LMI based Stability criteria for 2-D PSV system described by FM-2 Model

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Abstract

Stability analysis is the key issue for the two-dimensional (2-D) discrete time system. Linear state-space models describing 2-D discrete systems have been proposed by several researchers. A popular model, called Fornasini-Marchesini (FM) second model was proposed by Fornasini and Marchesini in 1978. An important key issue of practical importance in the two-dimensional (2-D) discrete system is stability analysis. This paper presents an analysis of the existing literature for the same. Also, we propose two results for sufficient conditions of stability criteria for 2 -D PSV Filters described by FM-2 Model. Periodically Shift Varying (PSV) filters are having numerous applications and hence its stability analysis has gained increased importance. LMI (Linear Matrix Inequality) based criteria has dual advantage of relaxed result as well as ease of implementation due to software tools like MATLAB LMI TOOLBOX. For that we have taken example of and analyzed stability criteria for FM2 model and various stability analyses have been done.

Keywords: *2-D periodically shift varying (PSV) filters, Givone–Roesser model, stability, Linear Matrix inequality, Fornasini-Marchesini (FM) model*

I. INTRODUCTION

Due to their applications in various important areas such as multi-dimensional digital filtering, seismographic data processing, thermal processes, gas absorption, water stream heating etc, there have been a continuously growing research interests in two-dimensional (2-D) systems [1-2]. In a 2-D discrete system, information propagates in two independent directions as a result of which the system dynamics may be represented as a function of two independent integer variables. Many researchers have made an attempt to describe the 2-D system dynamics in terms of linear state-space models for 2-D discrete systems [3-5]. The 2-D models that have received considerable attention are Givone-Roesser model [3], Fornasini-Marchesini (FM) first model [4] and FM-2 model [5].

The main issues in the design of any control system are stability analysis. With the introduction of state-space models of 2-D discrete systems, various Lyapunov equations have emerged as powerful tools for the stability analysis of 2-D discrete systems. Lyapunov based sufficient conditions for the stability of 2-D discrete systems have been studied in [6-8]. When the dynamics of practical systems are represented using state-space models, errors are inevitable as the actual system parameters would be different than the estimated system parameters, *i.e.*, the model parameters. The cause of errors are the approximations made during the process modeling, differences in presumed and actual process operating points, change in operating conditions, system aging etc. Control designs based on these models, therefore, may not perform adequately when applied to the actual industrial process and may lead to instability and poor performances. This has motivated the study of robust control for the uncertain 2-D discrete systems. The aim of robust control is to stabilize the system under all admissible parameter uncertainties arising due to the errors around the nominal system. Many significant results on the solvability of robust control problem for the uncertain 2-D discrete systems have been proposed in [9-11] .

There are two concerns in control: first is to design a robust controller to ensure the stability of uncertain systems and the other is to guarantee a certain performance level under the presence of uncertainties. The latter is called as guaranteed cost control problem which has the advantage of providing an upper bound on the closed-loop cost function (performance index). Consequently, a guaranteed cost controller not only stabilizes the uncertain system but also guarantees that the value of closed-loop cost function is not more than the specified upper bound for all admissible parameter uncertainties. Based on this idea, many significant results have been obtained for the uncertain 2-D discrete systems [12-14].

Study and analysis of 2-D discrete systems under the presence of noise is another research area of great interest where it is usually necessary to estimate the state variables from the system measurement data. One of the celebrated approaches is Kalman filtering [15] which is based on two fundamental assumptions that the system under consideration is exactly known and a priori information on the external noises (like white noise, etc.).

Rest of the paper proceeds as follows. In section II an analysis of the existing literature on the FM-2 model is presented. Section III presents a literature survey on the GR and FM-1 models. In section IV we have proposed results for stability criteria of PSV FM-2 systems whereas section V gives derivation of the same. Results have been discussed with example as well as stability analysis has been done in section VI. Section VII is the conclusion.

