HEAT SOURCE-INDUCED DYNAMIC THERMOELASTIC RESPONSE IN A SEMI-INFINITE CYLINDER

J. J. Tripathi[1]

[1] *Department of mathematics, Dr. Ambedkar College, Deekshabhoomi,Nagpur-440010, Maharashtra, India.*
E-mail: tripathi.jitesh@gmail.com

ABSTRACT

We address the problem of thermal stresses and temperature distribution in a semi-infinite cylinder with traction-free boundaries on both ends, exposed to a specified axisymmetric temperature distribution. The study applies the Lord-Shulman and classical coupled thermoelasticity theories, utilizing integral transforms for solving the problem. First, an exact solution is obtained in the transform domain, followed by analytical inversion of Hankel transforms and numerical inversion of Laplace transforms. The resulting thermal stresses are evaluated for copper material, and a comparison of results is made between the two theories.

Keyword: - *Lord-Shulman theory, Classical coupled theory, Axisymmetric Temperature Distribution, Traction-free.*

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1. INTRODUCTION

The study of thermoelasticity, which couples thermal and elastic fields, has significantly evolved since its inception. Biot [1] pioneered the theoretical framework by integrating thermodynamics with irreversible processes, laying the foundation for later developments. Lord and Shulman [2] further generalized the theory, introducing a dynamical perspective to capture non-classical behavior in thermoelastic systems. Over the years, advancements have led to the study of anisotropic media in thermoelasticity, as discussed by Dhaliwal and Sherief [3], and the formulation of theories for different material behaviors by Green and Lindsay [4].

Incorporating the concept of second sound, which describes the finite speed of heat propagation, Chandrasekariah [5] reviewed this phenomenon in detail, making it a vital aspect of contemporary thermoelasticity. The generalization of thermoelastic theories to account for various geometries and boundary conditions has been a significant focus in recent works. Youssef [6, 7, 8] extensively studied thermoelastic problems involving cylindrical and spherical cavities subjected to moving heat sources, providing analytical solutions for these cases.

Chen and Gurtin [30] developed a theory of heat conduction that incorporates two distinct temperature fields, addressing non-equilibrium thermodynamic processes in materials. Mallik and Kanoria [10] and El-Maghraby [11, 12] analyzed two-dimensional thermoelastic problems with heat sources in thick plates. These studies have contributed to a more comprehensive understanding of how heat fluxes influence stress fields within various materials. Furthermore, McDonald [13] explored the effects of laser-generated ultrasound waveforms on metals, offering insights into how thermal and elastic waves interact in these contexts.

Recent efforts have focused on the generalization of thermoelastic solutions for different geometries. For example, Baghri and Eslami [14, 24] provided unified solutions for cylinders and spheres, while Aquadi [15] and Sherief and El-Maghraby [16] dealt with axisymmetric and crack-related problems in thermoelastic solids. In numerical methods, the Gaver-Stehfast algorithm [17, 18, 19], as reviewed by Knight and Raiche [20], has emerged as a powerful tool for inverting Laplace transforms in thermoelastic problems, especially in stochastic processes and transient calculations.

The inversion of Laplace integrals remains a critical mathematical technique in solving complex thermoelastic problems, as demonstrated by Widder [21]. Numerical approaches, such as those outlined in Numerical Recipes by Press et al. [22], continue to play a significant role in developing solutions for practical applications.
Several studies have explored the effects of boundary conditions and heat supply mechanisms on thermoelastic materials. For example, Awad [23] examined spatial decay estimates in bounded domains, while Furukawa et al. [26] and Das and Lahiri [25] studied interactions in cylindrical and spherical geometries. Ghosh and Kanoria [27] applied the Green-Lindsay theory to functionally graded materials, further enriching the thermoelastic theory's applicability to advanced materials.
This body of work demonstrates the growing importance of advanced thermoelastic theories and their applications in modern engineering problems involving thermal stresses and heat transfer in complex materials and geometries. In the present problem, the effects of the induced temperature and heat source on the temperature distribution and stress fields in a homogeneous isotropic thermoelastic thick cylinder of height $2h$ and infinite extent have been studied. The analytic solutions are found in Laplace transform domain. Then numerical methods are used to invert the Laplace transforms and to evaluate the integrals involved, so as to obtain the solution in the physical domain. The derived expressions are computed numerically for copper material and the results are presented graphically.

