Generalization on Some Results on Multi-Valued of Fixed Point Results in Fuzzy Metric Space

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Abstract

This study basically looks at fixed point theorems and their applications. A fixed point theory taken over fuzzy metric spaces is a combination of fuzzy set theory and fixed point theory. Fuzzy sets and fuzzy logic are a study of the broader application of fuzzy set theory. This paper aims to derive a fixed point theorem on fuzzy metric spaces that looks to a contractible condition. This theorem considers the use of Finnish metric spaces. The results show that extensions of some results in ambiguous metric spaces have been presented.

Keywords: *Fixed point, Fuzzy set theory, Fuzzy metric spaces, F-contraction etc.*

Introduction

Based on the theory of fuzzy sets introduced by Zadeh [1], George and Veeramani [2] provided axioms for fuzzy metric spaces. One of the most influential theories regarding binary functions is the triangular norm (in short, the t-norm), which first appeared in the work of Schweizer and Scheler [3]. This is an important operation in many fields, such as fuzzy sets, fuzzy logic and their applications. Starting with the famous Banach contraction theory [4], a large number of mathematicians began to formulate various contraction conditions under which a fixed point exists. One of the most interesting inspirations is the fixed-point theory established in fuzzy metric spaces, which was introduced by Grabiak [5], where a fuzzy metric version of the Banach contraction theory was presented. Subsequently, Gregory and his co-authors introduced a variety of fuzzy contractive mappings in fuzzy metric spaces (see [6, 7, 8]). On the other hand, Mihet [9,10] proposed a fixed-point theorem for weak Banach contractions in W-complete fuzzy metric spaces and generalized previous results, including some new types of contractions, such as Edelstein fuzzy Contractive mapping, fuzzy sai-contractive mapping, etc. (for details, see [11]). Recently, Wardowski [12] introduced a new concept of fuzzy H-contractive mapping and derived some relevant fixed-point theorems. Additionally, Wardowski [13] introduced a contraction called F-contraction and proved a fixed-point theorem in metric spaces. Recently, [11,14] gave other contractions in fuzzy metric spaces. Throughout this paper, we present a new contraction called fuzzy F-contraction, which differs from [12, 15] because our contraction involves a simpler condition, that is, the mapping is only strictly increasing. Furthermore, we encounter fixed-point theorems for fuzzy F-contractions in the setting of fuzzy metric spaces. In particular, we first give a lemma regarding Cauchy sequences in fuzzy metric spaces. Second, we introduce the concept of fuzzy F-contraction, in which the function only requires a strictly increasing condition. Third, using the lemma mentioned above, we obtain some fixed-point theorems for fuzzy F-contractions with trivial conditions and straightforward proofs. Fourth, we present some examples to support our results. Our examples show that our findings are indeed true generalizations to the existing literature.

Generalization of Metric Spaces

There are many ideas about spaces whose structure is smaller than a metric space but larger than a topological space. Disordered place is where distance is not defined but there is a regular connection. A viewpoint space is a space that defines the point-to-point distance rather than the point-to-point distance. From a category standpoint, they are of exceptional quality. Continuity space is an extension of metric space and some determinism and can be used to combine the concepts of metric space and space. There are also many ways to relax metric theory, leading to different general theories of space. These generalizations can be combined. The terminology used to describe them is not yet fully standardized. In particular, pseudometrics in functional analysis often arise from quasi-norms of the vector space, so it is convenient to call them "quasi-measures". This is about the use of time in topology.

Multi-valued Function:

A function is multi-valued if it specifies more than one output value for a particular input. In other words, there are many possible outputs for a given input.

Fixed point:

In mathematics, a fixed point of a function is a point that is mapped by the function itself. In other words, if f(x)=x, then x is a fixed point of the function f.

