JUSTIFICATION OF JURIN'S LAW BY THE RESULT OF AVERAGE LEVEL (HEIGHT) OF THE TIDE: CASE OF THE ISLAND OF MADAGASCAR

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ABSTRACT

This paper presents a study that explores the application of Jurin's law (which describes the rise of liquids in capillary tubes) to analyze tidal height variations on the coasts of Madagascar. Although Jurin's law mainly applies to confined fluids, the study proposes that it could also be used to understand sea level variations influenced by geophysical and gravitational forces, such as those of tides.

The aim is to demonstrate how average tide levels on the coasts of Madagascar can validate this law. By comparing tide height variations in two distinct areas – the Mozambique Channel and the Indian Ocean – the study shows that tidal peaks are higher in the Mozambique Channel than in the Indian Ocean. This difference is explained by the size of the radius (r): the radius of the Indian Ocean (15,000 km) is larger than that of the Mozambique Channel (400 km), which justifies the application of Jurin's law.

In summary, the study proposes an innovative approach by using the principles of capillarity to interpret natural tidal variations, and by demonstrating that differences in the geometry of marine spaces (such as the size of basins) influence the observed tide levels.

Keyword : *Jurin's law, Tides, Tidal height, Mean tide levels, Capillarity.*

I. INTRODUCTION

Jurin's law, which describes the phenomenon of capillarity and the rise of liquids in capillary tubes, has long been a central subject in the study of fluid behavior. Although this law primarily applies to fluids in confined spaces, its analogy can be explored in natural systems such as tides. Tides, influenced by gravitational forces and various geophysical interactions, exhibit height variations that can be examined through the lens of fluid physics principles, including Jurin's law. In this context, the present study aims to justify the application of Jurin's law by analyzing the average tide height on the coasts of the island of Madagascar. This analysis offers a new perspective on the interpretation of sea level variations using principles from capillarity, paving the way for an in-depth understanding of the interactions between physical forces and natural tidal variations.

II. METHODOLOGY

II. 1 STUDY AREA

Madagascar is an island located in the Indian Ocean, approximately 400 km east of the coast of Africa, opposite Mozambique. It is the fourth largest island in the world, with an area of approximately 587,000 km². It is located between the following geographic coordinates:

• Latitude: approximately 12° South latitude (north of the island) to 25° South latitude (south of the island).
• Longitude: approximately 43° East longitude (west of the island) to 51° East longitude (east of the island).

The study was carried out on the coastal areas of Madagascar, including four ports located in the part of the Mozambique Channel which are in red dot in figure (1): Nosy-Be, Mahajanga, Maroantaly and Toliara, as well as four ports located in the part of the Indian Ocean which are in black spot: Vohémar, Fénérive Est, Toamasina and Fort-Dauphin. Their coordinates are presented respectively in table (1) below.

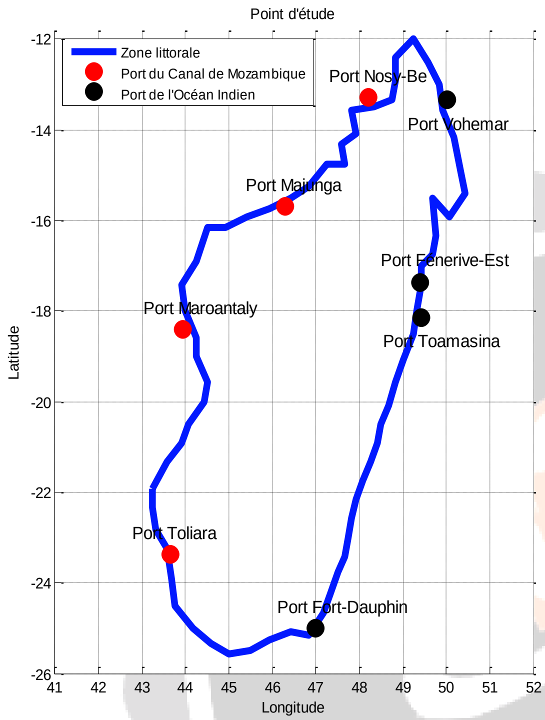


Figure 1: Point map of studies

Table 1: Coordinates of study points

PORT	LATITUDE	LONGITUDE
Nosy-be	- 13,314	48,261
Mahajanga	- 15,717	46,317
Maroantaly	- 18,250	43,261
Toliara	- 23,350	43,700
Vohemaro	- 13,221	49,800
Fenerive est	- 17,221	49,243
Toamasina	- 20,000	48,500
Fordauphin	- 25,157	46,600