2. FORMULATION OF THE PROBLEM

Consider an axisymmetric homogeneous isotropic thick plate of height $2h$ defined as $0 < r < \infty$, $-h \le z \le h$.We take the axis of symmetry as the z axis and the origin of the system of co-ordinates at the middle plane between the upper and lower faces of the plate. The problem is studied using the cylindrical polar co-ordinates (r,ϕ,z). Due to the rotational symmetry about the z axis of the problem all quantities are independent of the co-ordinate ϕ .

The displacement vector, thus, has the form

$$\vec{u} = (u,0,w)$$

The equations of motion can be written as [15]

$$\mu\nabla^2 u - \frac{\mu}{r^2}u + (\lambda+\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial T}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} - \gamma\frac{\partial T}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2} \tag{2}$$

The generalized equation of heat conduction has the form [15]

$$k\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho c_E T + \gamma T_0 e) - \rho\left(1 + \tau_0\frac{\partial}{\partial t}\right)Q \tag{3}$$

where T is the absolute temperature and e is the cubical dilatation given by the relation

$$e = \frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} \tag{4}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{5}$$

The following constitutive relations supplement the above equations

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma(T - T_0) \tag{6}$$

$$\sigma_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda e - \gamma(T - T_0) \tag{7}$$

$$\sigma_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{8}$$

We shall use the following non-dimensional variables

$r' = c_1\eta r\,,\; z' = c_1\eta z\,,\; u' = c_1\eta u\,,\; w' = c_1\eta w\,,\; t' = c_1^2\eta t$

$\tau_0' = c_1^2\eta\tau_0\,,\; \sigma_{ij}' = \dfrac{\sigma_{ij}}{\mu}\,,\; \theta = \dfrac{\gamma(T-T_0)}{(\lambda+2\mu)}\,,\; Q' = \dfrac{\rho\gamma Q}{k c_1^2\eta^2(\lambda+2\mu)}$

where $\eta = \dfrac{\rho c_E}{k}$, $c_1 = \sqrt{\dfrac{\lambda+2\mu}{\rho}}$, c_1 is the speed of propagation of isothermal elastic waves.

Using the above non-dimensional variables, the governing equations take the form (dropping the primes for convenience)

$$\nabla^2 u - \frac{u}{r^2} + \left(\beta^2 - 1\right)e - \beta^2\frac{\partial\theta}{\partial r} = \beta^2\frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\nabla^2 w + \left(\beta^2 - 1\right)\frac{\partial e}{\partial z} - \beta^2\frac{\partial\theta}{\partial z} = \beta^2\frac{\partial^2 w}{\partial t^2} \tag{10}$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\theta + \varepsilon e) - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\beta^2 Q \tag{11}$$

while the constitutive relations (6)-(8), becomes

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + \left(\beta^2 - 2\right)e - \beta^2\theta \tag{12}$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + \left(\beta^2 - 2\right)e - \beta^2\theta \tag{13}$$

$$\sigma_{rz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{14}$$

We note that the equation (4) retains the form

Also $\beta^2 = \dfrac{(\lambda+2\mu)}{\mu}$

Combining equations (9) and (11), we obtain upon using equation (5),

$$\nabla^2 e - \nabla^2\theta = \frac{\partial^2 e}{\partial t^2} \tag{15}$$

We assume that the initial state is quiescent, that is, all the initial conditions of the problem are homogeneous.

The boundary conditions are taken as

$$\begin{aligned} \theta &= f(r,t) \quad , \quad z = \pm h \\ \sigma_{zz} &= \sigma_{rz} = 0 \quad , \quad z = \pm h \end{aligned} \tag{16}$$

Where $f(r,t)$, is a known function of r and t.

2.1 Solution in the transformed Domain:

Applying the Laplace transform defined by the relation,

$$\bar{f}(r,z,s) = L\left[f(r,z,t)\right] = \int_0^\infty e^{-st} f(r,z,t)\,dt \tag{17}$$

to all the equations (9)-(15), we obtain,

$$\nabla^2\bar{u} - \frac{\bar{u}}{r^2} + \left(\beta^2 - 1\right)\bar{e} - \beta^2 \frac{\partial\bar{\theta}}{\partial r} = \beta^2 s^2\,\bar{u} \tag{18}$$

$$\nabla^2\bar{w} + \left(\beta^2 - 1\right)\frac{\partial\bar{e}}{\partial z} - \beta^2 \frac{\partial\bar{\theta}}{\partial z} = \beta^2 s^2\,\bar{w} \tag{19}$$