II. AN ANALYSIS OF THE EXISTING LITERATURE ON THE FM-2 MODEL

Consider the following 2-D discrete system represented by FM second model [5]:

$$\begin{aligned} x(i+1, j+1) = A_1 x(i, j+1) + A_2 x(i+1, j) \\ + B_1 u(i, j+1) + B_2 u(i+1, j) \end{aligned} \tag{1a}$$

$$z(i, j) = Cx(i, j) + Du(i, j) \tag{1b}$$

$$i \geq 0, j \geq 0 \tag{1c}$$

where $x(i, j)$ is an $n \times 1$ state vector, $A_1 \in R^{n\times n}, A_2 \in R^{n\times n}, u(i, j)$ is $m \times 1$ input vector.

$B_1 \in R^{n\times m}, B_2 \in R^{n\times m}$ z is a scalar output, $C \in R^{1\times n}$ and $D \in R^{1\times n}$.

It is understood that, the above system has a finite set of initial conditions [2] *i.e.*, there exist two positive integers r_1 and r_2 such that

$$x(i,0) = 0,\ i \geq r_1, x(0, j) = 0,\ j \geq r_2 \tag{1d}$$

The equilibrium $x(i, j) = 0$ of system (1) is said to be globally asymptotically stable [2] if

$$\lim_{i\to\infty\, and/or\, j\to\infty} x(i, j) = \lim_{i+j\to\infty} x(i, j) = 0 \tag{2}$$

The transfer function of system (1) is given as

$$H(z_1,z_2) = c(I_n - z_1A_1 - z_2A_2)^{-1} \times (z_1B_1 + z_2B_2) + D \tag{3}$$

If we define,

$$N(z_1, z_2) = \det(I_n - z_1 A_1 - z_2 A_2) \qquad (4a)$$

then the state-space model (1) is asymptotically stable [5] if and only if

$$N(z_1, z_2) \neq 0 \text{ for all } (z_1, z_2) \in \overline{U^2} \qquad (4b)$$

Where, $\overline{U^2} = \{(z_1, z_2) : |Z_1| \leq 1, |Z_2| \leq 1\}$.

In literature, we find the stability criteria analysis of FM-2 systems. Some of them have been presented for reference. Lyapunov based sufficient condition for the stability of system (1) has been investigated in [16] and it is proposed that the system (1) is asymptotically stable if there exist an n*n symmetric matrix **P** such that

$$\begin{pmatrix} \alpha P & 0 \\ 0 & \beta P \end{pmatrix} - (A_1 \quad A_2)^T P (A_1 \quad A_2) > 0 \qquad (5a)$$

Provided

$$\alpha > 0, \beta > 0, \alpha + \beta = 1 \qquad (5b)$$

Reference [17] presents a stability test for system (1) which states that

$$N(z_1, z_2) \neq 0 \text{ In } \bar{U}_2^{\,2} \; \| A_1 \| + \| A_2 \| < 1 \qquad (6)$$

where N(z_1, z_2)is defined in (4a). Further, based on the 2-D Lyapunov equation approach, the problem of stability margin has also been studied in [17]

Studies in [18] has illustrated that there are a large number of systems that are stable but their stability can - not be assured by (5). That is, no values of α and β can be found to satisfy (5) for a large number of systems to confirm their stability. The result proposed in [17] is made into more generalized form in [18] and it has been proposed that the system (1) is asymptotically stable if there exist n*n symmetric matrices, P>0 , W_1 >0 , W_2 >0 and R>0, such that.

$$\begin{pmatrix} P^{T/2} W_1 P^{1/2} & 0 \\ 0 & P^{T/2} W_2 P^{1/2} \end{pmatrix} - (A_1 \quad A_2)^T P^{T/2} R P^{1/2} (A_1 \quad A_2) > 0 \qquad (7a)$$

$$(R - W_1 - W_2) \geq 0 \qquad (7b)$$

In [19], sufficient conditions to guarantee the asymptotic stability of system (1) are presented. The first criterion states that for system (1) to be asymptotically stable it is sufficient that