II.2 METHOD

To justify Jurin's law in the context of tides, the study relies on a series of data collected on tide heights at eight points including four (4) on the Mozambique Channel and four on the Indian Ocean on along the Malagasy coast. A statistical analysis of tide level (height) averages will be carried out to identify recurring patterns and relevant variations. The results will then be compared to theoretical predictions based on Jurin's law to assess the relevance and validity of this law in a marine context.

II-2-1 Statistical methods

Arithmetic average

It is given by the formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (1)$$

Where n is the total number of observations in the sample to be studied.

Dispersion: the variance of a time series makes it possible to evaluate the dispersion around the mean.

We must distinguish two cases, depending on whether we are dealing with the entire population exhaustively or whether we only have a sample of achievements considered representative of the total population. The formula for the variance (Var) in the case of a completely known population (theoretical variance) is:

$$Var(x) = \frac{\sum_{t=1}^{t=n}\left(x_1 - \bar{x}\right)^2}{n} \quad (2)$$

The formula for the variance in the case of a sample (empirical variance) is given by:

$$Var(x) = \frac{\sum_{t=1}^{t=n}\left(x_1 - \bar{x}\right)^2}{n-1} \quad (3)$$

In the case of calculating the variance of a time series, the second formula is the only one used. Its empirical standard deviation is:

$$\sigma_x = \sqrt{Var(x)} \quad (4)$$

III – RESULTS AND INTERPRETATIONS

The results include a significant correlation between variations in mean tidal height and the principles described by Jurin's law. An in-depth discussion will focus on the implications of these results for the understanding of tides, taking into account the geographic and climatic specificities of Madagascar. The relevance of Jurin's law will be analyzed in this context, and the limits of its application will also be addressed.

III.1 – Study of daily data

Daily evolution of tides in this study dated from January 1, 1979 to December 31, 2023 including 16,436 observation days. We have observed this evolution in the 4 ports of the Mozambique Channel and 4 ports of the Indian Ocean.

The Figures illustrate the daily average height of high tide and low tide at ports in the Mozambique Channel and the Indian Ocean. On the axis the abscissa represents the number of days and the tide height marks on the ordinate. The blue curve represents the high tide level and the low tide level which is pink.

