INTERPOLATING DEFLECTION AND SLOPE OF A CANTILEVER THIN BEAM WITH POINT LOAD: A COMPREHENSIVE ANALYSIS

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ABSTRACT
In this paper, we have discussed important concept such as deflection and slope of a cantilever beam with a point load at free end with the help of interpolation. We required to calculate the slope anywhere on the using interpolation technique.

Keywords: - *Deflection, Slope, Double integration method, Moment area method and Macaulay's method.*

1. INTRODUCTION

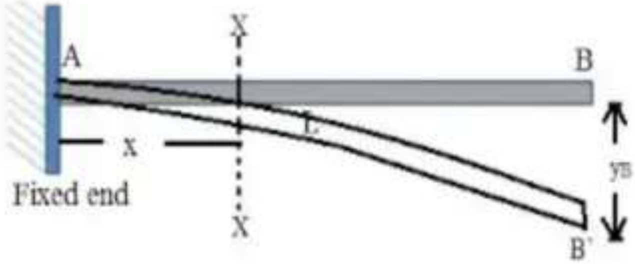
A cantilever beam is defined as a beam where one end of the beam will be fixed and the other end of the beam will be free. There are three main methods by which we can easily specify the deflection and slope at any section of a loaded beam. They are double integration method, moment area method and Macaulay's method. The double integration method and the moment area method are used to specify deflection and slope at any area of loaded beam when the beam will be loaded with a single load. Macaulay's method is used to specify deflection and slope at any section of a loaded beam when the beam will be loaded with multiple loads. We will apply the double integration method right here to specify the deflection and slope of a cantilever beam that's loaded with point load at the free end.

Differential equation for elastic curve of a beam

$$M = E.I.\frac{d^2y}{dx^2} \text{..........(i)}$$

After first integration of differential equation, we will get value of slope $\frac{dy}{dx}$. Similarly, after second integration of differential equation, we will get value of deflection that is y.

2. Methods



Let's consider a cantilever beam AB of length L which is fixed at support A and free at point B and loaded with a point load its free end as displayed in following figure.
From the above figure,
AB = position of the cantilever beam before loading
AB'= position of the cantilever beam after loading
θ_A = slope at support A
θ_B = slope at support B
The boundary conditions are
At point A, deflection will be '0'
At point A, slope will be '0'
At point B, deflection will be maximum
At point B, slope will also maximum
Let us consider one section XX at a distance X from end support A, let's calculate the bending moment of this section
M =-WL+WX …… (ii)
We have taken the concept of sign convention to provide the suitable sign for above calculated bending moment about section XX.

2. We can write the expression for bending moment at any section of beam as

$$M = E.I.\frac{d^2y}{dx^2}$$

From (ii), -WL+WX= $M = E.I.\frac{d^2y}{dx^2}$

$$-WLX + W\frac{X^2}{2} + c_1 = EI\frac{dy}{dx}$$

$$E.I(y) = -WL\frac{x^2}{2} + W\frac{x^3}{6} + c_1x + c_2$$

Where c_1& c_2 are the constants of integration.
At point A, i,e x=0, slope will be zero i.e $\frac{dy}{dx}$=0
At point A, i.e x=0, deflection will be zero i.e y=0
The boundary condition in above equation of slope and deflection of beam, we will have the following values of constant c_1& c_2 as mentioned below.

$$c_1 = 0 \; and \; c_2 = 0$$

Let us insert the values of c_1& c_2 in slope eq & in deflection & too. we will have the eq of the slope & also eq of deflection at any section of the loaded beam.

The eq of slope in $E.I\,\frac{dy}{dx} = -W\,Lx + W\frac{x^2}{2}$

$$\frac{dy}{dx} = y' = -WL$$

Slope at the free end at X=L, y=0 (no deflections)

$$E.I\,\frac{dy}{dx} = -W\,L + W\frac{x^2}{2}$$

$$y' = \frac{1}{E\,I}[-W\,\frac{L^2}{2} + W\frac{x^2}{2}]$$

$$y' = \frac{W}{2EI}[-x^2 + L^2]$$

Negative sign represents that tangent at end B makes an angle with beam axis AB in anti-clockwise direction.

In numerical analysis, calculating on the given data that a cantilever thin beam is 4m long and has a point load of 5KN at the free end.

The flexural stiffness in 53.3MN^2.

$$y' = \frac{W}{2EI}[-x^2 + L^2]$$

Put $x = 0m, L = 6m$

y^I_1=0.0016

again put x=2m, L=6m

$y^I_2=1.5X10^{-3}$=0.0015

put x=4m, L=6m

$y^I_3=9.38086X10^{-4}$=0.000938086

put x=6m, L=6m

y^I_4=0.000

we will get a table between slope &length of the cantilever beam,

X(m)	0	2	4	6
$Y'(\theta)$slope	0.0016	0.0015	0.000938086	0

Table: 1

From this table:

X(m)	$Y'(\theta)$slope	$\Delta Y'(\theta)$	$\Delta^2 Y'(\theta)$	$\Delta^3 Y'(\theta)$
0	0.0016			
2	0.0015	-0.0001		
4	0.000938086	-0.000561914	-0.000461914	
6	0	-0.000938086	-0.000376172	0.000085742

Table: 2

We calculate slope u at 1.8m

slope=$u = \frac{x-a}{h} = \frac{1.8-0}{2} = 0.9$

y = f(a) + u Δf(a)+$\frac{u(u-1)}{2}$ Δ^2f(a) + $\frac{u(u-1)(u-2)}{3!}$ Δ^3f(a)

= 0.0016 + 0.9(-0.0001) + $\frac{(0.9)(0.9-1)}{2}$ (-0.000461914) + $\frac{(0.9)(0.9-1)(0.9-2)}{3!}$ 0.000085742

= 0.0016 - 0.00009 + (0.00002078613) +0.000001414743=0.00153220087

3. CONCLUSION:

In this paper, the authors discussed the concepts of deflection and slope of a cantilever beam with a point load at the free end, utilizing interpolation techniques. They explored different methods for specifying these values, including the double integration method, moment area method and Macaulay's method. Specifically, the double integration method was applied to determine the deflection and slope of a cantilever beam loaded with a point load at the free end. The differential equation for the elastic curve of the beam was presented, and through integration, the slope $\frac{dy}{dx}$ and deflection(y) values were obtained.

4. REFERENCES:

[1]. Theory of Beam by T.lwinski.
[2]. Analysis of Beans on elastic foundation by GLYN JONES.
[3]. Structural Theory and Analysis by JD TODD, Second edition, Springer link.
[4]. Numerical Analysis theory and applications by Richard.L.Burden.
[5]. Classical Numerical Analysis by ABNER J SALGADO and STEVEN M WISE.