1) $$\det(I_n - e^{j\omega_1} A_1 - e^{j\omega_2} A_2) \neq 0 \qquad (9a)$$

and

2) there exist an n*n symmetric matrix **P**>0 such that

$$\begin{pmatrix} \alpha P & 0 \\ 0 & \beta P \end{pmatrix} - (A_1 \quad A_2)^T P (A_1 \quad A_2) \geq 0$$

(9b)

provided

$$\alpha > 0, \beta > 0, \alpha + \beta = 1$$

Here ω_1 and ω_2 are the horizontal and vertical radian frequencies, respectively.

The second criterion states that the system (1) is asymptotically stable if there exist n*n symmetric matrices, P>0, $W_1 > 0$ and $W_2 > 0$ such that

$$\begin{pmatrix} P^{T/2}W_1P^{1/2} & 0 \\ 0 & P^{T/2}W_2P^{1/2} \end{pmatrix} - (A_1 \quad A_2)^T P^{T/2} R P^{1/2} (A_1 \quad A_2) \geq 0 \tag{10a}$$

and

$$(I_n - W_1 - W_2) > 0 \tag{10b}$$

All the above discussed results are applicable for FM-2 stability criteria in general.

To the best of author's knowledge, no results for FM-2 PSV systemare existing in literature.

III. A LITERATURE SURVEY ON THE GR AND FM-1 MODELS

As per Givone-Roesser (GR) model sufficient condition for LSIV 2-D systemstability is

$$\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} - A^T \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} A > 0 \tag{11}$$

GR PSV system stability criteria are found in [22]. LMI based criteria for the same in line with the results for FM-2 in this paper is an open ended problem.

We are presenting results of FM-1 for the sake of completeness of literature review. The four stability criteria for FM-1 are as follows. Last two are the results for stability criteria that have been suggested by the corresponding author with different co-authors.

Theorem 1:

If $\left\|\boldsymbol{A}_1(h,k)\right\| < 1$ and $\left\|\boldsymbol{A}_2(h,k)\right\| < 1$, $\forall$ (h, k) and

$$\prod_{n=Q-1}^{0} \boldsymbol{F}_n < 1 \text{ and } \prod_{n=P-1}^{0} \boldsymbol{G}_m < 1 \tag{12}$$

Then, the FM-1 PSV system is asymptotically stable [20].

Theorem 2 [21]: FM-1 LSIV System is globally asymptotically stable, provided there exist n×n positive definite symmetric matrices $\boldsymbol{P_1}$, $\boldsymbol{P_2}$, and $\boldsymbol{P_3}$ such that

$$Q = \begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix} - A^T(h,k)(P_1 + P_2 + P_3)A(h,k) > 0 \ \forall \ (h,k) \tag{13}$$

Theorem-2 is used to derive results for PSV and is available in the form of Theorem-3 & Theorem-4 .

Theorem 3: FM-1 PSV System is globally asymptotically stable provided there exist n × n positive definite symmetric matrices $\boldsymbol{P}_1, \boldsymbol{P}_2$, and $\boldsymbol{P}_3$ and a positive scalar ε such that [23]

$$\begin{pmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \varepsilon E^T & 0 \\ 0 & \varepsilon E & -\varepsilon I & 0 \\ H^T P & 0 & 0 & -\varepsilon I \end{pmatrix} < 0,$$

, (14)

where $P = [P_1 \ P_2 \ P_3]$, $Q = \begin{pmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{pmatrix}$, $A = (A_1 \quad A_2 \quad A_3)$, $H = \begin{pmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{pmatrix}$,

$E = (E_1 \quad E_2 \quad E_3)$, *0* represents null matrix, *I* represents unity matrix of appropriate dimension and a notation of the form $\boldsymbol{M} < \boldsymbol{0}$ implies that the matrix $\boldsymbol{M}$ is negative definite.

