Isomorphism Check of Structurally Identical kinematic Chains using Coding Technique

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ABSTRACT

The kinematic structure of mechanisms is basic to understanding their function[1]. The structure or topology of a mechanism is defined by the number of links, number and type of joints, the connectivity of the links and joints and also the consideration as to which links are to be considered as frame and input. Decimal codes of kinematic chains are readily amenable to retrieval of parent chain and, as such, they hold the key to enumeration process. An optimum selection of range of Decimal codes of chains of a given type, which permits all possible types of connectivity to the link with highest degree (i.e., with highest number of elements), can provide a computerized method of enumeration. The concept is also shown useful in identifying Structurally identical kinematic chains. As these identification codes (Decimal codes)are decodable, they enable reconstruction of topology of kinematic chains/mechanisms on computer.

Keywords : Kinematic Chain, Adjacency Matrix, UTAM(Upper Triangular Adjacency Matrix), Decimal codes, Decodability.

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1. Introduction :

The Advent of robots and manipulators, medical applications, in recent years, has added a new dimension to the scope of kinematic chains. For kinematic chains, with $n_i \leq 8$, identification of structurally identical mechanisms is relatively simple. The problem however becomes involved for chains with larger number of links. Graph theoretic representation of a chain, with a link represented by a vertex and a joint by an edge, leads to a link-link connectivity matrix (also called the zero-one adjacency matrix) for a labeled kinematic chain with simple joints. It turns out to be a powerful tool in structural analysis and is well suited to computer processing. Ambekar and Agrawal [2] proposed to represent a kinematic chain and mechanisms through a set of max codes. However, for identifying structurally equivalent mechanisms, computation of max-code is unnecessary and time consuming at the same time. Also, Graph theoretic literature [3], shows that method of calculating the characteristic polynomial is neither simple nor does it provide any insight into the relation between the graph structure and different coefficients. Read and Corneil [4] remarked that a good solution to the coding problem provides a good solution to the isomorphism problem, though, the converse is not necessarily true. Hassan[5]; Dargar A, Khan RA, Hasan A [6] have matrix method to check the isomorphism of kinematic chain.This goes to suggest that a suitable coding procedure can be relied upon for structural identification of chains and mechanisms.

2. Codes and Canonical Numbers : A Brief Introduction.

A kinematic chain of n-links, can be represented by a link-link adjacency matrix for a given scheme of labeling and a binary code can be readily extracted from the Upper triangular adjacency matrix (abbreviated by UTAM). To establish a binary code, for the given scheme of labeling, one considers the UTAM, extracted from the link-link adjacency matrix of the kinematic chain. A binary sequence is established by laying strings of 'zeros ' and 'ones 'in rows '1_i' through (i < 1) of the UTAM, one after the other, in a sequence 'from top to bottom'. This binary sequence may be looked upon as a binary number/code. From the binary code, so obtained, a decimal number can be readily established giving an Identification code, for the given kinematic chain, which may not be unique.

For an n-link kinematic chain, theoretically, a total of n! schemes of labeling are possible. The zero-one link-link adjacency matrix is a square symmetric matrix with a total of n^2 number of elements in it. Thus, accounting for 'n_i' number of diagonal elements, the total number of elements in UTAM are n(n-1)/2. Thus, the length (i.e. number of digits) of the binary sequence is also, n (n-1) / 2. It follows from preceding paragraph that a total of n! binary/decimal numbers are possible for the same chain. All the binary/decimal numbers, in general, are not distinct. When all the n! decimal numbers of an n-link kinematic chain are laid down in say, descending order, the greatest and the smallest one are easily recognized as the unique or canonical numbers for the given chain.

The labeling scheme for a kinematic chain, for which the decimal number is in some (either maximum or minimum) canonical form, is called canonical numbering (labeling) of the chain. The corresponding adjacency matrix, which represents canonical numbering, is said to be in some canonical form. An important property of binary number is that an entry of '1_i ' as an (i+1)th digit in a binary sequence, counted from the right hand end, has a contribution to the decimal code equal to 2^i . Also, the joint contribution from all the remaining i elements on the right hand side of this element, assuming all these elements to be occupied by '1_i ' , is given by a summation

$$2^0 + 2^1 + 2^2 + 2^3 + \ldots\ldots\ldots\ldots + 2^{(i-1)} \qquad (1)$$

The summation of terms in geometric progression, with first term as 2^0 and the common ratio of 2, can be shown to be equal to (2^i -1). This goes to prove that a contribution of any '1_i ', in any element of a UTAM, is more significant than even the joint contribution of all the 'ones' in subsequent elements of adjacency matrix. This feature is fundamental to a basic understanding of any algorithm on identification codes of chains.