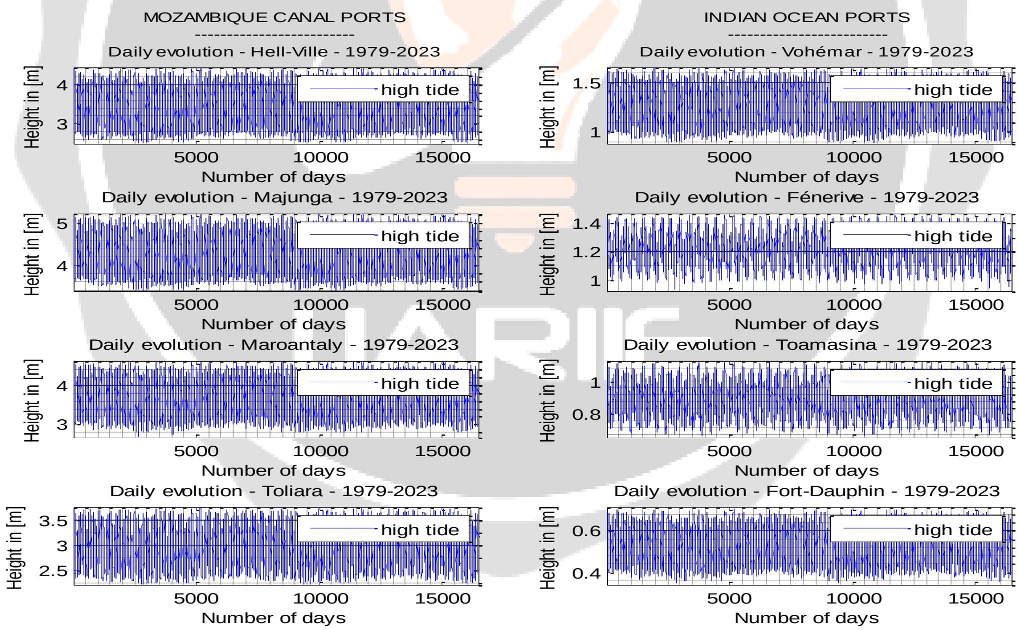


Figure 2: Daily average high tide curves on the coasts of Madagascar.

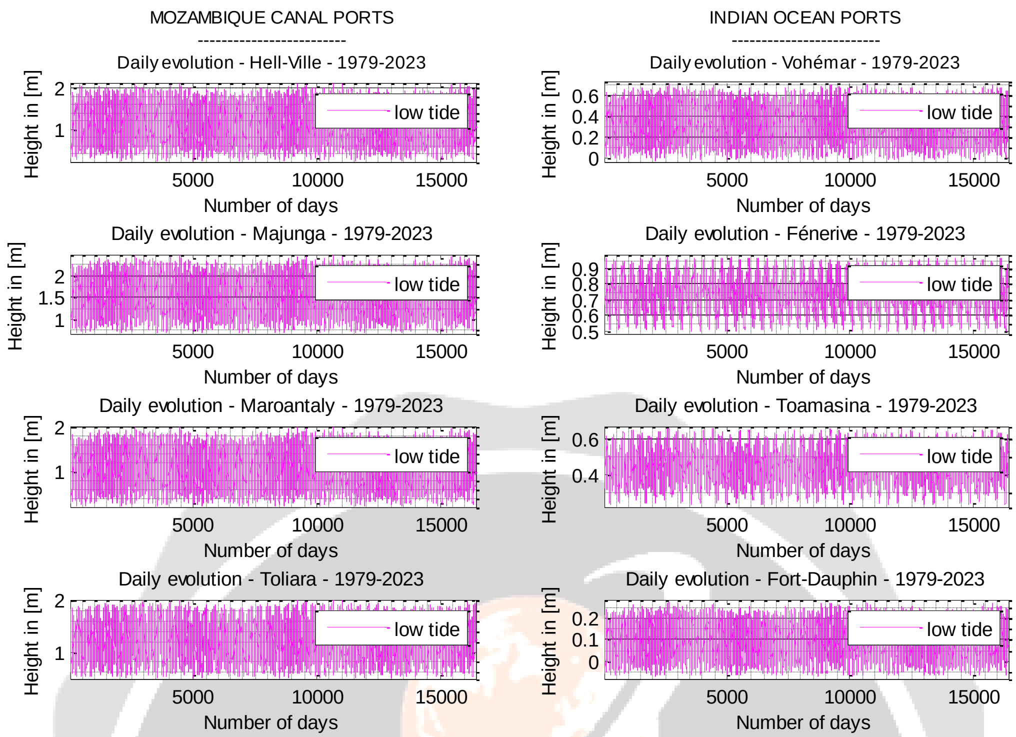


Figure 3: Curves of daily average low tides in the coastal areas of Madagascar.

Table No. 2 Peak Summary (Daily Average)

		LOCATION							
		MOZAMBIQUE CHANNEL				**INDIAN OCEAN**			
		Nosy Be	Majunga	Maroantaly	Toliara	Vohemar	Fenerive	Toamasina	Fort Dauphin
HIGH TIDE	Height (m)	4,45	5,22	4,62	3,75	1,64	1,46	0,67	0,71
	Period (th day)	11 680th	15 050th	11 680th	13 209th	southern summer	13200th	southern summer	southern summer
LOW TIDE	Height (m)	0,23	0,68	0,22	0,49	-0,025	-0,07	0,23	-0,025
	Period (th day)	14 600th	14 880th	14 600th	13209h	Southern Winter	end of Winter	Month of August	every September