Now, if you are referring to a multi-valued fixed point, it may indicate a situation where a given input has multiple outputs, and some of those outputs are fixed points. In mathematical terms, if $f(x)=x_1$ and $f(x)=x_2$, where x_1 is not equal to x_2, and both x_1 and x_2 are fixed points, then you have a multi-valued fixed point.

However, it is important to note that the concept of a multi-valued fixed point may not be commonly used or well defined in standard mathematical discussions.

Fixed point theorem on fuzzy metric space reference (CLRG) object

The concept of fuzzy sets was proposed by Zadeh in 1965 as another way to resolve uncertainty in everyday life. Problems in mathematics refer to the pursuit of some possible goals, albeit with some limitations, and there are also problems of thinking about different goals. As a result, it is very difficult to achieve success that will allow us to achieve the best situation among all business goals. One possible technique that may be useful for your purpose is the use of fuzzy loss. It has been widely developed and used by many designers to apply this concept in topology and theory. George and Veeramani modified the fuzzy metric space concept introduced by Kramosil and Michalek and obtained the Hausdorff topology. Jungk shared some great ideas for personal maps. The importance of the CLRG property ensures that there is no need to expand the subspace.

2008 Altun I showed the basic concepts of fixed-connected fuzzy metric space. Sintunavarat announced the discovery of other objects (CLRG). Chauhan and his colleagues used the idea of general limits to simultaneously prove the stability theorem for nonlinear fluids in fuzzy metric spaces. The verifiable link sum (CLRG) product is used as a tool to find stable content of compressed images. The purpose of this article is E.A. It is a concept developed by. Feature of the combined report (CLRG) and the same answer to the question from Rhodes. Character values (CLRG) ensure continuity without the need for multiple subspaces.

Preliminaries

Nadler (1969) A point x 0 ∈X

Joseph (2013) Every valued mapping can be seen as a multi-valued mapping. ,

Banach (1922) LED(X, D) is a metric space.

Nadler (1969) Let x and y be nonempty sets. t is called a multi-valued mapping from We denote a multi-valued map as:

$t{:}x=2^y$

Aydi et al. (2012) Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d{:}X \times X \to \mathbb{R}^+$ is called a b-metric provide that, for all $x, y, z \in X$,

1. $d(x,y) = 0$ if and only if $x = y$
2. $d(x,y) = d(y,x)$
3. $d(x,z) \leq s[d(x,y) + d(y,z)]$.

A pair(X, d) is called a b-metric space.

Boriceanu (2009) The space $l_p\,(0 < p < 1)$, $l_p = \{(x_n{:} \sum_{n=1}^{\infty} |x_n|^p < \infty\}$, together with the function $d{:}l_p \times l_p \to \mathbb{R}^+$.

Boriceanu (2009) The space $L_p(0 < p < 1)$ for all real function $x(t), t \in [0,1]$ such that $\int_0^1 |x(t)|^p dt < \infty$, is b-metric space if we take $d(x,y) = (\int_0^1 |x(t) - y(t)|^p dt)^{\frac{1}{p}}$.

Definition1.[16] Amapping$T$: [0,1]×[0,1] →[0,1]iscalledatriangularnorm (t-norm) if the following conditions are satisfied:

(T1)$T(a,1)=a, a\in[0,1]$,

(T2)$T(a,b)=T(b,a), a,b\in[0,1]$,
(T3)$a\geq b, c\geq d \Rightarrow T(a,c)\geq T(b,d), a,b,c,d\in[0,1]$,
(T4)$T(a,T(b,c))=T(T(a,b),c), a,b,c \in[0,1]$.

Results

Theorem *Let (X, M, T) be a complete fuzzy metric space and $f : X \to X$ be a continuous mapping. Let $F, G : X \to C(X)$ are weakly commuting with f and F or G is f-strongly demicompact. If, for some $k : (0,\infty) \to (0,1)$ and altering distance function φ, the following condition is satisfied:*

$$\varphi(\widetilde{M}(Fx, Gy, t)) \leqslant k(t) \cdot \varphi(M(fx, fy, t)), \quad x, y \in X,\ x \neq y,\ t > 0,$$

then there exists $x \in X$ such that $fx \in Fx \cap Gx$.

