{ since } \zeta_n \in [F(e_n,\zeta_n)]_{\alpha_L}.$$

So there exists some $\zeta_k \in [F(\mathring{e},\mathring{\zeta})]_{\alpha_L}$, such that

$$d(\zeta_n,\zeta_k) \leq ad(\mathring{\zeta},\zeta_n).$$

Consider

$$d(\mathring{\zeta},\zeta_k) \leq d(\mathring{\zeta},\zeta_n) + d(\zeta_n,\zeta_k)$$

$$\leq d(\mathring{\zeta},\zeta_n) + ad(\mathring{\zeta},\zeta_n)$$

$$\frac{c}{2} + \frac{c}{2} = c \text{ for all } n \geq n_0.$$

Thus $\zeta_k \to \mathring{\zeta} \in [F(\mathring{t},\mathring{\zeta})]_{\alpha_L}$ and hence $\mathring{\zeta} \in U$, implies that $(\mathring{e},\mathring{\zeta}) \in$ Q. Thus (e,ζ) 4 $(\mathring{e},\mathring{\zeta})$ for all $(e,\zeta) \in \mathbf{M}$ which implies that $(\mathring{e},\mathring{\zeta})$ is an upper bound of $\mathbf{M}$. By **Zorn's**

Conclusion

In this study, some fuzzy fixed point and fuzzy coincidence theorems are proved in various spaces to illustrate the utility of fuzzy fixed point theory. Some interesting notions are introduced in the context of expansion which is helpful in the generalizations of classical results. This study is distributed among seven chapters and each one of the research-oriented sections promotes the diversity of ideas in metric spaces. The intentions behind the first two chapters are to give a brief introduction of fuzzy fixed point theory, research motivation and the illumination of the elementary concepts to be used in the entire study.

Cone metric spaces hold a very strong position amongst all the emerging branches of metric spaces, where the distances are considered in the form of vectors from an ordered Banach space. In [71], Jankovi´c et. al. prove that every fixed point result in cone metric spaces, for which the conjecture that the underlying cone is normal and solid holds, may be reduced to the corresponding result for metric spaces. But the situation is different for non-normal solid cones. In chapter six, some L-fuzzy fixed point results for local and global contractions in the context of cone metric spaces, by exempting the normality on cone, are achieved. A homotopy result is also obtained as an application. Some generalized coincidence points, common fixed points and fixed point theorems in various spaces, like metric spaces, quasi-pseudo-metric spaces and cone metric spaces by using different contractive conditions for fuzzy mappings. For the development of the theory some notions, namely α-commuting, α-weakly compatible mapping, L-fuzzy mappings for L-fuzzy sets, βFL -admissible for a pair of L-fuzzy mappings are also established. On the basis of the above definitions some interesting results are obtained, which not only generalize many important results of fuzzy mappings and multivalued mappings in the current literature, but also deduce few existence theorems for the solution of generalized class of nonlinear integral equations.

References

[1]. R. Full_er and H. J. Zimmermann. On computation of the compositional rule of inference under triangular norms. Fuzzy Sets and Systems, 51(3):267 { 275, 1992.

[2]. A. I. Galushkin. Neural Networks Theory. Springer, 2007.

[3]. S. Gottwald. Fuzzy Sets and Fuzzy Logic: The foundations of application from a Mathematical Point of View. Vieweg-Verlag, 1993.

[4]. S. Guadarrama, S. Mu~noz, and C. Vaucheret. Fuzzy prolog: a new approach using soft constraints propagation. Fuzzy Sets and Systems, 144(1):127{150, 2004.

[5]. P. H_ajek. Metamathematics of Fuzzy Logic. Kluwer, 1998.

[6]. F. Herrera and J. L. Verdegay, editors. Genetic Algorithms and Soft Computing. Physica-Verlag, 1996.

[7]. E. Herrera-Viedma and O. Cord_on. Special issue on soft computing applications to intelligent information retrieval on the internet. International Journal of Ap- proximate Reasoning, 34(2):89{95, 2003.