III.2 – Study of monthly data

The monthly tide level variation in this analysis starts from January 1979 to December 2023, which is precisely 540 Months. For each figure the curve in blue represents high tide and in pink that of low tide

III.2.1 High tide

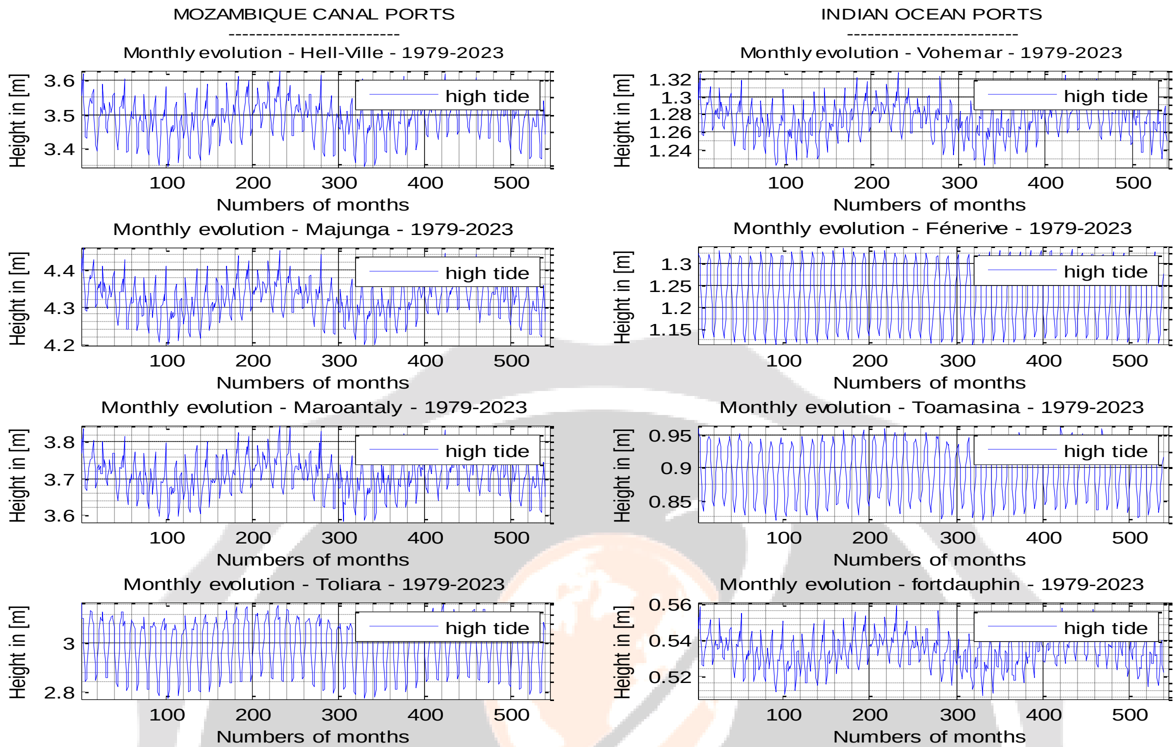


Figure 4: Curves of monthly average high tides in the coastal areas of Madagascar