Theorem 4: FM-1 PSV System is globally asymptotically stable provided there exist n x n positive definite symmetric matrices P_1, P_2, and P_3 and positive scalar2s $\varepsilon_1, \varepsilon_2$,and ε_3 such that, [23]

$$\begin{pmatrix} P_1 - \varepsilon_1 E_1^T E_1 & 0 & 0 & A_1^T P & PH_1 & PH_2 & PH_3 \\ 0 & P_2 - \varepsilon_2 E_2^T E_2 & 0 & A_2^T P & 0 & 0 & 0 \\ 0 & 0 & P_2 - \varepsilon_3 E_3^T E_3 & A_3^T P & 0 & 0 & 0 \\ PA_1 & PA_2 & PA_3 & P & 0 & 0 & 0 \\ H_1^T P & 0 & 0 & 0 & \varepsilon_1 I & 0 & 0 \\ H_2^T P & 0 & 0 & 0 & 0 & \varepsilon_2 I & 0 \\ H_3^T P & 0 & 0 & 0 & 0 & 0 & \varepsilon_3 I \end{pmatrix} > 0$$

(15)

where $\boldsymbol{P} = \boldsymbol{P_1} + \boldsymbol{P_2} + \boldsymbol{P_3}$,a notation of the form $\mathbf{M} > \mathbf{0}$ implies that the matrix **M** is positive definite,**0** represents null matrix and *I* represents unity matrix of appropriate dimension.

In [20], it has been justified with numerical example that, for PSV FM-1 Theorem-3 and 4 gives better results as compared to Theorem-1.

As they are LMI based criteria, it has two implicit advantages:

[1] They are implementable by tools such as, MATLAB LMI TOOL BOX.

[2] By hundreds of LMI based results existing in literature, It is evident that LMI based stability criteria are more relaxed. That means that, in 2-d case there cannot be necessary and sufficient condition. But, Gap between them may have been minimized as compared to the results existing in literature [22].

IV. PROPOSED RESULTS FOR FM-2 PSV SYSTEM

Follwing results are proposed by us for stability conditions of a 2-D PSV system represented by FM-2 model.

The first result can be stated as follows.

Theorem-5:

A 2-D PSV System represented by FM-2 model is globally asymptotically stable provided there exist n* n positive definite symmetric matrices P_1 and P_2 and positive scalar ε such that, for $A=[\,A_1 \quad A_2\,]$, $H=[\,H_1 \quad H_2\,]$, $E= E_1 \oplus E_2$

$$\begin{pmatrix} -P & PA & 0 & PH \\ A^T P & -P & kE^T & 0 \\ 0 & kE & -kI & 0 \\ H^T P & 0 & 0 & -kI \end{pmatrix} > 0 \qquad (16a)$$

Where

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{P_1} + \boldsymbol{P_2} \\ \boldsymbol{Q} &= \boldsymbol{P_1} \oplus \boldsymbol{P_2} \end{aligned} \qquad (16b)$$

The second result can be stated as follows.

Theorem-6:

2-D PSV System represented by FM-2 model is globally asymptotically stable provided there exist n x n positive definite symmetric matrices P1, P2 and positive scalars ε1, ε2 such that

$$\begin{pmatrix} \boldsymbol{P_1 - \varepsilon_1 E_1{}^T E_1} & \boldsymbol{0} & \boldsymbol{A_1{}^T P} & \boldsymbol{PH_1} & \boldsymbol{PH_2} \\ \boldsymbol{0} & \boldsymbol{P_2 - \varepsilon_2 E_2{}^T E_2} & \boldsymbol{A_2{}^T P} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{PA_1} & \boldsymbol{PA_1} & \boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{H_1{}^T P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\varepsilon_1 I} & \boldsymbol{0} \\ \boldsymbol{H_2{}^T P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\varepsilon_2 I} \end{pmatrix} \boldsymbol{>0}$$