$$\left(\nabla^2 - s - \tau_0 s^2\right)\bar{\theta} = \left(1 + \tau_0 s\right)(\varepsilon s \bar{e} - \bar{Q}) \tag{20}$$

$$\left(\nabla^2 - s^2\right)\,\bar{e} = \nabla^2\bar{\theta} \tag{21}$$

$$\bar{\sigma}_{rr} = 2\frac{\partial\bar{u}}{\partial r} + \left(\beta^2 - 2\right)\bar{e} - \beta^2\bar{\theta} \tag{22}$$

$$\bar{\sigma}_{zz} = 2\frac{\partial\bar{w}}{\partial z} + \left(\beta^2 - 2\right)\bar{e} - \beta^2\bar{\theta} \tag{23}$$

$$\bar{\sigma}_{rz} = \left(\frac{\partial\bar{u}}{\partial z} + \frac{\partial\bar{w}}{\partial r}\right) \tag{24}$$

The boundary conditions (17), in the transformed domain, take the form

$$\bar{\theta} = \bar{f}(r,s) \qquad , \qquad z = \pm h \tag{25}$$

$$\bar{\sigma}_{zz} = \bar{\sigma}_{rz} = 0 \qquad , \qquad z = \pm h$$

Eliminating $\bar{e}$ between the equations (20) and (21), we get,

$$\left\{\nabla^4 - \left(s^2 + s(1+\tau_0 s)(1+\varepsilon)\right)\nabla^2 + s^3(1+\tau_0 s)\right\}\bar{\theta} = -\left(1+\tau_0 s\right)(\nabla^2 - s^2)\bar{Q} \tag{26}$$

After factorization the above equation becomes,

$$\left(\nabla^2 - k_1^2\right)\left(\nabla^2 - k_2^2\right)\bar{\theta} = -\left(1+\tau_0 s\right)(\nabla^2 - s^2)\bar{Q} \tag{27}$$

where k_1^2 and k_2^2 are the roots with positive real parts of the characteristic equation

$$k^4 - \left(s^2 + s(1+\tau_0 s)(1+\varepsilon)\right)k^2 + s^3(1+\tau_0 s) = 0 \tag{28}$$

The solution of Eq. (27) can be written in the form,

$$\bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2 + \bar{\theta}_p \tag{29}$$

where . $\bar{\theta}_i$ is a solution of the homogenous equation,

$$\left(\nabla^2 - k_1{}^2\right)\bar{\theta}_i = 0 \quad ,i = 1,2. \tag{30}$$

and $\bar{\theta}_p$ is a particular solution of equation (28)

In order to solve the problem, we shall use the Hankel transform of order zero with respect to r . This transform of a function $\frac{\partial T(r,z,t)}{\partial r} = 0$ is defined by the relation,

$$\bar{f}^*(\alpha,z,s) = H\left[\bar{f}(r,z,s)\right] = \int_0^\infty \bar{f}(r,z,s)\, r\, J_0(\alpha r)dr \tag{31}$$

where J_0 is the Bessel function of the first kind of order zero.

The inverse Hankel transform is given by the relation

$$\bar{f}(r,z,s) = H^{-1}\left[\bar{f}^*(\alpha,z,s)\right] = \int_0^\infty \bar{f}^*(\alpha,z,s)\,\alpha\, J_0(\alpha r)d\alpha \tag{32}$$

Applying the Hankel transform to equation (30), we get

$$\left\{D^2 - \left(k_1{}^2 + \alpha^2\right)\right\}\bar{\theta}_i^* = 0 \quad ,i = 1,2. \quad \text{, where } D = \partial/\partial z$$

The solution of the above equation can be written in the form

$$\bar{\theta}_i^* = A_i(\alpha,s)\left(k_i{}^2 - s^2\right)\cosh(q_i z) \tag{33}$$

where $q_i = \sqrt{\alpha^2 + k_i{}^2}$

Applying the Hankel transform to both sides of equation (27) ,we get

$$\left(D^2 - q_1{}^2\right)\left(D^2 - q_2{}^2\right)\bar{\theta}_p{}^* = -(1+\tau_0 s)\left(D^2 - q^2\right)\bar{Q}^* \tag{34}$$

where $q = \sqrt{\alpha^2 + s^2}$

We take the heat source $Q(r,z,t)$ in the following form

$$Q(r,z,t) = \frac{\delta(t)\,\delta(r)\cosh z}{2\pi r} \tag{35}$$

This is a cylindrical shell heat source releasing heat instantaneously at $t = 0$ and situated at the centre $r = a$ varying in the axial direction.