Proof. The proof is similar with that of the Theorem 1, except in the part related to Cauchy sequence. Namely, since F or G is f-strongly demicompact, $fx_{2n+1} \in Fx_{2n}$ or $fx_{2n+2} \in Gx_{2n+1}$ and $\lim_{n\to\infty} M(fx_{2n}, fx_{2n+1}, t) = 1, t > 0$, we conclude that there exist convergent subsequence $\{fx_{2n_p}\}_{p\in\mathrm{N}}$ or $\{fx_{2n_p+1}\}_{p\in\mathrm{N}}$, respectively, such that

$$\lim_{p\to\infty} fx_{2n_p} = x.$$

The last part of the proof is analogous as in Theorem 1, where instead of sequence $\{fx_n\}_{n\in\mathrm{N}}$, we deal with subsequences $\{fx_{2n_p}\}_{p\in\mathrm{N}}$ and $\{fx_{2n_p+1}\}_{p\in\mathrm{N}}$.

If in Theorems 1 and 2, we take that $F = G$ and that f is the identity mapping, we get the following corollary.

Corollary 1. *Let (X, M, T) be a complete fuzzy metric space, $F : X \to C(X)$, and one of the following conditions is satisfied:*

(a) *F is weakly demicompact mapping,*

or

(b) *(X, M, T) is strong fuzzy metric space and T is t-norm of H-type.*

If there exist $k : (0,\infty) \to (0,1)$ and altering distance function φ such that:

$$\varphi(\widetilde{M}(Fx, Fy, t)) \leqslant k(t) \cdot \varphi(M(x, y, t)), \quad x, y \in X,\ t > 0,$$

then there exists $x \in X$ such that $x \in Fx$.

Moreover, if the mapping F in Corollary 1 is single-valued we got the result in [30].

Example 1.

(a) Let $X = [0,2], T = T_P, M(x,y,t) = t/(t + d(x,y))$, where d is Euclidian metric. Then (X, M, T) is a fuzzy metric space. Let $F(x) = \{1,2\}$, $x \in X$. Since F is weakly demicompact and condition (25) is satisfied, by Corollary 1(a) follows that there exists $x \in X$ such that $x \in Fx$.

(b) Let $X = [0,2], T = T_M, M^*(x,y,t) = t/(t + d^*(x,y))$, where d^* is ultrametric. Ultrametric space is metric space, where instead of triangle inequality condition, the following is satisfied: $d^*(x,z) \leqslant \max\{d^*(x,y), d^*(y,z)\}$. Then (X, M^*, T) is a strong fuzzy metric space [11]. For $F(x) = \{1,2\}$, $x \in X$, condition (25) is satisfied and by Corollary 1(b) follows that there exists $x \in X$ such that $x \in Fx$.

Conclusion

In this contribution, we present a new concept of TFP results using management in the FCM environment. Additionally, some special TFP theorems are explained by the triangular form of FCM using different conventions. The management function is an extended one-to-one self-mapping that is then collected in the FCM environment. The existence of the VIE system and its special solutions were examined in more detail. The authors use various methods in the VIE field, such as Riemann integral equations, Lebesgue integral equations, and nonlinear integral equations, to support their results. This experiment focuses on some new optimization concepts and their application to fuzzy metric space. In this test we present the highlights of the descriptive questions related to the research questions. It is inevitable that these theorems will be used in proofs. We look forward to using this technique in future trials. In this work, we propose another way of self-mapping by using the - function to set the distance between two points in the blur region. Based on this self-representation, some point theorems are proven in incomplete fuzzy metric space and small fuzzy metric space. Clearly, the present analysis improves our view of stable points in the M-fuzzy metric space.

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