III.2.2 Low tides

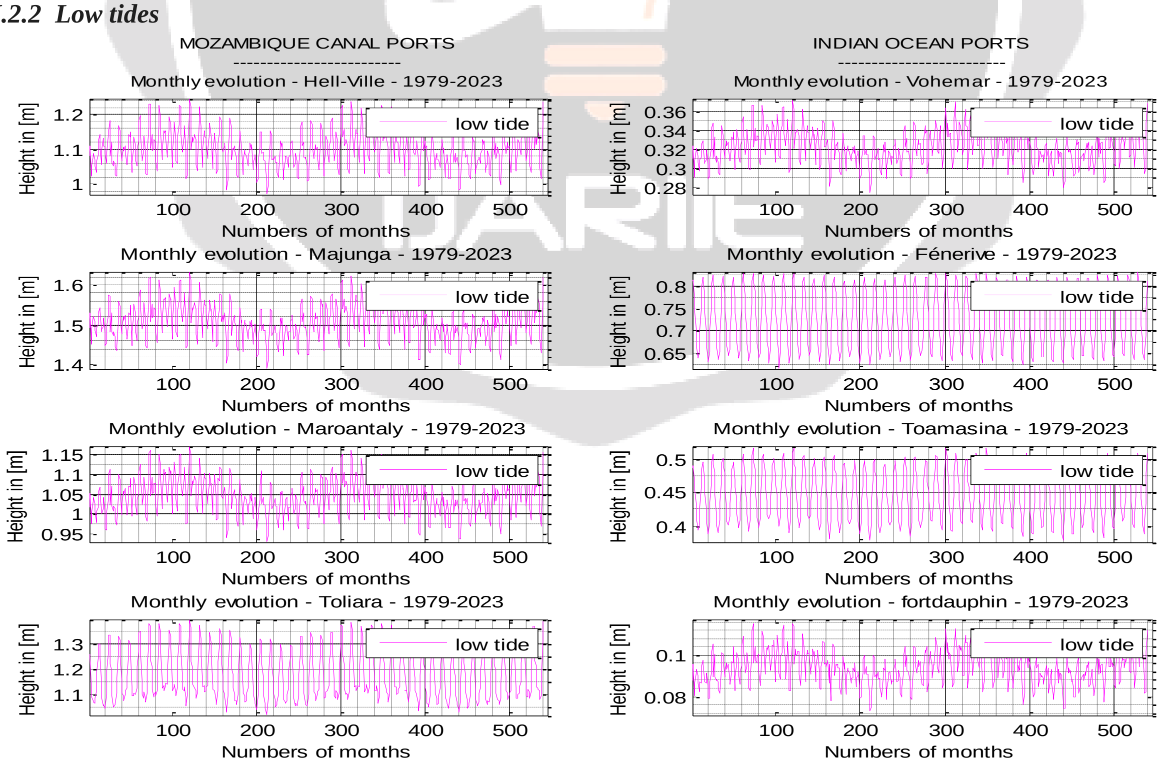


Figure 5: Curves of monthly averages of high tides on the coasts of Madagascar

Table No. 3 Peak Summary (Monthly Average)

		LOCATION							
		MOZAMBIQUE CHANNEL				**MOZAMBIQUE CHANNEL**			
		Nosy Be	Majunga	Maroantaly	Toliara	Vohemar	Fenerive	Toamasina	Fort Dauphin
HIGH TIDE	Height (m)	3,63	4,45	3,84	3,17	1,33	0,83	0,37	0,51
	Period (th day)	423[e]	389[e]	384[e]	421[e]	423[e]	384[e]	420[e]	330[e]
LOW TIDE	Height (m)	3,63	4,45	3,84	3,17	0,28	0,62	0,38	0,08
	Period (th day)	423[e]	435[e]	420[e]	421[e]	243[e]	182[e]	436[e]	243[e]

III.3 – Study of annual data

The monthly tide level variation in this analysis starts from the year 1979 to the year 2023, which is precisely 44 years. For each figure the curve in blue represents high tide and in pink that of low tide.