On taking Laplace transform and Hankel transform, we get,

$$\bar{Q}^* = \cosh z \tag{36}$$

The solution of the equation (35) has the form,

$$\bar{\theta}_p{}^* = \frac{(1-\tau_0 s)(1-q^2)}{(1-q_1{}^2)(1-q_2{}^2)} \cosh z \tag{37}$$

Then the complete solution in the transformed domain can be written as

$$\bar{\theta}^*(\alpha, z, s) = \sum_{i=1}^{2} A_i(\alpha, s)\left(k_i{}^2 - s^2\right)\cosh q_i z - \frac{(1+\tau_0 s)(1-q^2)}{(1-q_1{}^2)(1-q_2{}^2)} \cosh z \tag{38}$$

On taking the inverse Hankel transform of both sides, we get,

$$\bar{\theta}(r, z, s) = \int_0^\infty \left\{ \sum_{i=1}^{2} A_i(\alpha, s)\left(k_i{}^2 - s^2\right)\cosh q_i z - \frac{(1+\tau_0 s)(1-q^2)}{(1-q_1{}^2)(1-q_2{}^2)} \cosh z \right\} \alpha\, J_0(\alpha r)\, d\alpha \tag{39}$$

Similarly eliminating θ between equations (20) and (21), we get,

$$\left(\nabla^2 - k_1{}^2\right)\left(\nabla^2 - k_2{}^2\right)\bar{e} = -(1+\tau_0 s)\nabla^2 \bar{Q} \tag{40}$$

On taking Hankel transform of equation (40), we get,

$$\left(D^2 - q_1{}^2\right)\left(D^2 - q_2{}^2\right)\bar{e}^* = -(1+\tau_0 s)\left(D^2 - \alpha^2\right)\bar{Q}^* \tag{41}$$

Complete solution of equation (41) is of the form,

$$\bar{e}^*(\alpha, z, s) = \sum_{i=1}^{2} A_i(\alpha, s) k_i{}^2 \cosh q_i z - \frac{(1+\tau_0 s)(1-\alpha^2)}{(1-q_1{}^2)(1-q_2{}^2)} \cosh z \tag{42}$$

Taking the inverse Hankel Transform, we get,

$$\bar{e}(\alpha, z, s) = \int_0^\infty \left\{ \sum_{i=1}^{2} A_i(\alpha, s) k_i{}^2 \cosh q_i z - \frac{(1+\tau_0 s)(1-\alpha^2)}{(1-q_1{}^2)(1-q_2{}^2)} \cosh z \right\} \alpha\, J_0(\alpha r)\, d\alpha \tag{43}$$

Taking Hankel transform of equation (19) and using equations (39) and (43), the complete solution can be written as,

$$\bar{w}^*(\alpha, z, s) = B(\alpha, s)\sinh q_3 z + \sum_{i=1}^{2} A_i(\alpha, s) q_i \sinh q_i z - \frac{(1+\tau_0 s)}{(1-q_1{}^2)(1-q_2{}^2)} \sinh z \tag{44}$$

where $q_3 = \sqrt{\alpha^2 + \beta^2 s^2}$

On inverting the Hankel transform, we get,

$$\bar{w}(\alpha,z,s)=\int_0^\infty\left\{B(\alpha,s)\sinh q_3 z+\sum_{i=1}^{2}A_i(\alpha,s)q_i\sinh q_i z-\frac{(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\sinh z\right\}\alpha J_0(\alpha r)\,d\alpha \tag{45}$$

Taking the Hankel and Laplace transform of both sides of equation (4) and using equations (43) and (45), we get,

$$H\left[\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u})\right]=\left\{-B(\alpha,s)q_3\cosh q_3 z-\alpha^2\left[\sum_{i=1}^{2}A_i(\alpha,s)\cosh q_i z-\frac{(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\cosh z\right]\right\} \tag{46}$$

On applying inverse Hankel Transform on both sides of equation (46), we get,

$$\bar{u}=\int_0^\infty\left\{-B(\alpha,s)q_3\cosh q_3 z-\alpha^2\left[\sum_{i=1}^{2}A_i(\alpha,s)\cosh q_i z-\frac{(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\cosh z\right]\right\}J_1(\alpha r)\,d\alpha \tag{47}$$