(17a)

Where

$$\boldsymbol{P} = \boldsymbol{P_1} + \boldsymbol{P_2} \qquad (17b)$$

V. DERIVATION OF PROPOSED RESULTS FOR FM-2 PSV SYSTEM

The above result has been derived as follows.

$$\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} - (A + HFE)^T (P_1 + P_2)(A + HFE) > 0, \quad (18)$$

$$(A + HFE)^T (P_1 + P_2)(A + HFE) - \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} < 0 \quad (19)$$

$$\begin{pmatrix} \frac{-(P_1+P_2)^{-1}}{P} + \epsilon H H^T & A \\ A^T & \varepsilon^{-1} E^T E - \frac{\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}}{Q} \end{pmatrix} < 0, \quad (20)$$

Where,

$$H = (H_1 \quad H_2)$$

$$E = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$A = (A_1 \quad A_2)$$

Using equations 13, 14 and 15 we have the following expression,

$$\begin{pmatrix} -P^{-1} + \varepsilon P H H^T P^T & PA \\ A^T P & \varepsilon^{-1} E^T E - Q \end{pmatrix} < 0$$

(21)

Segregating the above matrix into two parts and recombining the decomposed terms we get the following

$$\begin{pmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \varepsilon E^T & 0 \\ 0 & \varepsilon E & -\varepsilon I & 0 \\ H^T P & 0 & 0 & -\varepsilon I \end{pmatrix} < 0,$$

(22)

Above is the same as the first proposed result. Similar steps can be followed for deriving the second result as shown below.

$$\begin{pmatrix} P_1 & 0 & (A_1 + H_1 F_1 E_1)^T \\ 0 & P & (A_2 + H_2 F_2 E_2)^T \\ A_1 + H_1 \mathcal{E}_1 E_1 & A_2 + H_2 \mathcal{E}_2 E_2 & (P_1 + P_2)^{-1} \end{pmatrix} > 0$$

(23)

The above result can be written as

$$\begin{pmatrix} 0 & {A_1}^T \\ A_1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & (\varepsilon_1 E_1 H_1)^T \\ 0 & 0 & (A_2 + H_2 \varepsilon_2 E_2)^T \\ 0 & 0 & (P_1 + P_2)^{-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ H_1 \varepsilon_1 E_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ H_1 \end{pmatrix} \varepsilon_1 (E_1 \quad 0 \quad 0) > 0,$$

(24)

Hence,

$$\begin{pmatrix} P_1 - {\varepsilon_1}^{-1} {E_1}^T E_1 & 0 & {A_1}^T \\ 0 & P_2 - {\varepsilon_2}^{-1} {E_2}^T E_2 & {A_1}^T \\ A_1 & A_2 & (P_1 + P_2)^{-1} \end{pmatrix} > 0, \qquad (25)$$

Finally decomposing the above result into two matrices and recombining individual terms we get the second proposed result of FM-2 for PSV that is

$$\begin{pmatrix} P_1 - \varepsilon_1 {E_1}^T E_1 & 0 & {A_1}^T P & PH_1 & PH_2 \\ 0 & P_1 - \varepsilon_1 {E_1}^T E_1 & {A_2}^T P & 0 & 0 \\ PA_1 & PA_1 & P & 0 & 0 \\ {H_1}^T P & 0 & 0 & \varepsilon_1 I & 0 \\ {H_2}^T P & 0 & 0 & 0 & \varepsilon_2 I \end{pmatrix} \qquad (26)$$

VI. DISCUSSIONS ON RESULTS

As they are LMI based criteria, it has two implicit advantages:

[1] They are implementable by tools such as, MATLAB LMI TOOL BOX.

[2] By hundreds of LMI based results existing in literature, It is evident that LMI based stability criteria are more relaxed. That means that, in 2-D case there cannot be necessary and sufficient condition. But, Gap between them may have been minimized as compared to the results existing in literature [22]. Of course, this claim need to be validated by numerical example as done in [23] for similar results of the system represented by FM-1.