Consider the Stephenson's Chain, as shown in Fig. 1, with 4 binary and 2 ternary links. The graphical representation of the same chain is shown at Fig. 2. For the arbitrarily labeled graph as shown in Fig 2., the adjacency matrix, "Aj" and the corresponding UTAM (Upper Triangular Adjacency Matrix) is also written

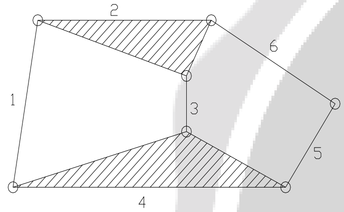


Fig. 1. Stephenson's Chain

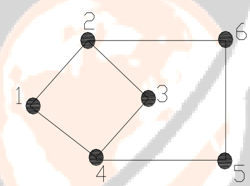


Fig.2.Graph of Chain(arbitrary Labeling)

For the arbitrarily labeled graph as shown in Fig.2, the adjacency matrix and the corresponding UTAM (Upper Triangular Adjacency Matrix)is as under

$$\mathbf{A} = \begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{vmatrix}$$

$$\text{Corresponding UTAM} = \begin{vmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ & & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ & & & \mathbf{1} & \mathbf{0} \\ & & & & \mathbf{1} \end{vmatrix}$$

There are fifteen entries in the UTAM. Binary sequence is obtained if the rows of UTAM are laid down as a single row, starting from top to bottom. The resulting binary sequence for the UTAM given above is as under.

101001001100101.

The decimal code, for this binary sequence is,

$$(1)2^{14}+(0)2^{13}+(1)2^{12}+(0)2^{11}+(0)2^{10}+(1)2^{9}+(0)2^{8}+(0)2^{7}+(1)2^{6}+(1)2^{5}+(0)2^{4}+(0)2^{3}+(1)2^{2}+(0)2^{1}+(1)2^{0}$$

Briefly, it may be rewritten as

$$(1)2^{14}+(1)2^{12}+(1)2^{9}+(1)2^{6}+(1)2^{5}+(1)2^{2}+(1)2^{0} = 21093$$

Hence the chain at Fig.(2) is identified by the decimal number 21093.

Table 1. Table illustrating decodability

	21093		
2	10546	1	**Least significant bit**
2	5273	0	
2	2636	1	
2	1318	0	
2	659	0	
2	329	1	
2	164	1	
2	82	0	
2	41	0	
2	20	1	
2	10	0	
2	**5**	0	
2	2	1	
2	1	0	
2	0	1	**Most significant bit**

Hence, $(21{,}093)_{10} = (101001100100101)_2$
The decodability property of an identification number is an important property which makes the identification code method reversible.

It can be verified that the same chain, with changed labeling as at Fig.(3),has the same identification code.

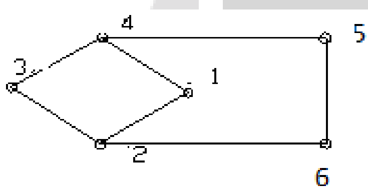


Fig 3: Arbitrary labeled chain

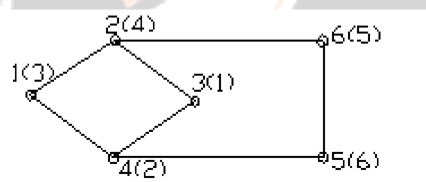


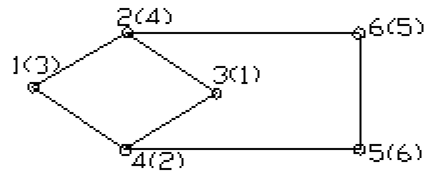
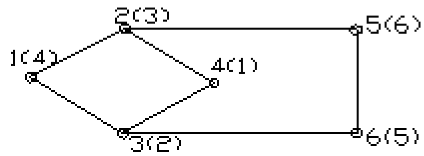
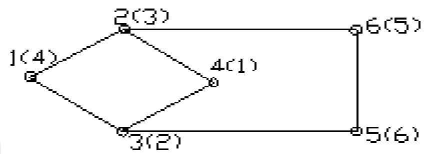
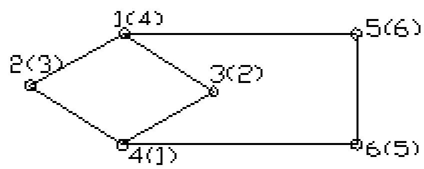
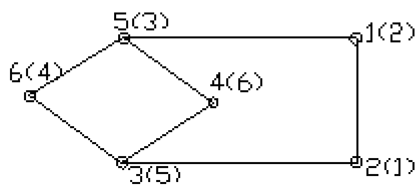
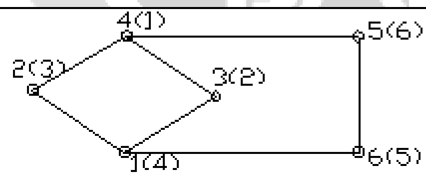
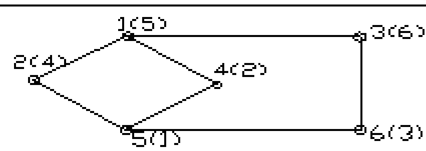
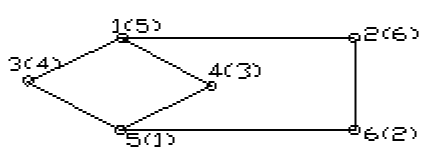
Fig 4:Chain labeled showing identical links