The stress tensor components are in the form

$$\bar{\sigma}_{zz}=\int_0^\infty\left\{2B(\alpha,s)q_3\cosh q_3 z+\left(\alpha^2+q_3^2\right)\left[\sum_{i=1}^{2}A_i(\alpha,s)\cosh q_i z-\frac{(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\cosh z\right]\right\}\alpha J_0(\alpha r)\,d\alpha \tag{48}$$

$$\bar{\sigma}_{rz}=\int_0^\infty\left\{-\left(\alpha^2+q_3^2\right)B(\alpha,s)\cosh q_3 z-2\alpha^2\left[\sum_{i=1}^{2}A_i(\alpha,s)\,q_i\sinh q_i z-\frac{(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\sinh z\right]\right\}J_1(\alpha r)\,d\alpha \tag{49}$$

After applying the Hankel transform, the boundary become,

$$\bar{\theta}^*(\alpha,z,s)=\bar{f}^*(\alpha,s)\quad,\qquad z=\pm h \tag{50}$$

$$\bar{\sigma}_{zz}^*=\bar{\sigma}_{rz}^*=0\quad,\qquad z=\pm h \tag{51}$$

On applying the boundary conditions (50) and (51) to determine the unknown parameters, we get,

$$\sum_{i=1}^{2}A_i(\alpha,s)\left(k_i^2-s^2\right)\cosh q_i h-\frac{(1+\tau_0 s)\left(1-q^2\right)}{(1-q_1^2)(1-q_2^2)}\cosh(h)=\bar{f}^*(\alpha,s) \tag{52}$$

$$\left(\alpha^2+q_3^2\right)\sum_{i=1}^{2}A_i(\alpha,s)\cosh q_i h-2q_3\,B(\alpha,s)\cosh q_3 h=\frac{(1+\tau_0 s)\left(\alpha^2+q_3^2\right)}{(1-q_1^2)(1-q_2^2)}\cosh(h) \tag{53}$$

$$2\alpha^2\sum_{i=1}^{2}A_i(\alpha,s)\,q_i\sinh q_i h+\left(\alpha^2+q_3^2\right)B(\alpha,s)\cosh q_3 h=\frac{2\alpha^2(1+\tau_0 s)}{(1-q_1^2)(1-q_2^2)}\sinh(h) \tag{54}$$

On solving equations (52)-(54) numerically, we get the complete solution of the problem in the transformed domain.

3. INVERSION OF DOUBLE TRANSFORMS:

Due to the complexity of the solution in the laplace transform domain, the inverse of the Laplace transform is obtained using the Gaver-Stehfast algorithm [17],[18],[19]. A detailed explanation can be found in Knight and

Raiche (1982) [19].The final formula based on the work done by Widder[21] (1934) who developed an inversion operator for the Laplace transform is given here. Gaver [17] and Stehfast [18, 19] modified this operator and derived the formula

$$f(t)=\frac{\ln 2}{t}\sum_{j=1}^{K}D(j,K)F\left(j\frac{\ln 2}{t}\right) \tag{55}$$

With

$$D(j,K)=(-1)^{j+M}\sum_{n=m}^{\min(j,M)}\frac{n^{M}(2n)!}{(M-n)!n!(n-1)!(j-n)!(2n-j)!} \tag{56}$$

where K is an even integer, whose value depends on the word length of the computer used. $M=K/2$ and m is the integer part of the $(j+1)/2$. The optimal value of K was chosen as described in Stehfast algorithm, for the fast convergence of results with the desired accuracy. The Romberg numerical integration technique [22] with variable step size was used to evaluate the integrals involved. All the programs were made in mathematical software MATLAB.

4. NUMERICAL RESULTS AND DISCUSSION:

For numerical calculations we take

$$f(r,t)=\theta_0 H(a-r)H(t)$$

where θ_0 is a constant.