Comment 1:

As we have two results for stability analysis of 2-D PSV systems, we need to take decision on which of these criterion is to be used for stability analysis. So, the proposed flow chart of the proposed work with two criteria can be shown as follows.

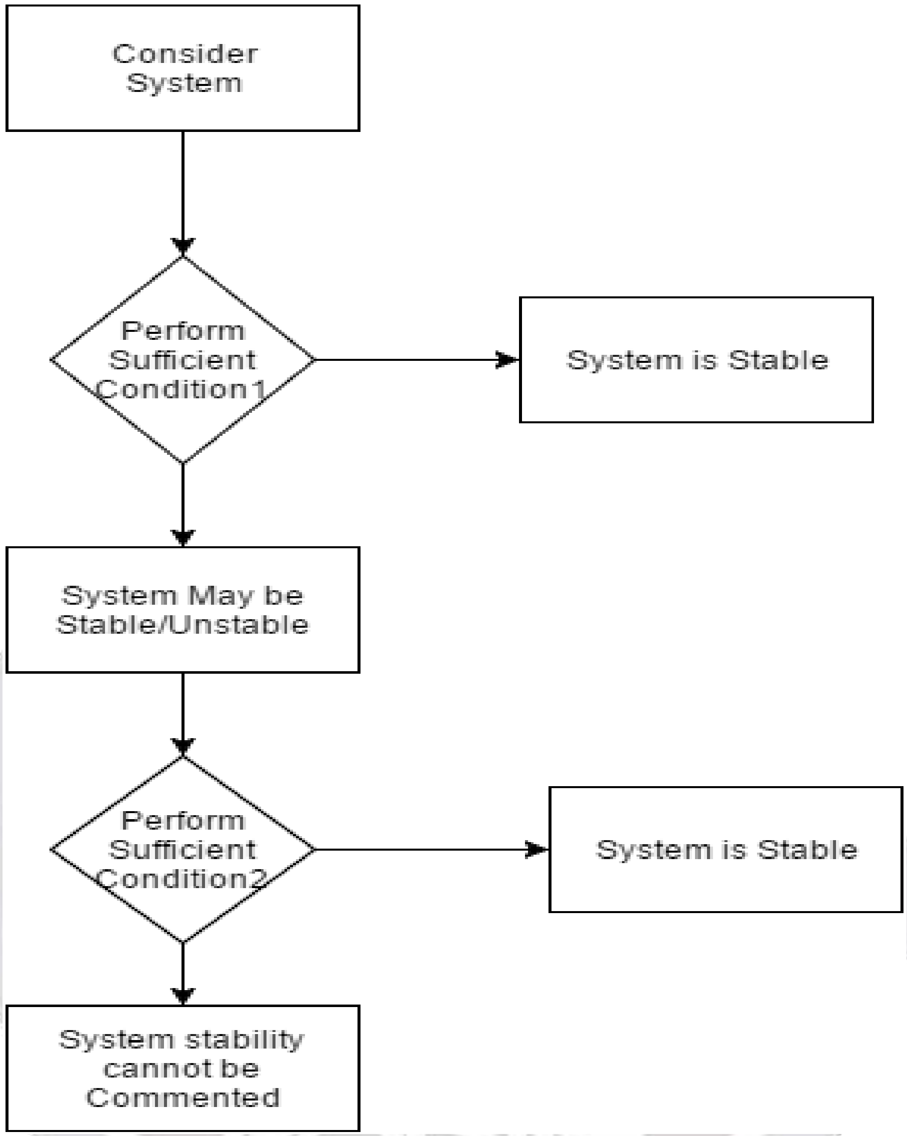

Figure 1 Flow chart of the proposed work with two criteria

By using the above flow chart as shown in figure 1, the two step stability analysis criteria can be implemented.