[8]. Z.-G. Hou, M. M. Polycarpou, and H. He. Special issue on neural networks for pattern recognition and data mining. Soft Computing, 12(7), 2008.

[9]. M. Jamshidi. Special issue on neural networks and fuzzy logic: theory and applications in robotics and manufacturing. Comput. Electr. Eng., 19(4), 1993.

[10]. P. Juli_an, G. Moreno, and J. Penabad. On the declarative semantics of multiadjoint logic programs. In Proc of IWANN'09 (this issue), 2009.

[11]. P. Juli_an and C. Rubio-Manzano. A similarity-based wam for bousi-prolog. In Proc of IWANN'09 (this issue), 2009.

[12]. S.N. Mishra, N. Sharma, S.L. Singh, Common fixed points of maps on fuzzy metric spaces, Int. J. Math. Math. Sci. 17 (1994) 253–258.

[13]. B. Singh, S. Jain, Semicompatibility and fixed point theorems in fuzzy metric space using implicit relation, Int. J. Math. Math.Sci. (2005) 2617–2629.

[14]. R.P. Pant, Common fixed points of contractive maps, J. Math.Anal. Appl 226 (1998) 251–258.

[15]. M. Aamri, D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal.Appl. 270 (2002) 181–188.

[16]. B.E. Rhoades, A common fixed point theorem for sequence of fuzzy mappings, Int. J. Math. Math. Sci. 8 (1995) 447_450.

[17]. H.M. Ko, Y.H. Tasi, Fixed point theorems for localized property, Tamkang J. Math. 8 (1) (1977) 81_85.

[18]. S. Reich, Fixed points of contractive functions, Boll. Unione Mat. Ital. 4 (5) (1972) 24_26.

[19]. S.B. Nadler, Multivalued contraction mappings, Pacific J. Math. 30 (1969) 475_488.

[20]. T. Hu, Fixed point theorems for multivalued mappings, Canad. Math. Bull. 23 (1980) 193_197.

[21]. M. Edelstein, An extension of Banach's contraction principle, Proc. Amer. Math. Soc. 12 (1961) 7_12.

[22]. D.F. Bailey, Some theorems on contractive mappings, J. Lond. Math. Soc. 41 (1996) 101_106.

[23]. I. Beg, A. Azam, Fixed points of multivalued locally contractive mappings, Boll. Unione Mat. Ital. (4A) 7 (1990) 227_233.

[24]. R.D. Holmes, On fixed and periodic points under certain set of mappings, Canad. Math. Bull. 12 (1969) 813_822.

[25]. T. Hu, H. Rosen, Locally contractive and expansive mappings, Proc. Amer. Math. Soc. 86 (1982) 656_662.

[26]. C. Waters, A fixed point theorem for locally nonexpansive mappings in normed space, Proc. Amer. Math. Soc. 97 (1986) 695_699.

[27]. Ya.I. Alber, S. Guerr-Delabriere, Principle of weakly contractive maps in Hilbert spaces, in: I. Gohberg, Yu. Lyubich (Eds.), New Results in Operator

[28]. Theory and its Applications, in: Oper. Theory Adv. Appl., vol. 98, Birkhäuser, Basel, 1997, pp. 7_22.

[29]. B.E. Rhoades, Some theorems on weakly contractive maps, Nonlinear Anal. 47 (4) (2001) 2683_2693.

[30]. S. Bae, Fixed point theorems for weakly contractive multivalued maps, J. Math. Anal. Appl. 284 (2003) 690_697.

[31]. I. Beg, M. Abbas, Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition, Fixed Point Theory

[32]. Appl. 2006 (2006) 1_7 (Article ID 74503).

[33]. L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.

[34]. I. Kramosil, J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika 11 (1975) 336–344.
[35]. K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. 28 (1942) 535–537.
[36]. B. Schweizer, A. Sklar, Statiscal metric spaces, Pacific. J. Math.10 (1960) 314–334.