On taking Hankel and Laplace transform of the above function, we get,

$$\bar{f}^{*}(\alpha,s)=\frac{a\theta_0 J_1(\alpha a)}{\alpha s} \tag{57}$$

Copper material was chosen for purposes of numerical evaluations. The constants of the problem are shown below

$k=386 J.K^{-1}.m^{-1}.s^{-1}$	$\alpha_t=1.78\times10^{-5}K^{-1}$	$c_E=383.1 J.Kg^{-1}.K^{-1}$	$\eta=8886.73 s.m^{-2}$	$\mu=3.86\times10^{10}N.m^{-2}$
$\lambda=7.76\times10^{10}N.m^{-2}$	$\rho=8954\ kg.m^{-3}$	$c_1=4.158\times10^{3}m.s^{-1}$	$\tau_0=0.02s$	$T_0=293\ K$
$\varepsilon=0.0168 N.m.J^{-1}$	$\beta^2=4$			
$a=1$	$\theta_0=1$	$h=1$		

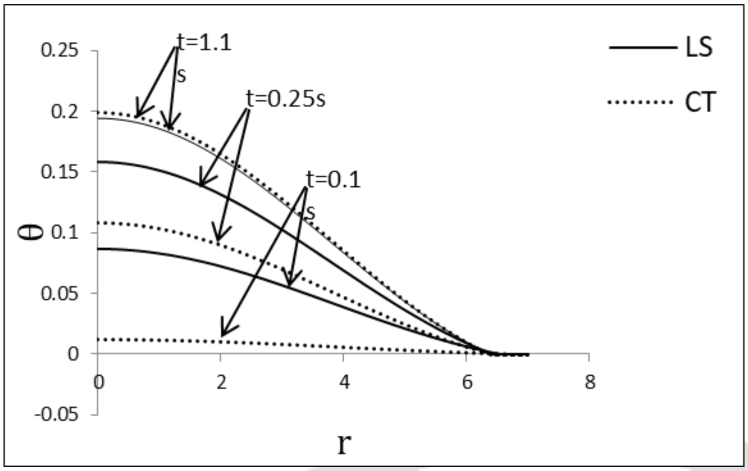


Fig.1. Temperature distribution θ in the middle plane.

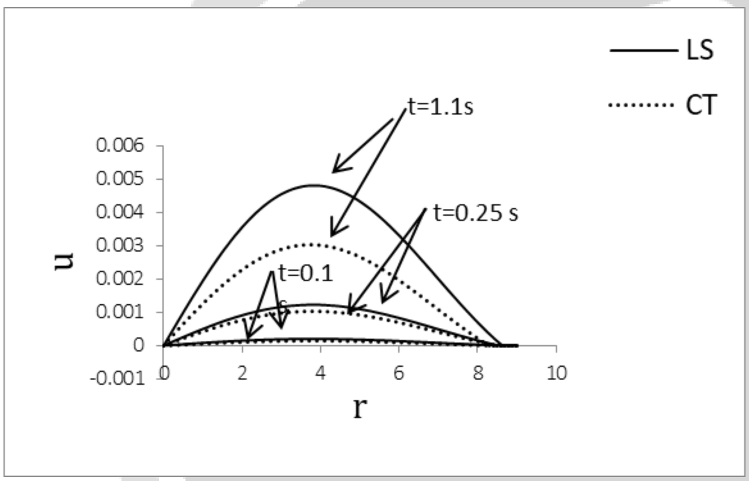
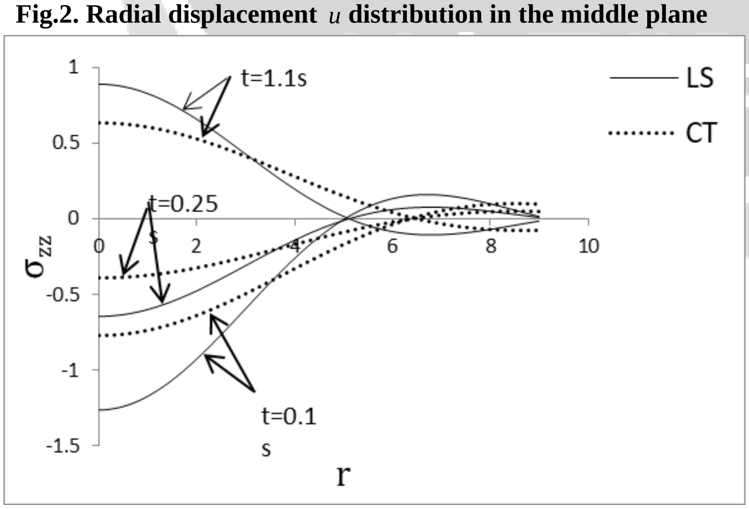


Fig.2. Radial displacement u distribution in the middle plane



Fig, 3. Axial stress component σ_{zz} along the radial direction in the middle plane.