Comment 2:

The intention is to minimize the gap between necessary and sufficient condition for stability of 2-D discrete system. The criteria happen to be more relaxed and is less computationally complex due to efficient implementation using LMI toolbox . Also, this is the new work which has never been attempted.

Table 1 .Stability analysis of 2-D PSV system for FM2 model

	Proposed Method Result(I)	**Proposed Method Result(II)**	**Comment**
Stability criteria	$\begin{pmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \varepsilon E^T & 0 \\ 0 & \varepsilon E & -\varepsilon I & 0 \\ H^T P & 0 & 0 & -\varepsilon I \end{pmatrix} > 0$	$\begin{pmatrix} P_1-\varepsilon_1 E_1^T E_1 & 0 & A_1^T P & PH_1 & PH_2 \\ 0 & P_1-\varepsilon_1 E_1^T E_1 & A_2^T P & 0 & 0 \\ PA_1 & PA_1 & P & 0 & 0 \\ H_1^T P & 0 & 0 & \varepsilon_1 I & 0 \\ H_2^T P & 0 & 0 & 0 & \varepsilon_2 I \end{pmatrix} > 0$	Existing Result is not available
Computational complexity, memory	Less	More	Result -I is better (please ,refer next slide)
Speed	More	Less	
Gap Between Necessary and sufficient conditions	More	Less	Result-II is better
Ease of handling algorithm for stability criteria	yes	yes	Better due to Matlab LMI toolbox

We have derived stability criteria for FM1 [23],FM2 and GR to check it and for applications purpose we have Borrowed example of [24] LSIV 2-D Attasi's model filter (PSV system) using circulant matrices to test our derived stability criteria.

$$\overline{A_1} = \begin{pmatrix} 0.5 & -0.5 & 0.125 & -0.125 \\ -0.125 & 0.5 & -0.5 & 0.125 \\ 0.125 & -0.125 & 0.5 & -0.5 \\ -0.5 & 0.125 & -0.125 & 0.5 \end{pmatrix},$$

$$\overline{A_2} = \begin{pmatrix} 0.5 & 0 & -0.015 & 0.25 \\ 0.25 & 0.5 & 0 & -0.015 \\ -0.015 & 0.25 & 0.5 & 0 \\ 0 & -0.015 & 0.25 & 0.5 \end{pmatrix},$$

$\bar{B} = (1 \quad 0.39 \quad -1 \quad 0.45)^T$,

$\bar{C} = (1 \quad -1 \quad -1 \quad 1)$,

$$A_1 = \overline{A_1} \text{ and } C = \bar{C}$$

$$A_2(0,0) = \begin{pmatrix} 0.5 & 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & -0.5 & 0.5 \end{pmatrix},$$

$$A_2(0,1) = \begin{pmatrix} 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0.5 \end{pmatrix},$$

$$B(0,0) = (-1 \quad -1 \quad -0.25 \quad -1)^T$$

$$B(0,1) = (1 \quad -0.25 \quad -1 \quad -0.5)^T$$

$A_0 = A_1 A_2$.

Period of PSV in this case is 1:2.

for FM2:

$A = (A_1 \quad A_2)$,

where $A_1 = \overline{A_1}$,

$$A_2(0,0) = \begin{pmatrix} 0.5 & 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & -0.5 & 0.5 \end{pmatrix}, A_2(0,1) = \begin{pmatrix} 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0.5 \end{pmatrix},$$

$\Delta A = A(A_1, A_2(0,0))$- $A(A_1, A_2(0,1))$;

$\Delta A = HFE$.

$$\Delta A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 & 0 \end{pmatrix},$$

after getting ,H F, E we can follows steps available in[23] for stability analysis.

for FM1:

$\mathrm{A} = (A_0 \quad A_1 \quad A_2)$,

for R:

$$A = \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix},$$

values are available for FM1 and for FM2 as shown previously, to convert FM1 matrices values in GR Form we have taken reference of [14] and converted in suitable matrices form. FM1 and GR are not independent and can be mutually recasted.