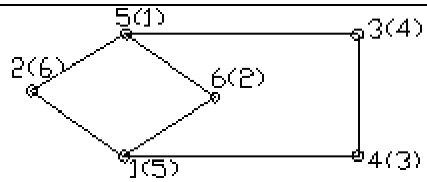
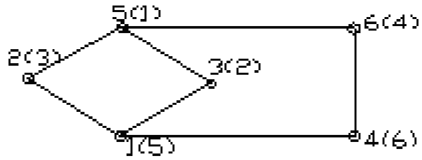
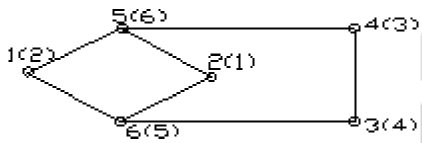
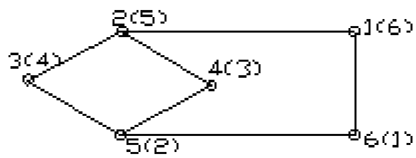
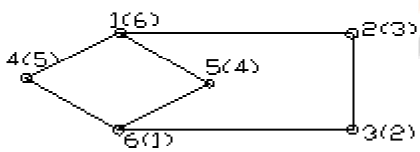
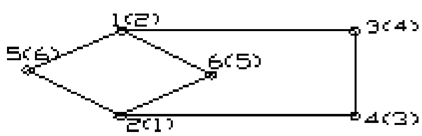
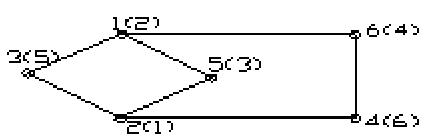
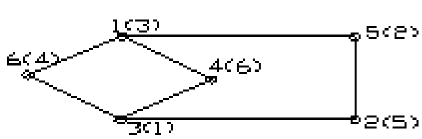
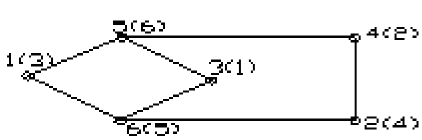
Comparing the two chains at Fig. (2) and Fig. (3) it may be concluded that links 2 and 4 are structurally identical in combination with links 6 and 5 respectively. Thus link pairs 2 and 6 can be exchanged with link pairs 4 and 5 mutually. Physically this means that with link 2 as frame and link 6 as input link the mechanism will be this, when link labels 1 and 3 are exchanged in isolation, the UTAM and the binary code/ decimal code remain unchanged. The links 1 and 3 are therefore called structurally identical links. The numbers in the bracket, as shown at Fig.(4), indicate the alternative scheme of labeling which yields the same Identification code.

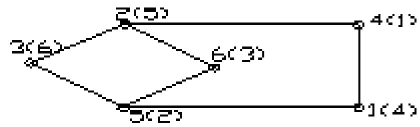
An important property of identification number lies in its decodability. For the given identification code, it is possible to reconstruct the linkage topology on the basis of these identification codes alone. The method consists in dividing the identification code by 2. The reminders are arranged sequentially to get the binary number with which the linkage topology can be reconstructed. For example consider the identification code of 21,093. The chains, as shown at Figures 2 and 3, can be established from this identification code as under.

Table 2 shows all the distinct labeling schemes possible for Stephenson's chain. The table also shows binary codes (BC) and decimal codes (DC) for each distinct scheme of labeling.

Table 2. Different Labeling Schemes possible for Stephenson's chain.

Sl. No.	Graphical representation of Stephenson Chain (Arbitrary Labeling)	Binary code (BC) and Decimal code (DC)
1		BC : 101001001100101 DC : 21093
2		BC : 110000110101001 DC : 25001
3		BC : 110000101110001 DC : 24945
4		BC : 110100100100011 DC : 26915
5		BC : 100101000101101 DC : 18989
6		BC : 110010100100101 DC : 25893
7		BC : 111000010001101 DC : 28813
8		BC : 111000001010101 DC : 28757

9		BC : 101010010110001 DC : 21681
10		**BC : 111000010010011** **DC : 28819**
11		**BC : 000110011101100** **DC : 3308**
12		BC : 100011100010101 DC : 18197
13		BC : 101101000001011 DC : 23051
14		BC : 010110111100000 DC : 11744
15		BC : 010111010000100 DC : 11906
16		BC : 001111010101000 DC : 7848
17		BC : 000110101011100 DC : 3420

18		BC : 001101101000001 DC : 6977

From the Table 2**,** it follows that, for the same chain, different identification codes are possible. By arranging these identification codes in descending order Max code and Min code turns out to be **28819(Sl.No 10 from Tanle 2)** and **3308 (Sl.No 11 from Tanle 2)** respectively.

Conclusion:

The concept of identification codes is claimed to be important from the point of view of a computerized method of enumeration of kinematic chains. This is particularly true from the point of view of decodability of identification codes. The concept of identification code is also shown to be useful in identifying structurally identical links or pair of links in a chain, which in turn, helps in identifying structurally equivalent mechanisms. The concept identification codes also known as decimal code of mechanisms, when extended, is likely to be of great help in developing a system of mechanisms classification and indexing.

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