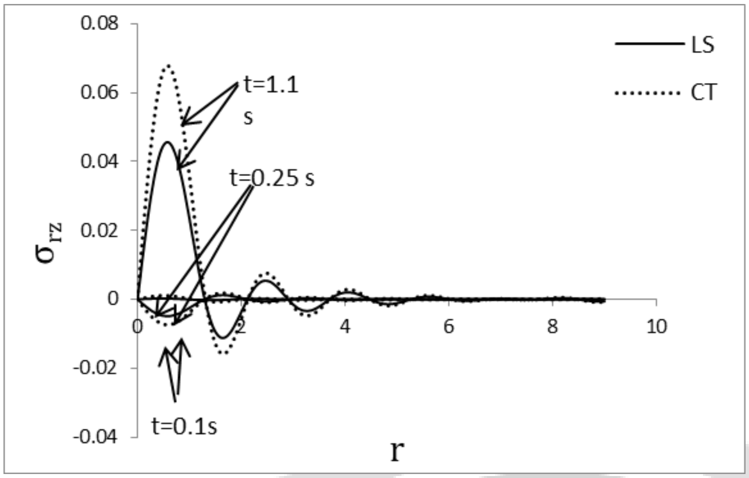


Fig. 4. Shear Stress Component σ_{rz} along the radial direction in the middle plane.

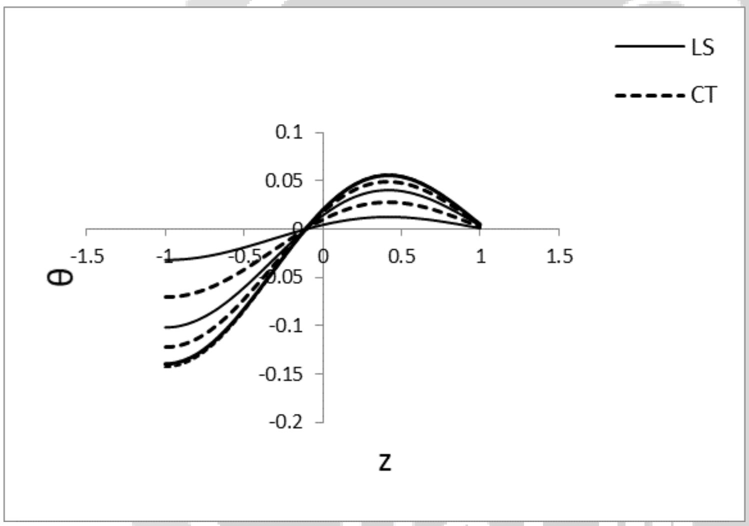
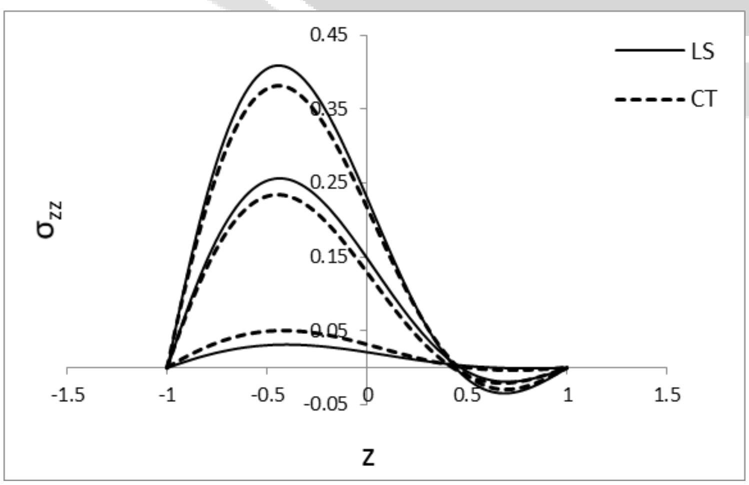


Fig.5. Temperature distribution θ along z axis at $r = 1m$.



Fig, 6. Axial stress component σ_{zz} along z axis at $r = 1m$.