FM1

$E'x(i+1,j+1) = A_1x(i+1,j+1) + A_2x(i+1,j) + A_0(i,j) + Bu(i,j) + B_1u(i,j+1) + B_2u(i+1,j),$

GR

$E\begin{pmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{pmatrix} = A\begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + Bu(i,j)$,

where $E = I_N$, $N = t_1 + t_2$.

To convert FM1 in to GR model we are assuming horizontal vector $\xi(i,j) = E'x(i,j+1) - A_1x(i,j)$. and equation of FM1 in R form would be

$$\begin{pmatrix} I_n & -A_2 \\ 0 & E' \end{pmatrix}\begin{pmatrix} \xi(i+1,j) \\ x(i,j+1) \end{pmatrix} = \begin{pmatrix} 0 & A_3 \\ I_n & A_1 \end{pmatrix}\begin{pmatrix} \xi(i,j) \\ x(i,j) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u(i,j)$$

Taking this concept we get matrix 'A' for R which is,

$$A= \begin{pmatrix} 0 & 0 & 0 & 0 & 0.5000 & 0 & -0.0150 & 0.2500 \\ 0 & 0 & 0 & 0 & 0.2500 & 0.5000 & 0 & -0.0150 \\ 0 & 0 & 0 & 0 & -0.0150 & 0.2500 & 0.5000 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0150 & 0.2500 & 0.5000 \\ 1.0000 & 0 & 0 & 0 & 0.7500 & -0.5000 & 0.1231 & -0.1563 \\ 0 & 1.0000 & 0 & 0 & -0.1563 & 0.7500 & -0.5000 & 0.1231 \\ 0 & 0 & 1.0000 & 0 & 0.1231 & -0.1563 & 0.7500 & -0.5000 \\ 0 & 0 & 0 & 1.0000 & -0.5000 & 0.1231 & -0.1563 & 0.7500 \end{pmatrix}$$

Using these matrices for FM1, FM2 and GR stability analysis has been done using previous steps [18] for PSV system.

TABLE 2. STABILITY ANALYSIS FOR ABOVE EXAMPLE

	FM1		FM2		GR	
	TH1	TH2	TH1	TH2	TH1	TH2

Stability condition	×	√	√	-	√	-

Such types of analysis have been done for check stability criteria using our derived criteria to check our stability analysis. Table 1 is over all stability analysis of the FM1 stability criteria; where for two derived theorem's analysis has been done in terms of computational complexity speed, gap between necessary and sufficient conditions etc. When we use Matlab LMI toolbox for solving derived stability criteria, at that time computational complexity of stability criteria means how much effort has been taken by processor to solve it. i.e. suppose if we have derived two stability criteria and also we have taken stable system to check it, if the Matlab LMI result shows for derived stability criteria 1 is 'feasible' but in 2nd criteria marginally feasible. 'Marginally feasible' criteria would be more complex in terms of computational complexity because even though system is stable but its shows marginally feasible due to more processing tasks

VII. CONCLUSION

A review on the stability criteria of 2-D discrete systems has been presented in this paper. The Lyapunov based approach has emerged as a popular approach to study the stability properties of such systems. The 2-D Lyapunov based stability conditions discussed so far in literature are only sufficient conditions. Impossibility of the Lyapunov based necessary and sufficient condition for the stability of 2-D discrete systems keeps it as an open and challenging research direction. Also proposed results for stability are also presented in theorem-5 and theorem-6.

In this paper, we have presented two important results for the FM-2 model of 2-D PSV system. Two sufficient conditions are established for the stability which is an LMI expressing stability criterion. So our first condition for stability is relaxed as far as Computational complexity is concerned and second condition could be applied if first condition for stability fails. Hence this makes the algorithm more robust and useful for the stability analysis of FM-2 model systems. In future, numerical examples can be taken up for verifying relative performance of these two and may be other future propositions.

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