III.3.1 High tide

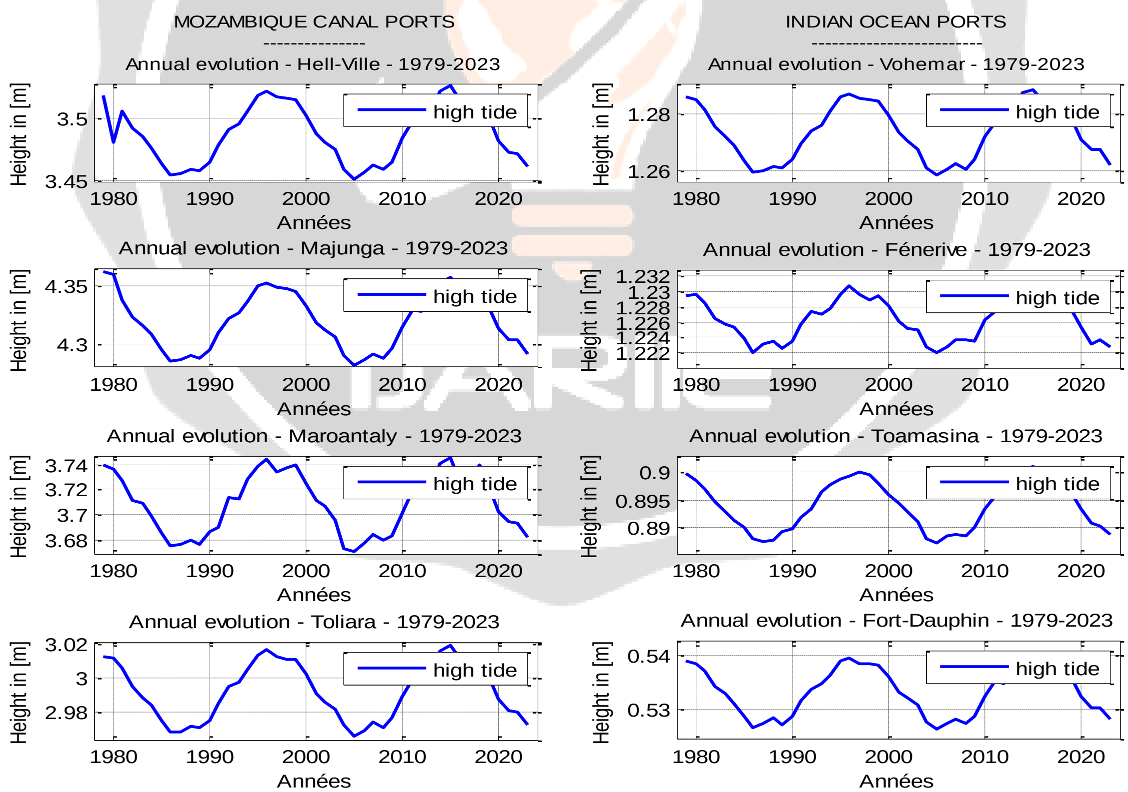


Figure 6: Curves of annual average high tides in the coastal areas of Madagascar

III.3.2 Low tides

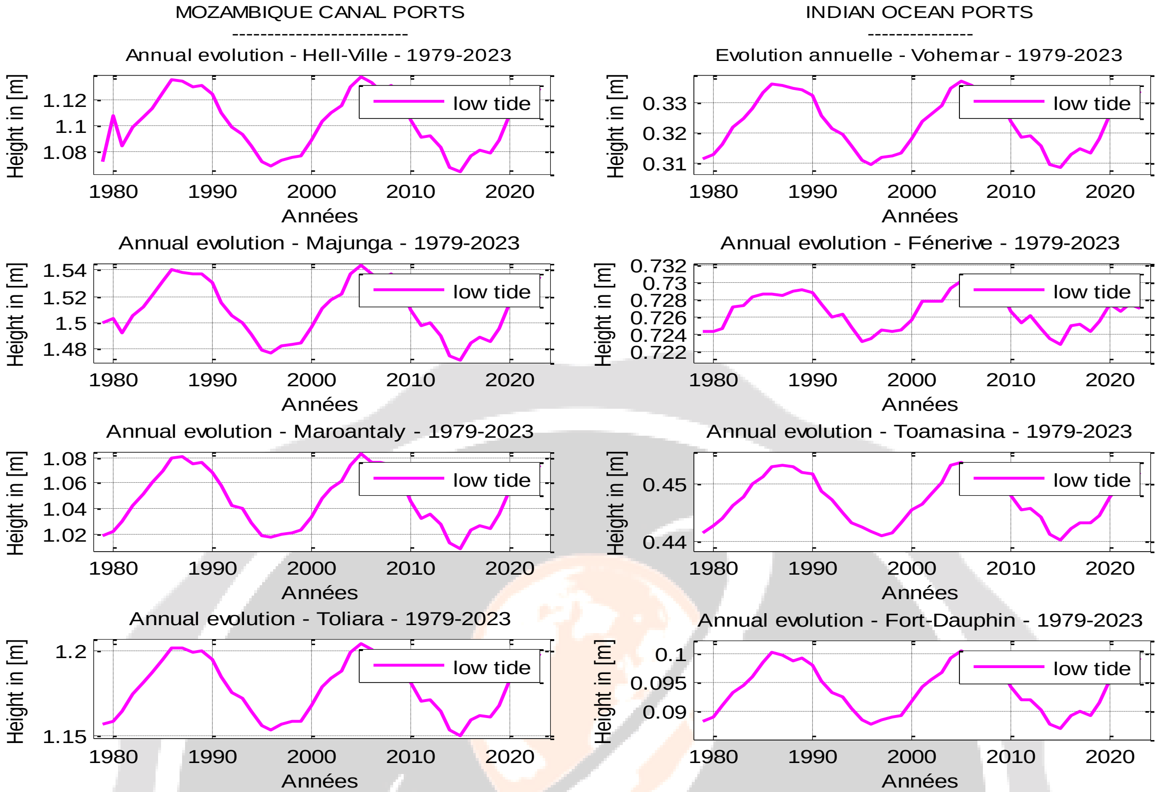


Figure 7: Curves of annual average high tides on the coasts of Madagascar

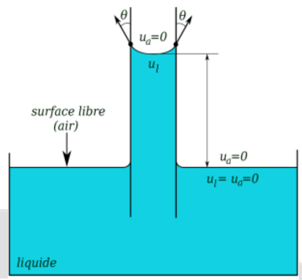
Table No. 4 Peak Summary (Annual Average)

		LOCATION							
		MOZAMBIQUE CHANNEL				**MOZAMBIQUE CHANNEL**			
		Nosy Be	Majunga	Maroantaly	Toliara	Vohemar	Fenerive	Toamasina	Fort Dauphin
HIGH TIDE	Height (m)	3,53	4,36	3,75	3,02	1,89	1,23	0,90	0,54
	Period (th day)	2015	2015	2015	2015	2015	2015	2015	2015
LOW TIDE	Height (m)	1,07	1,48	1,02	2.25	0.31	0.72	0,72	0.09
	Period (th day)	1995	1996	1979	1996	1996	1995	2015	1979

IV. JURIN law

Reminder of Jurin law

In physics, Jurin's law gives the height of rise or depression of a liquid in a capillary tube. This law was stated in 1717 by the English physician James.



Liquids in small diameter (capillary tube) tend to rise (wetting liquid) relative to the level of the surrounding liquid.

Therefore, the law of Jurin is therefore:

$$h = \frac{2\sigma \cos\theta}{r\rho g} \quad \text{d'où} \quad h = \frac{2A}{r\rho g} \qquad (5)$$

With

σ cos θ = A: surface tension of the liquid

ρ: density

g: acceleration of gravity.

r: distance between two extremes

Which comes from:

⁎ law of hydrostatics

$$P_{\text{int}} = P_0 + \rho g h \qquad (6)$$

$$P_{\text{int}} = P_{ext} + \rho g h \qquad (7)$$

⁎ and the Laplace relation

$$P_{\text{int}} - P_{ext} = \frac{2\sigma}{R} \qquad (8)$$

Assumption of calculation of justification

Take the case of Madagascar which is an island in the Indian Ocean, separated by the Mozambique Channel from the African continent.

rc = dc: distance Madagascar – Africa (Mozambique Channel): 400 km

ri = di: distance Madagascar – Borneo (Indian Ocean): 15,000 km

We know that A, ρ, g are unchangeable variables (constant); then the value of h depends on the value of r, that is to say: ***The height h of the liquid lifted is inversely proportional, for the same liquid, to the radius r of the tube at the level of the meniscus***.

Conclusion

According to the results obtained above: the value of the peaks of the daily, monthly and annual average tide in the Mozambique Channel ***is higher than*** that of the Indian Ocean which is justified by the Jurin Law, with the r (15,000 Km) of the Indian Ocean larger than r (400 Km) Mozambique Channel.

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