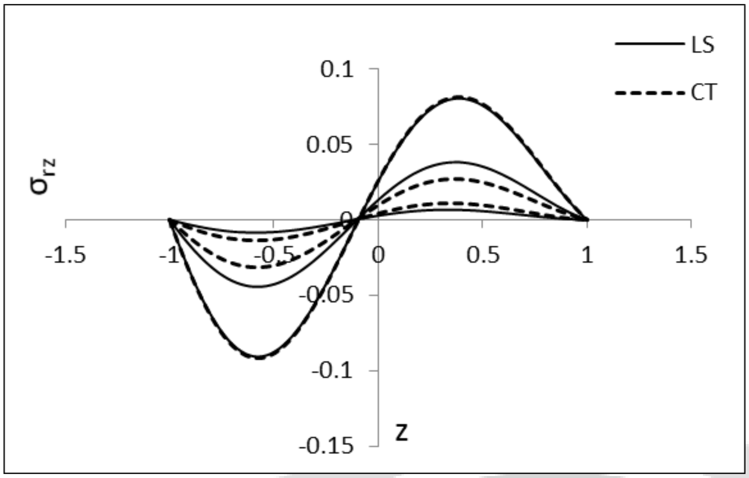


Fig. 7. Shear Stress Component σ_{rz} along z axis at $r = 1m$.

The numerical values for temperature θ, the radial displacement component u, the axial stress component σ_{zz} and the shear stress component σ_{rz} have been calculated at the middle of the plane ($z = 0$) for different time instants $t = 0.1, 0.25, 1.1$ along the radial direction and are displayed graphically for Lord-Shulman theory (L-S theory) and Classical Coupled Thermoelasticity (CT theory) as shown in fig 1,2,3 and 4 respectively . Since the displacement function w is an odd function of z, its value on the middle plane is always zero and it is not represented graphically here.

Fig. 1 , shows the nondimensional temperature distribution along the radial direction at the middle plane ($z = 0$) at different time instants $t = 0.1, 0.25, 1.1$. Classical Coupled Theory of thermoelasticity (CT) predicts an infinite speed of wave propagation, whereas the Lord-Shulman (LS) model of generalized thermoelasticity involves the introduction of one relaxation time τ_0 , due to which the waves assume finite propagation speeds. Hence the variation in values is clearly seen for the two theories in the plots. But the nature of the curve seems to be the same in both the theories. It is also observed that the nondimensional temperature θ drops gradually along the radial direction.

Fig 2 . shows the plot of radial displacement u along the radial direction at the middle of the plane ($z = 0$) at different time instants. It is observed that the radial displacement increases from zero and becomes maximum near $r = 4m$, then it decreases as r increases and becomes again zero near $r = 9$.

Fig. 3. shows the variation of axial stress σ_{zz} along the radial direction in the middle plane ($z = 0$). A difference in profiles of axial stress is seen at small times (i.e. at $t = 0.1, 0.25$) and large times (i.e. at $t = 1.1$) . The difference in results for L S and C T can also be seen in the plot.

Fig. 4. shows the shear stress σ_{rz} distribution along the radial direction of the cylinder in the middle plane ($z = 0$) at different time instants. Shear stress shows sinusoidal nature in the plots with high peaks in the middle of the plane and gradually reducing as the radius increases.

Fig.5.,fig.6. and fig.7 shows the plots of non-dimensional temperature distribution , axial stress σ_{zz} and the shear stress σ_{rz} distribution respectively, along z axis at different time instants $t = 0.1, 0.25, 1.1$. It is observed from the plots of axial stress and Shear stress that the mechanical boundary conditions are satisfied at $z = \pm h$.

Clearly the difference between the L S and CT theory of thermoelasticity is observed in the plots.

5. CONCLUSIONS:

In this problem we have used the generalized theory of thermoelasticity (L-S model) to solve the problem for semi-infinite cylinder with heat source and compared the model with Classical coupled theory (C T). We have directly found the solution for the field equations without using the potential functions .This helps in eliminating the well known problems associated with the solutions using potential functions. The numerical inversion methods are very fast and accurate as compared to any other methods. Due to the presence of one relaxation time in the field equations the heat wave assumes finite speed of propagation. From the graphs we can clearly observe that the results obtained using the generalized theory of thermoelasticity (L S model) with one relaxation time are different from the results obtained by using the Classical coupled theory (C T model) of thermoelasticity. We may conclude that the system of equations in this paper may prove to be useful in studying the thermal characteristics of various bodies in important engineering problems using the more realistic Lord-Shulman model of thermoelasticity predicting finite speeds of wave propagation.

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